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NONLINEAR PROPOGATION OF ALFVÉN WAVES IN COMETARY PLASMAS *

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ABSTRACT

Large amplitude Alfvén waves propagating along the guide magnetic field in a three-component plasma are shown to be modulationally unstable due to their nonlinear interaction with nonresonant electrostatic density fluctuations. A new class of subsonic Alfvén soliton solutions are found to exist in the three-component plasma. The Alfvén solitons can be relevant in explaining the properties of hydromagnetic turbulence near the comets.

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Hydromagnetic turbulence in the frequency range of 10^{-2} - 10^{-3} Hz has been observed near the comet Halley and the comet Giacobini-Zinner (Riedler et al 1986 , Tsurutani and Smith 1986 A strong similarity between the fluctuations in the wave fields and the spectra of cometary ion fluxes has been observed; thus suggesting that the free energy in the picked-up cometary ion distribution might be responsible for the low-frequency turbulence.

Tsurutani and Smith (1986) and Yumoto et al. (1986) have invoked a resonant ion- beam instability (Wu and Davidson 1972) to explain the hydromagnetic turbulence near the comets. Sagdeev et al (1986) have suggested the Alfvén wave instability, excited by the beam like motion of the cometary ions along the ambient magnetic field, as an alternative mechanism for the hydromagnetic turbulence near the comets.

Recently, Lakhina (1987) has proposed a model consisting of a gas of Alfvén solitons to explain the observed properties of the hydromagnetic turbulence near the comet Halley. However, his results are based on the two-component (ie, electrons and protons) plasma. Since the plasma in the cometary environment is essentially multi-component, at least three components namely, electrons, protons, and the water group cometary ions, it would be interesting to study the nonlinear propagation of Alfvén waves in such a plasma.

Nonlinear propagation of the Alfvén ion cyclotron (AIC) waves in electron - proton plasmas has been studied extensively (Spangler and Sheerin 1982 , Sakai and Sonnerup 1983. Ovenden et al 1983 , Wong and Goldstein 1986, Shukla and Stenflo 1985, Shukla et al 1986, Longtin and Sonnerup 1986).

In this paper, we investigate the propagation of large amplitude Alfvén ion cyclotron waves in a plasma consisting of electrons, protons and heavier ions. We consider the nonlinear interaction of these waves with nonresonant electrostatic density fluctuations taking into account the ion inertia effects and the time derivative ponderomotive force (Shukla and Stenflo 1985).

The frequency ω and the wavenumber k of the finite amplitude circularly polarized electromagnetic wave propagating parallel to $B_0\hat{z}$ are governed by the dispersion relation

$$\omega^2 = k^2 c^2 + \sum_j \frac{\omega\,\omega_{pj}^2}{\omega + \omega_{cj}} , \qquad (1)$$

where $|\omega + \omega_{cj}|/k$ has been assumed to be much larger than the thermal velocities, $\omega_{pj} = (4\pi N_{0j} q_j^2/m_j)^{1/2}$ and $\omega_{cj} = q_j B_0/m_j c$, with c and m_j being the speed of light and the mass of the j th species. Here, j = e for electrons, p for protons, and i for the heavier (or the cometary) ions. Furthermore, we note that for positive (negative) $\vec{B}_0$, Eq.(1) represents the right (left) - hand circularly polarized AIC waves.

The amplitude of the AIC waves get modulated due to their interaction with slow electrostatic plasma motion parallel to $\vec{B}_0$. The envelope is governed by the nonlinear Schrödinger equation (Shukla and Stenflo 1985) :

$$i\left(\partial_t + v_g \partial_z\right) E + \frac{1}{2} v_g' \partial_z^2 E - \Delta E = 0, \qquad (2)$$

where $v_g = \partial\omega/\partial k$ and $v_g' = \partial^2\omega/\partial k^2$ are the group velocity and the group dispersion, respectivly. The nonlinear frequency shift Δ is given by

$$\Delta = \frac{v_g}{2kc^2} \sum_j \frac{\omega\,\omega_{pj}^2}{\omega + \omega_{cj}} \left(N_j - \frac{k v_{jz}\,\omega_{cj}}{\omega(\omega + \omega_{cj})} \right), \qquad (3)$$

where $N_j = \delta N_j / N_{0j}$, and N_{0j} is the equilibrium density of the jth component with $N_{0e} = N_{0p} + N_{0i}$, and δN_j and v_{jz} are the density and the velocity perturbations associated with the slow plasma motion and are related to each other by their respective continuity equations.

When the phase velocity of the slow plasma motion is much smaller than the electron thermal velocity and the modulating frequency is much smaller than electron gyro-frequency $|\omega_{ce}|$, the slow response of the electrons is described by (Shukla and Stenflo 1984) :

$$\partial_z N_e = -\frac{q_e}{T_e}\partial_z\phi - \frac{q_e^2}{m_e\,\omega\,\omega_{ce}}\left(\partial_z + \frac{k}{\omega}\partial_t\right)\frac{|E|^2}{T_e}, \qquad (4)$$

where T_e is the electron temperature and the second term on the right-hand side of Eq.(4) arises due to the ponderomotive force on the electrons. The electrons are coupled to the protons and the heavier ions by the ambipolar potential ϕ. The ion momentum equations

$$\partial_t v_{\sigma z} = -\frac{q_\sigma}{m_\sigma}\partial_z\phi - \frac{\gamma_\sigma T_\sigma}{m_\sigma}\partial_z N_\sigma - \frac{q_\sigma^2}{m_\sigma^2}\frac{1}{\omega(\omega + \omega_{c\sigma})}\left(\partial_z + \frac{k\,\omega_{c\sigma}}{\omega(\omega + \omega_{c\sigma})}\partial_t\right)|E|^2, \qquad (5)$$

together with the respective continuity equations and Eq. (4) give the complete dynamics of the slow plasma motion. Here. σ = p for protons and i for the heavier ions. Assuming quasi-neutrality $\delta N_e = \delta N_p + \delta N_i$, one can derive the equations for δN_e and δN_i (or δN_p) from the above system. The nonlinear frequency shift Δ can then be evaluated by substituting perturbed densities in Eq.(3). Let us consider that the nonlinear wave travels with a velocity V along z- axis. Then in the wave frame $z' = z - Vt$, we have for $\omega \ll \omega_{ci}$

$$\Delta = \left[\frac{v_g \, \omega_{pp}^2 \, q^2 |E|^2 \omega^2}{2 k c^2 \omega_{cp}^4 m_p^2 (V^2 - c_{se}^2)} \right] \left(1 - \frac{2kV}{\omega}\right)^2 P, \tag{6}$$

where

$$P = \left[1 + \frac{N_{oi}}{N_{op}} \left(\frac{m_i V^2}{m_p} - c_{sp}^2 - c_{se}^2 Q \right) \right] \times \left[1 - \frac{N_{oi}}{N_{oe}} \frac{T_e}{m_i} \frac{c_{si}^2 - c_{sp}^2}{(V^2 - c_{se}^2)(V^2 - c_{si}^2)} \right]^{-1}, \tag{7}$$

$$Q = \left[1 - \frac{N_{op} T_e}{c_{se}^2 N_{oe} m_i} \left(1 + \frac{N_{oi}}{N_{op}} \right) \right] \left(\frac{m_i}{m_p} - 1 \right), \tag{8}$$

and $$c_{se} = \left[\frac{\gamma_p T_p}{m_p} + \left(\frac{N_{op}}{m_p} + \frac{N_{oi}}{m_i} \right) \frac{T_e}{N_{oe}} \right]^{1/2},$$

$$c_{si} = (\gamma_i T_i / m_i)^{1/2} \text{ and } c_{sp} = (\gamma_p T_p / m_p)^{1/2}.$$

On putting $N_{oi} = 0$, we recover Eq.(8) of Shukla and Stenflo (1985) in the limit of $\omega \ll \omega_{cp}$.

For $\omega \ll \omega_{ci}$ we get from Eq.(1) for the left - hand Alfvén mode :

$$\omega = k v_A \left[1 - \frac{k v_A (1 + N_{oi} m_i^2 / n_{op} m_p^2)}{2 (1 + N_{oi} m_i / N_{op} m_p) |\omega_{cp}|} \right], \tag{9}$$

where $v_A = B_0 / [4\pi (N_{op} m_p + N_{oi} m_i)]^{1/2}$.

It is clear from Eq.(9) that $v_g > 0$ and $v_g' < 0$. Hence the wave would be subjected to the modulational instability provided $\Delta > 0$. For V= 0 (ie. quasi-static modulations) , we note that Δ is always negative (cf. Eqs. (6) - (8)). Therefore, the localized solutions for the left-hand Alfvén mode cannot exist for this case. Shukla and Stenflo (1985) showed that super-sonic soliton solution (ie., $V > c_{se}$) for the left-hand Alfvén waves can exist in a two-component plasma; the same is true for the three-component plasma as seen from Eqs.(6)-(8) . It is interesting to note that for $c_{si}^2 < V^2 < c_{s*}^2 \leq c_{se}^2$, where

$$c_{s*}^2 = \left[c_{si}^2 + (N_{oi}/N_{op})(c_{sp}^2 + c_{se}^2 Q) \right] / (1 + N_{oi} m_i / N_{op} m_p), \tag{10}$$

Δ becomes positive, thus allowing the sub-sonic soliton solution to exist in the three-component plasma. For the case of cometary plasmas, the condition $c_{si}^2 < V^2 < c_{se}^2$ is easily satisfied as the temperature of the cometary ions is much smaller than the electron temperature. Thus, we conclude that in the cometary environment , large amplitude Alfvén waves could have sub-sonic as well as super-sonic localized structures. We feel that the sub-sonic Alfvén solitons could contribute significantly to the observed properties of the hydromagnetic turbulence near comets.

In summary, we have considered the nonlinear interaction of a large amplitude Alfvén wave with nonresonant density fluctuations in a cometary plasma, and have shown that both the sub-sonic and the super-sonic Alfvén solitons can exist. Our results may be relevant to the coherent structures found embedded in the hydromagnetic turbulence observed near the comets (Galeev 1986). The results may also have relevance to the propagation of hydromagnetic waves in the magnetosphere during the disturbed times when significant fluxes of O^+ ions are found intermixed with the electrons and the protons in the outer magnetosphere

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