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ABSTRACT : Dispersions around mean values of one-body observables are obtained by restoring classical many-body correlations in Vlasov and Landau-Vlasov dynamics. The method is applied to the calculation of fluctuations in mass, charge and linear momentum in heavy-ion collisions. Results are compared to those obtained by the Balian-Veneroni variational principle in semi-classical approximation.

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1. Introduction

One among the most striking features revealed a decade ago by first studies of heavy ion collisions is the existence of large dispersions of observables like mass, charge or kinetic energy around their mean values[1]. In this field, numerous theoretical works were made to follow the dynamical evolution of either the one-body reduced density matrix (TDHF i.e. Time Dependent Hartree Fock) or a collective reduced density matrix in transport statistical approaches.

TDHF, which is founded at a microscopic level, is known to underestimate dispersions[2], especially those associated with the mass distributions of outgoing fragments. The variational principle developed by Balian and Vene- roni (BV) leads to the conclusion that TDHF is the best mean field approach to single particle observables but is unable to provide a correct answer as far as dispersions around mean values are concerned[3]. In a time dependent process, the boundary conditions are given at the initial time t_0 for the many-body density operator and at a later time t_1 for the observables. Therefore, if one applies a variational principle and reduces the problem to one-body dynamics, the evolution of the system depends explicitely on the observable one is interested in. To obtain dispersions within this framework, TDHF equations for the one-body density matrix elements should be solved in coupling with time - dependent RPA equations for the operator matrix elements. In other words, the BV variational principle establishes an intimate connection between dynamical two-body correlations and fluctuations around mean values. One should mention that this theory predicts properly the spreading of wave packets. In Ref. 4, it is shown also that fluctuations of single particle observables in the Lipkin-Meshkov-Glick model are reasonably well described by applying the

BV method. Applications to nuclear collisions[4,5] indicate that main defect of TDHF in the determination of dispersion can be cured.

Transport statistical approaches[6] give a time evolution of collective variables (defined a priori in most theoretical studies) in complete analogy with a diffusion process which could be formulated as a random walk problem[7]. Coupling between macroscopic variables and microscopic degrees of freedom is responsible for dispersions through the fluctuation-dissipation theorem. Description of this coupling requires some models of nucleon transport : dispersions around mean values are found hitherto compatible with experimental findings. Nevertheless, these approaches are only semi - microscopic, some macroscopic models being needed in order to define relevant dynamical observables.

In this paper, we revisite the problem of determination of dispersions and we discuss it in the framework of semi-classical approximation to TDHF (VE i.e. Vlasov Equation) and its extension obtained by taking into account residual two body collisions(LVE i.e. Landau Vlasov Equation). VE can be obtained by a truncation of the expansion in the Wigner transform of the TDHF equations. Dynamical evolution of one-body distribution is identical to the one obtained from TDHF if one neglects pure quantal effects. Phenomenological collision terms can be also easily calculated, extending VE into LVE[8]. In the determination of approximate solutions of VE or LVE, the distribution function has been decomposed in Ref.9 into a sum of elementary gaussians, each of them representing a part of nucleonic distribution function. This ensemble of elementary packets can be viewed as a statistical ensemble ; therefore the mean value of each single particle observable can be considered as an ensemble-averaged value. It is now appealing to take an advantage of the statistical interpretation of this decomposition for the

computation of higher order moments. This corresponds to the "backwards" version of the procedure proposed in Ref.10 : there, the authors pluged Slater determinants built on a cascade statistical ensemble into TDHF equations. As a consequence of the introduced fluctuations, they obtained a dynamical multifragmentation process. Our procedure is similar to the one given in Ref.11 where the authors pick up and gather the elementary packets in phase space in order to introduce some fluctuations into a collision term. These fluctuations are due to the fact that a nuclear system is a mixture of Slater determinants in LVE. The selection of single Slater determinants by choosing as many elementary packets as the number of orbitals allows us to extract classical many body correlations from the distribution function. In order to get mean values as well as dispersions, the statistical ensemble of elementary packets will be used to describe heavy ion collisions.

In section 2 we give a brief description of Vlasov and Landau-Vlasov dynamics. Section 3 contains a dicussion of dispersions in TDHF and in Balian-Veneroni formalism along with their semi-classical analogs calculated with Vlasov equation. In section 4 we present a method of restoring many body classical correlations (RMBC) and discuss the results obtained for mass, charge and momentum dispersions. We also make a comparison with transport theory.

2- Vlasov and Landau-Vlasov dynamics

The Vlasov and Landau-Vlasov equations can be written as :

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}_r f - \vec{\nabla}_r U \vec{\nabla}_p f = \begin{cases} 0 & \text{VE} \\ I_{coll} & \text{LVE} \end{cases} \qquad (1)$$

Here $f = f(\vec{r}, \vec{p}, t)$ is a one-body distribution function, $\vec{r}$ and $\vec{p}$ represent the space and momentum coordinates and the single particle potential U is treated in a self-consistent way. The collision term I_{coll} is calculated with the use of an effective nucleon-nucleon cross section with explicit requirements for enegy and momentum conservation and with an appropriate Pauli blocking factor[9].

In order to solve equation (1) we write the distribution function $f(\vec{r}, \vec{p}, t)$ as a linear combination of the distribution functions for a large number NG of pseudo-particles which behave like classical particles in the sense of the Ehrenfest theorem :

$$f(\vec{r}, \vec{p}, t) = \sum_{i=1}^{NG} w_i (\vec{r}_i, \vec{p}_i) f_i (\vec{r}, \vec{r}_i, \vec{p}, \vec{p}_i, t) \qquad (2)$$

The pseudo-particle distribution functions $f_i(\vec{r}, \vec{r}_i, \vec{p}, \vec{p}_i, t)$ are written in a form of uncorrelated isotropic gaussian wave packets :

$$f_i (\vec{r}, \vec{r}_i, \vec{p}, \vec{p}_i, t) = N \exp \left(-\frac{1}{2}\left[\frac{(\vec{r} - \vec{r}_i)^2}{\sigma_r^2} + \frac{(\vec{p} - \vec{p}_i)^2}{\sigma_p^2}\right]\right) \qquad (3)$$

with N being a normalization constant and $\vec{r}_i(t)$ and $\vec{p}_i(t)$ describing the mean position of the gaussian "i" in phase space[9].

The weighting factors w_i were treated in the Thomas-Fermi approximation, so :

$$w_i = \theta(\varepsilon_F - \varepsilon_i) \qquad (4)$$

where ε_i is the mean single particle energy for the coherent state and ε_F denotes the Fermi energy. The Pauli exclusion principle is satisfied by requiring that no more than 4 particles occupy an h^3 phase-space cell. At

time t=0, one starts with random selection of NG pseudo-particles (coherent states) in the available phase space. The spreadings of the gaussian wave packets were determined for this static configuration (t=0) by requiring that the distribution function yields the proper values for the nuclear binding energy and for the root mean square radius of the nucleus.

The time evolution of the system is then obtained by following the semi-classical trajectories of the randomly chosen pseudo-particles (in the self-consistent way). The width of the gaussians and the weighting factors are kept constant. It is important to notice that, in this procedure, the initial distribution function should be understood as an ensemble averaged quantity. This property of the distribution function will be used in section 4.

3- TDHF and Balian-Veneroni dispersions in semi-classical approximation

In the TDHF formalism, if one applies Wick's theorem to calculate the fluctuations of a single particle observable Q at time t_1, one gets :

$$(\Delta Q)^2/_{t_1} = (\langle Q^2\rangle - \langle Q\rangle^2)/_{t_1} = Tr\,[Q\rho(t_1)\,Q\,(1 - \rho(t_1))] \qquad (5)$$

The values obtained by this procedure strongly underestimate the experimental results because, in the mean-field treatment, the fluctuations which are caused by many-body correlations are washed out . One should, restore them in order to obtain a realistic estimate of dispersions of single particle observables.

In analogy to the last equation one could look at an estimate of $(\Delta Q)^2$ in the VE dynamics by calculating :

$$(\Delta Q)^2/_{t1} = \int Qf(\vec{r},\,\vec{p},\,t_1)\,Q\,(1 - f\,(\vec{r},\,\vec{p},\,t_1))d\,\vec{r}d\,\vec{p} \qquad (6)$$

One could expect to find too small a value of $(\Delta Q)^2$ because of the same arguments as in TDHF. The function f being represented by a sum of gaussians differs from the discrete values of 0 or 1, and Eq.(6) yields therefore mainly a spurious contribution to the dispersion which depends only on the width of the gaussians.

The method explained above was applied to study mass distribution in the reaction of ^{16}O on ^{16}O at the laboratory energy of 160 MeV. The LV dynamics leads to fusion for angular momentum below l = 33. In Vlasov dynamics one finds a region of transparency for low values of l. A comparison of standard deviation of the mass distribution obtained in different approaches is presented in table I. In our calculations presented in this table we choose l = 36. To get an estimate of the predictions of eq.(6) we took the observable $Q = \theta(z)$ and therefore performed the integrations for the half-space $z>0$ only. To evaluate the spurious effect of width of the gaussians, we performed the calculations with σ_r = 0.61 fm, σ_p = 0.15 fm^{-1} and with the double widths σ_r = 1.22 fm, σ_p = 0.30 fm^{-1} (in the later case the system becomes underbound). The extracted value of ΔA is less than 0.2 u.

Application of the BV variational principle can be performed after a transposition of the procedure of Ref.4 into semi-classical dynamics. In quantum mechanics, the variance associated with a one-body observable Q is shown to be[4] :

$$(\Delta Q)^2/_{t_1} = \mathrm{Tr}\,\{Q(t_o)\,\rho(t_o)\,Q(t_o)\,(1-\rho(t_o))\} \qquad (7)$$

where $Q(t_o)$ is obtained by solving a set of coupled equations for the one-body density ρ and the operator between the time t_o and t_1. It differs completely from the TDHF result obtained by application of the Wick theorem at the time t_1 (eq.5). It can be shown[4,12] that if $\rho=\rho^2$ the results obtained from eq. 7 are identical to those obtained from the following formula :

$$(\Delta Q)^2/_{t_1} = \lim_{\varepsilon \to 0} \frac{1}{2} \mathrm{Tr} \left[\frac{\rho(t_o) - \sigma(t_o, \varepsilon)}{\varepsilon}\right]^2 \qquad (8)$$

where $\sigma(t,\varepsilon)$ is a solution of TDHF equations with the boundary condition :

$$\sigma(t_1, \varepsilon) = e^{i\varepsilon Q} \rho(t_1) e^{-i\varepsilon Q} \qquad (9)$$

In semi-classical dynamics, the distribution function f(t) is calculated by solving Vlasov equations. If the condition $f = f^2$ is fulfilled at time t_o, one can use eq(8) with the unitary transformation (9) being replaced by :

$$f(t_1, \varepsilon) = e^{L_Q \varepsilon} f(t_1) \qquad (10)$$

where $L_Q = \{Q, \ \}$ is an effective Liouvillian.

In Ref.9, f is decomposed to a sum of gaussians. The unitary transformation (10) is nothing but an elementary motion in phase space determined by the effective Liouvillian L_Q. For instance if $Q \equiv \theta(z)$ where $\theta(z) = 1$ for $z \geq 0$ and zero everywhere else, one can determine the mass dispersion. The transformation (10) can be achieved by performing :

$$k_i^z \to k_i^z - \varepsilon . \frac{1}{\sqrt{2\Pi\sigma_r^2}} e^{(-z_i^2/2\sigma_r^2)} \qquad (11)$$

where (z_i, k_i^z) is the mean phase space location of each gaussian i, σ_r is the variance of the gaussians in configuration space. The Vlasov code already carried from t_o to t_1, is performed back from t_1 to t_o in order to estimate the phase-space integral corresponding to the equation (8). For sake of comparison with Ref. 4 and 5 and with our method, we computed mass dispersions for the $^{16}O + ^{16}O$ system at 10 MeV per nucleon bombarding energy with an impact parameter b = 6.5 fm. The convergence of the limit in eq.(8)

is not found in our calculation in spite of the high checked accuracy of the numerical method. For ε values smaller than 10^{-3}, the behaviour of the numerator in eq.(8) is rather logarithmic than quadratic in ε, a situation similar to calculations presented in Ref.13. Consequently, we determined $(\Delta A)^2$ by varying ε from 10^{-3} to 5.10^{-1} and by performing least square fit of the results to a parabola. The result is $\Delta A = 3.3$ u. It compares reasonably with Ref.5 and is two times larger than in Ref.4. Since effective forces are not identical in the three calculations, the deviation between results cannot be used as an indication that our semi-classical approximation introduces some bias in the determination of ΔA.

4- Restoring classical many body correlations

In order to restore classical many-body correlations, we start with the VE or LVE continuous distribution function at a given moment of time. Among all NG pseudo-particles, we randomly choose Z_p (resp. Z_t) pseudo-particles wihich at t=0 correspond to the proton distribution in the projectile (resp. target), Np (resp. N_t) which belonged to the neutrons in the projectile (resp. target). We then identify their location (within a projectile-like or target-like fragment) and momenta for time t in the solution of VE or LVE and discretize the corresponding occupation probabilities to 0 or 1. This procedure, which corresponds to a random selection of a single Slater determinant out of the mixture of very many Slater determinants, is repeated for all combinations. The first and second moments of the obtained distributions of fragment mass, charge or momentum are then calculated. This method is a way to simulate, in a statistical ensemble, the classical many-body corrrelations which were washed out in te VE or LVE formalisms. Since NG is large, the number of random choices of sets of pseudo-particles is very large. We can therefore get a statistically reliable estimate of the first and second moments of the distributions of different single-particle observables.

The method of restoring classical many-body correlations (RMBC) has been applied to study mass, charge and momentum distributions of the reaction products in Landau-Vlasov dynamics and for comparison in Vlasov dynamics for a broad range of impact parameters leading to deep inelastic scattering. Number of random configurations was taken to be 10^4 and the stability of the results with a change of this number has been checked. To illustrate the difference between Vlasov and Landau-Vlasov dynamics, we present the time evolution of ΔA for a central collision along with the time evolution of the relative distance between the centers of the ions (see fig.1). LVE leads to fusion while VE to a deep inelastic process. The crucial role of dynamics in a determination of width of single particle observables can be seen by comparing the results of (RMBC) for VE and LVE dynamics for l = 36, where both cases lead to a deep inelastic scattering (see table I). The value of ΔA obtained with VE is considerably larger than in LVE, because for VE the interacting system comes to a much more compact configuration and reseparates much more slowly. For all values of impact parameters one observes that during the deep phase when the interacting ions amalgamate, the width of mass and charge distributions become larger than the final values after the reseparation of the fragments. The same behaviour has been observed in calculations bases on transport theories[14].

In figure 2 we present the dependence of the standard deviations of mass and charge distributions as function of the initial orbital angular momentum. The values of ΔA and ΔZ decrease monotonically with the values of the angular momentum, because, in the deep phase, the overlap of the density distributions is smaller and the time of interaction shorter. We have also calculated the dispersion of the distribution of fragments linear momentum along the beam direction. At this low energy the distribution is strongly dominated by the Fermi motion and therefore does not show any considerable variations with time or impact parameter. The values of ΔP are close to $\approx$ 2 fm^{-1} for each of the colliding ions.

The fact that the BV value that we obtain is close to the value derived from our method (RMBC) (see Table I) can be interpreted as a dominance of two-body correlations in the determination of dispersions. One should mention at this point that our method is definitively more flexible for pratical purposes than the BV method, the main results being rather similar.

Because of the classical nature of TDHF (see Ref.2), it is worth to compare the evolution of observables with those deduced from transport theories , as it was shown in Ref.15 devoted to tangential friction in nuclear dynamics. Since VE and LVE are obtained from TDHF and extended TDHF in semi-classical approximation, the comparison between VE or LVE and transport theories turns out to be even more relevant. In transport theories, dispersion can be obtained on account of stochastic nucleon exchanges. For instance, in a pure random-walk process the variance in mass σ_A^2 is equal to the number of exchanged particles N_{ex} (Ref.16). Our gaussian decomposition allows us to estimate an average value of N_{ex} :

$$\langle N_{ex} \rangle = \int d\Gamma \, (\Theta_p \, f_T(t_1) + \Theta_T \, f_p(t_1)) \qquad (12)$$

where Θ_p(resp. Θ_T) is unity in the projectile-like (resp. target-like) fragment at time t_1 and zero elsewhere ; f_p (resp. f_T) is defined by $f(t) = f_p(t) + f_T(t)$ at each time t and $f_p(t_o) = F_p$ (resp. $f_T(t_o) = F_T$) with F_p(resp. F_T) being the static projectile (resp. target) initial distribution function, $d\Gamma$ is the phase space elementary volume element.
$\langle N_{ex} \rangle$ is calculated for the $^{16}O + ^{16}O$ system at 10 MeV/u bombarding energy after a time laps t_1, when the reseparation is achieved. $\sqrt{\langle N_{ex} \rangle}$ is drawn in figure 2, together with σ_A and σ_Z, as a function of l, the initial orbital

angular momentum. $\langle N_{ex} \rangle$ is roughly equal to σ_A^2, this later quantity being smaller by around 15%. As a matter of fact, this difference could be explained by a squeezing of the fluctuations resulting from the divergence of the inertia parameter associated with the mass asymmetry degree of freedom at the reseparation[14]. In spite of this small deviation, the results obtained from our self-consistent semi-classical description show that some features of transport theories are included in (RMBC), at least for fluctuations around mean values of one-body observables.

4. Conclusion

We have proposed a method of restoring classical many-body correlations in semi-classical Landau-Vlasov dynamics. It allows us to determine dispersions around mean values of physical observables. The Balian-Veneroni variational principle applied into the context of the Vlasov equation provides us with another estimate of these dispersions. The results indicate strongly that two-body correlations produce the main contribution to the dispersions. A comparison between our method and transport theory exhibits also the statistical nature of the dispersions in our approach. Then, at half way between microscopic self-consistent quantal models and macroscopic transport approaches, semi-classical dynamics with restored classical many body correlations can be considered as a very flexible and powerful tool for understanding basic features of low and intermediate energy nucleus-nucleus collisions.

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TABLE AND FIGURE CAPTIONS

Table I : Mass, charge and momentum dispersions calculated for $^{16}O + ^{16}O$ at E_{lab} = 160 MeV using different methods discussed in the text.

Figure 1 :

Mass dispersion and distance between the centers of the interacting ions as a function of time for a central collision of two ^{16}O ions at E_{lab} = 10 MeV/nucleon. In the lower part the solid line shows the relative distance between the centers as obtained in Vlasov dynamics (VE) while the dashed line was obtained with LVE. In the upper part the solid line corresponds to the mass dispersion obtained with VE, the dashed one with LVE.

Figure 2 :

Dispersion in mass and charge as a function of the angular momentum for $^{16}O + ^{16}O$ at E_{lab} = 10 MeV/nucleon obtained in Landau-Vlasov dynamics. The solid line corresponds to the dispersion in mass, the dashed-dotted line to the dispersion in charge and the dashed one represents a square root of the average number of exchanged nucleons.

METHOD	TDHF (Ref.4)	TDHF (Ref.5)	Eq.6	RMBC VE (l=36)	RMBC LVE (l=36)	$\sqrt{\langle N_{ex} \rangle}$ VE (l=36)	$\sqrt{\langle N_{ex} \rangle}$ LVE (l=36)	BV (Ref.4)	BV (Ref.5)	BV semi-classical
ΔA (u)	0.01 (l=30)	0.495 (l=30)	<0.2 (l=36)	2.7	1.9	3.2	2.0	1.42 (l=30)	2.5 (l=30)	3.3 (l=36)
ΔZ (charge-units)	-	0.400 (l=0)	-	1.8	1.2	2.3	1.5	-	0.52 (l=0)	-
ΔP (fm^{-1} ħ)	-	1.17 (l=0)	-	2.0	2.0	-	-	-	2.5 (l=0)	-

TABLE I

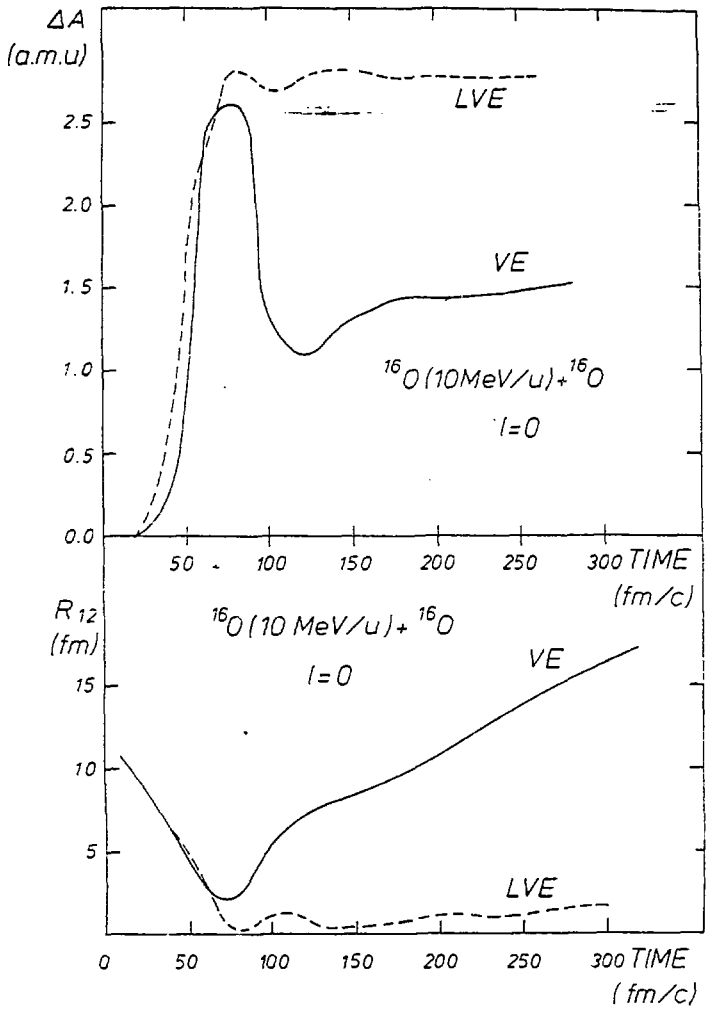
ΔA
(a.m.u)
2.5
2.0
1.5
1.0
0.5
0.0
LVE
VE
^{16}O (10MeV/u)+ ^{16}O
l=0
50
100
150
200
250
300 TIME
(fm/c)
R_{12}
(fm)
^{16}O (10 MeV/u)+ ^{16}O
l=0
VE
LVE
15
10
5
0
50
100
150
200
250
300 TIME
(fm/c)

Fig. 1

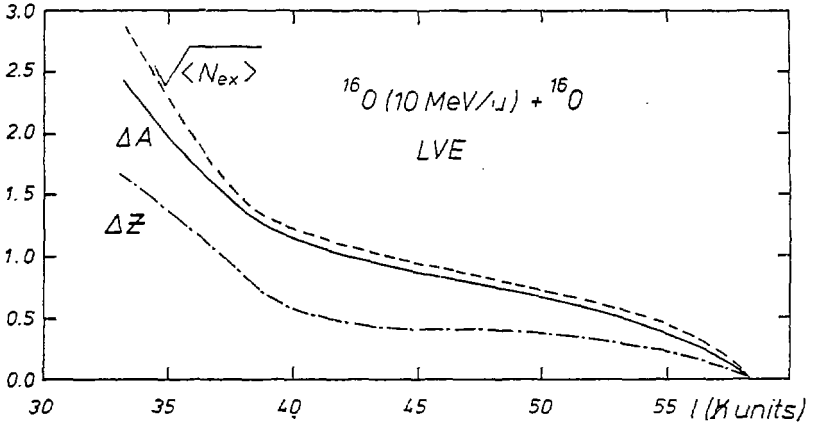
3.0
2.5
2.0
1.5
1.0
0.5
0.0
30
35
40
45
50
55
I (K units)
√⟨Nex⟩
ΔA
ΔZ
16O (10 MeV/u) + 16O
LVE

Fig. 2