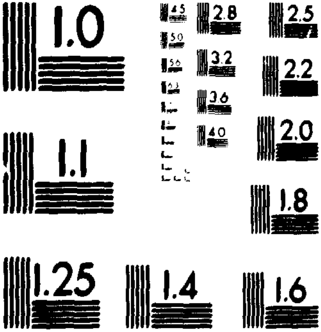

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COMITATO NAZIONALE PER LA RICERCA E PER LO SVILUPPO DELL'ENERGIA NUCLEARE E DELLE ENERGIE ALTERNATIVE

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ENEA

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SUMMARY

The hollow fiber systems are well known and developed in the scientific literature because of their applicability in the process separation units.
The authors approach to a mathematical model for a particular hollow fiber system, using liquid membranes.
The model has been developed in order to obtain a suitable tool for a sensitivy analysis and for a scaling-up.
This kind of investigation is very usefull from an engineering point of view, to get a spread range of information to build up a pilot plant from the laboratory scale.

RIASSUNTO

I sistemi a membrane liquide supportate sono ben noti e sviluppati nella letteratura scientifica a causa della loro applicabilità nelle unità dei processi separativi che richiedono elevate selettività come, ad esempio, per il recupero di metalli pregiati da scarti industriali, o nel ciclo del combustibile nucleare.
In questo lavoro si affronta lo studio di un modello matematico per un sistema di membrana liquida supportata su fibra cava.
Il modello è stato sviluppato allo scopo di ottenere uno strumento in grado di consentire un'analisi di sensività ed uno scaling-up del sistema. Questo tipo di analisi è molto importante da un punto di vista ingegneristico per utilizzare le informazioni ottenute in laboratorio nella progettazione di un impianto pilota.

Phisical model description.

The system investigated is aliquid feed passing through a fiber lumen (fig.1.)
The liquid membrane is supported in the fiber wall.The carrier complexes the extracting chemical species,and the extraction process takes place at interface beteween the liquid feed and the liquid membrane.The boundary conditions take into account the mass flux at the wall by mean of the membrane permeability /1/2/3/.
The concentration value at fiber end involves the radial diffusion and the axial convection (axially the diffusion is neglectible because the ratio beteween the axial diffusion time and the the axial convection time §§1).
The fluid flow is laminar therefore the velocity profile will be

$$W(r) = \bar{w}(1 - r^{*2}) \qquad 0 \leq r^{*} \leq 1 \qquad (1)$$

Concentration profilesin the fiber lumen are developed as the feed proceeds through the lumen. The mathematical approach is used to obtain some information on the effect of various fluid dynamic conditions on the concentration profiles.
The mathematical model for attacking this program is stated below,
starting from the mass balance equation:

$$W(r) \frac{\partial c}{\partial z} = D \frac{\partial^2 c}{\partial r^2} \qquad (2)$$

with boundary conditions

$$\begin{aligned} z &= 0 & c &= c_0 \\ r &= 0 & \partial c / \partial r &= 0 \\ r &= R & c &= c\,[\,P(Kd)\,] \end{aligned} \qquad (3)$$

We introduce as boundary conditions for r=R (r*=1) that the flux at fiber wall is a function of Kd .

the flux at wall is the product beteween the membrane permeability P and the interface concentration (Ci+1).

$$J_w = P(\Delta c)_w \qquad (4)$$

since the rate controllin step of the mass transfer phenomena is the chemical reaction at the interface the condition (4) becames

$$J_w = P c_w = P c_{i+1} \qquad (5)$$

but

$$J_w = -D \left. \frac{\delta c}{\delta r} \right|_w \qquad (6)$$

Then,using a finite difference approximation the derivative condition is:

$$P c_{i+1} = -D \frac{c_{i+1} - c_{i-1}}{2 \Delta r} \qquad (7)$$

Then:

$$c_{i+1} = c_{i-1} / (1 + 2\Delta r \cdot P/D) \qquad (8)$$

where:

z = distance along the length of the fiber away from the entrance

r = radial distance away from the center line of the fiber

R = fiber radius

W(r) = fluid linear velocity

C = transporting species concentration

L = fiber length

D = diffusion coefficent of the transporting species

P = membrane permeability

Kd = distribution coefficent

Introducing the following dimensionless value:

$$r^* = r/R$$

(9) $$z^* = z/L$$

$$P_e'(r) = \frac{W(r)\,R}{D}$$ (modified peclet number)/4/

the equation becames:

$$\frac{\partial c}{\partial z^*} = \frac{L}{R}\,\frac{1}{P_e'(r)}\,\frac{\partial^2 c}{\partial r^2} \qquad (10)$$

This equation has been solved with finite difference method:

$$\frac{c(i,k+1) - c(i,k)}{\Delta(z)} = \frac{L}{R}\,\frac{1}{P_e'(i)}\,\frac{c(i+1,k) - 2c(i,k) + c(i-1,k)}{\Delta r^2} \qquad (11)$$

with $0 \le z^* \le 1$

$$0 \le r^* \le 1 \qquad (12)$$

Discussion and results.

In figures 2 to 11 areshown the concentration profiles of the transporting species as function of the fiber radius (r/R) for different values of the axial coordinate (z/L).
The axial coordinate step is one cm .
The results are obtained for different values of the linear average velocity w ,radius R, distribution coefficent Kd,and fiber length (L).
The value of the axial coordinate closest to the inlet is on the top of the diagrams.

The low linear velocity is providing the contact time necessary for a large reduction of the valuable material feed concentration.
Figures 4 to 11show the effect of a higher linear velocity.
At this flow rate and with the specific permeability coeffIcent used,essentially no concentration reduction in the feed was noted as it passed through the fiber.
We can see from the computer results that increasing the flow rate tends to make the concentration (on the fiber wall) of the membrane transporting species almost independent of the fiber length.This means for high values of the flow rate we can assume that the concentration is constant on the fiber wall;on the other hand,for low flow rates the concentration on the fiber wall is a function of the axial coordinate.From the chemical point of view,this means that for high flow rates all the membrane works in the same condition ,there is no concentration change along the fiber wall. Instead for low flow rate,the concentration on the fiber wall is a function of the axial coordinate,and the interfacial flux is higher .
From figures 2 to 12 it is very intersting to see that by using low values for Kd very little material is lost from the feed.Therefore the concentration profile in fiber lumen are strongly dependent
from Kd.For the permeability we used the well known expression:

$$P = \frac{Kd}{Kd\,\frac{\delta_a}{D} + \frac{\delta_o}{D_o}}$$

This expression was obtined by P.R.Danesi et Al./1/2/ .
The diffusion thickness has been evaluated by estimating the distance from the membrane surface where the axial convective transport becames neglectible compared with the radial diffusive mass transport because of the radial velocity profile. In the model previously shown the flux at wall is not constant along the fiber length as consequence of the concentration variation.

Symbol list

Ci+1 = concentration at wall

W(r) = linear velocity function of the radius

Jw = flux at wall

D = diffusion coefficent 6.E-6 cm^2/sec

D_o = diffusion coefficent organic phase 1.E-6 cm^2/sec

δ_w = thickness of the diffusion film 1E-3 - 1.E-4 cm

δ_o = membrane hickness 35E-3 cm

Kd = distribution coefficent

P = permeability

z = axial coordinate

r = radial coordinate

Δ r = radial finite increment

Sbscript

i = position in the integration grid

w = wall

REFERENCES

1 P.R.Danesi,E.P.Horwitz,G.F.Vandegrift,R.Chiarizia, Mass transfer rate through liquid membranes: interfacial chemical reactions and diffusion as simultaneous permeability controlling factors,Sep.Sci.and Tech.16,201,(1981).

2 P.R.Danesi,R.Chiarizia,and P.Rickert,Transport of Eu+3 through a bis (2-ethyl-hexyl) phosphoric acid,N-dodecane solid supported liquid membrane,Sep.Sci. and Tech. 17,1183 (1982).

3)P.R.Danesi,C.Cianetti and V.Violante,Induction time in metal permeation processes through supported liquid membranes,Journal of Membrane Science,14,175-186,(1983).

4 K.Schugerl,H.G.Blaschke,U.Brunke,R.Streicher,Interaction of fluid dynamics,interfacial phenomena,and mass transfer in extraction processes,Recent Developments in Separation Science, vol III,part A,75-77,CRC Press 1977.

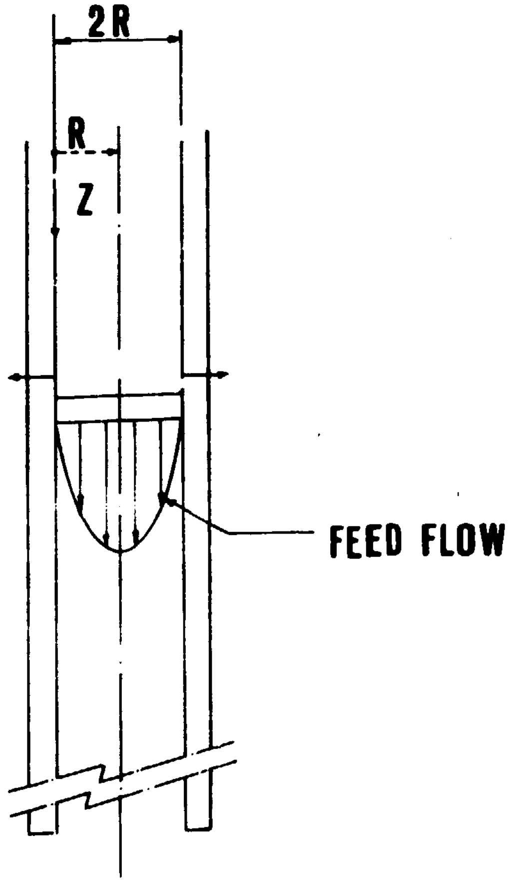

FIG. 1

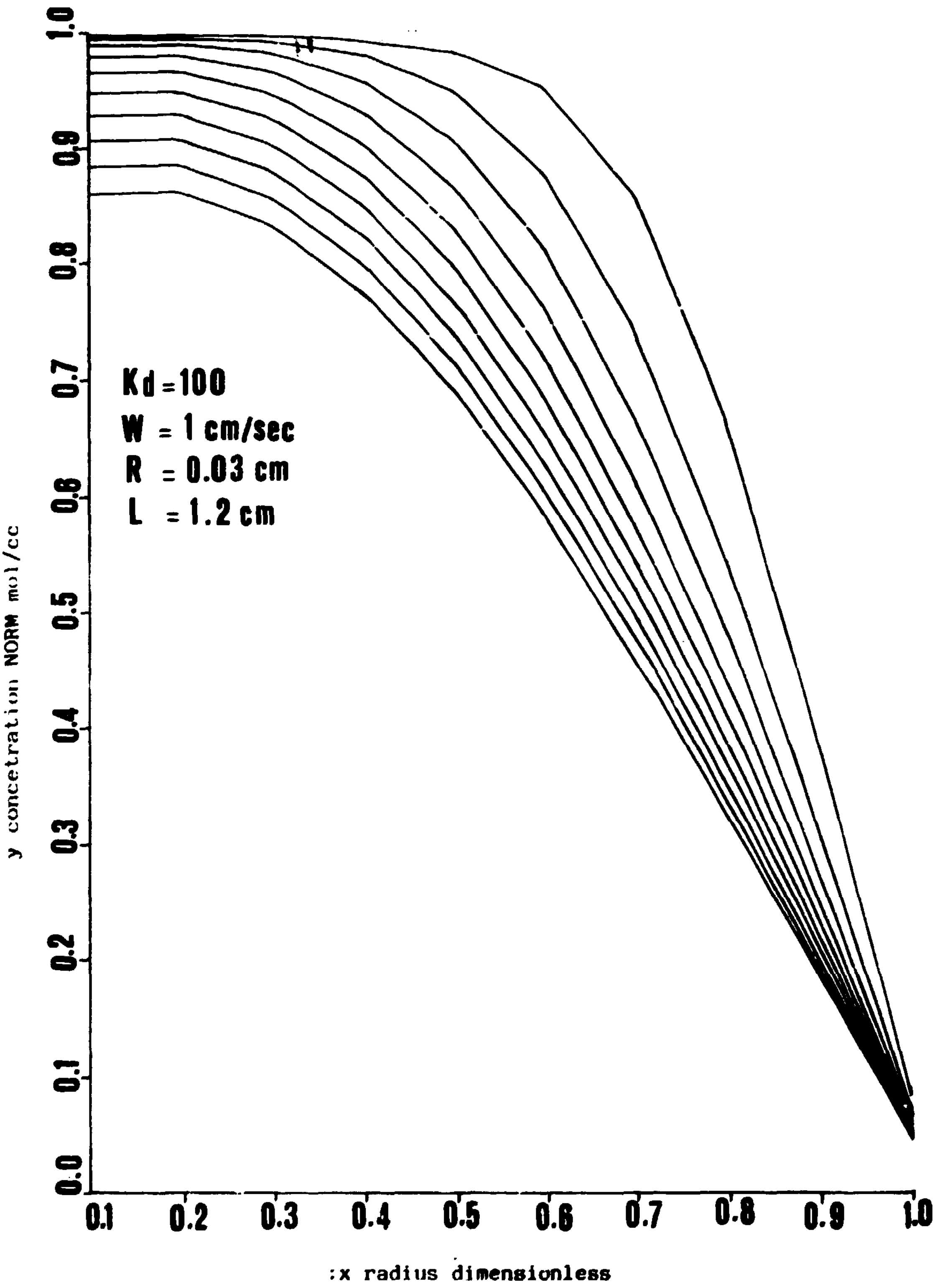

Fig. 2: **Concetration Gradient for the SLM Permeable Material in the Feed vs. It's Position from the Center Line of a Hollow Fiber. Each line (from top to bottom) is for a travel distance of 1 mm. further into the fiber; the first line is 1 mm. from the beginning of the fiber, which is 1.2 cm. long.**
Important variable values are listed on the figure.

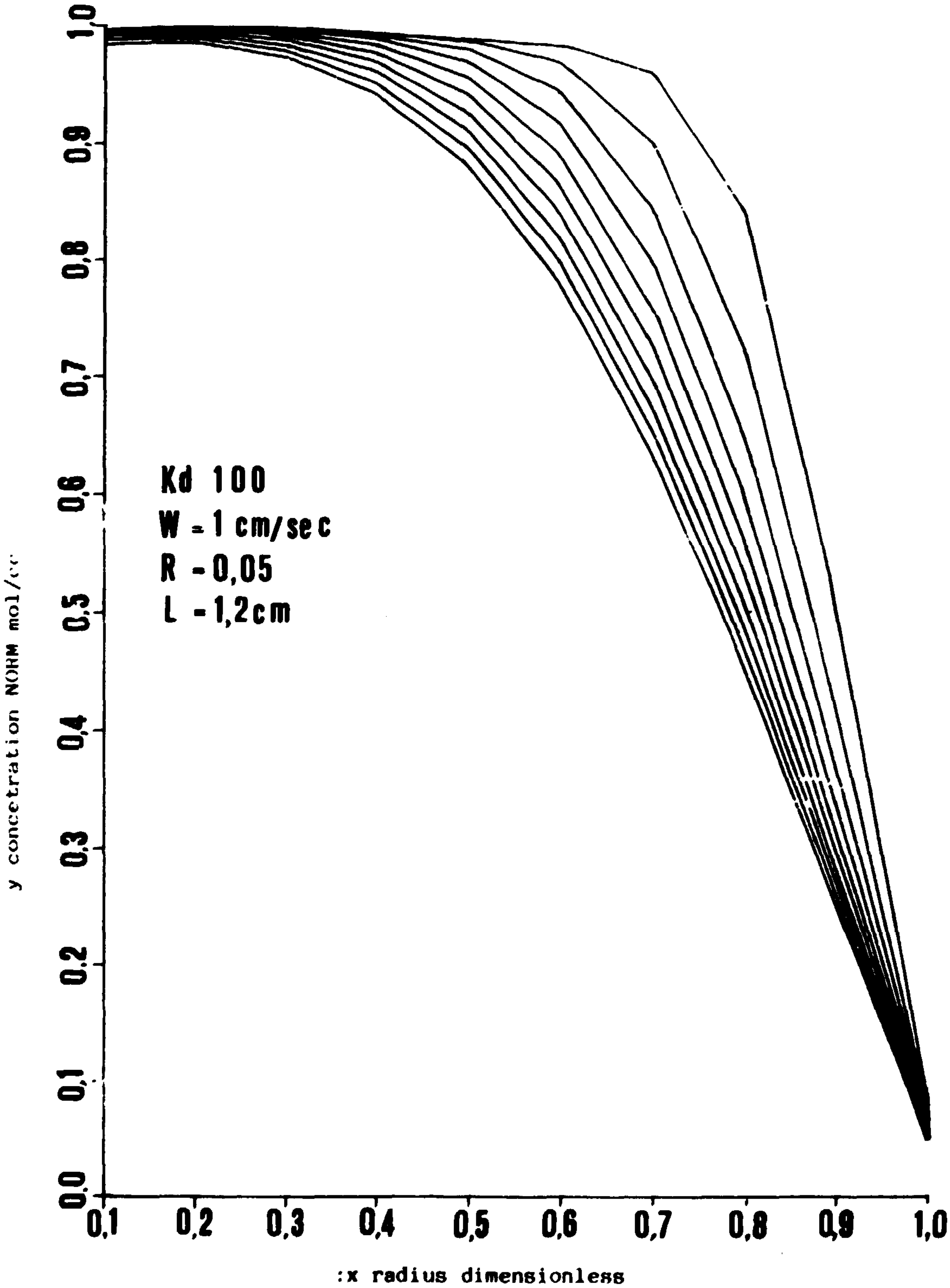

Fig. 3: **Concetration Gradient for the SLM Permeable Material in the Feed <u>vs</u>. It's Position from the Center Line of a Hollow Fiber. Each line (from top the bottom) is for a travel distance of 1 mm. firther into the fiber; the first line is 1 mm. from the beginning of the fiber, which is 1.2 cm. long.**
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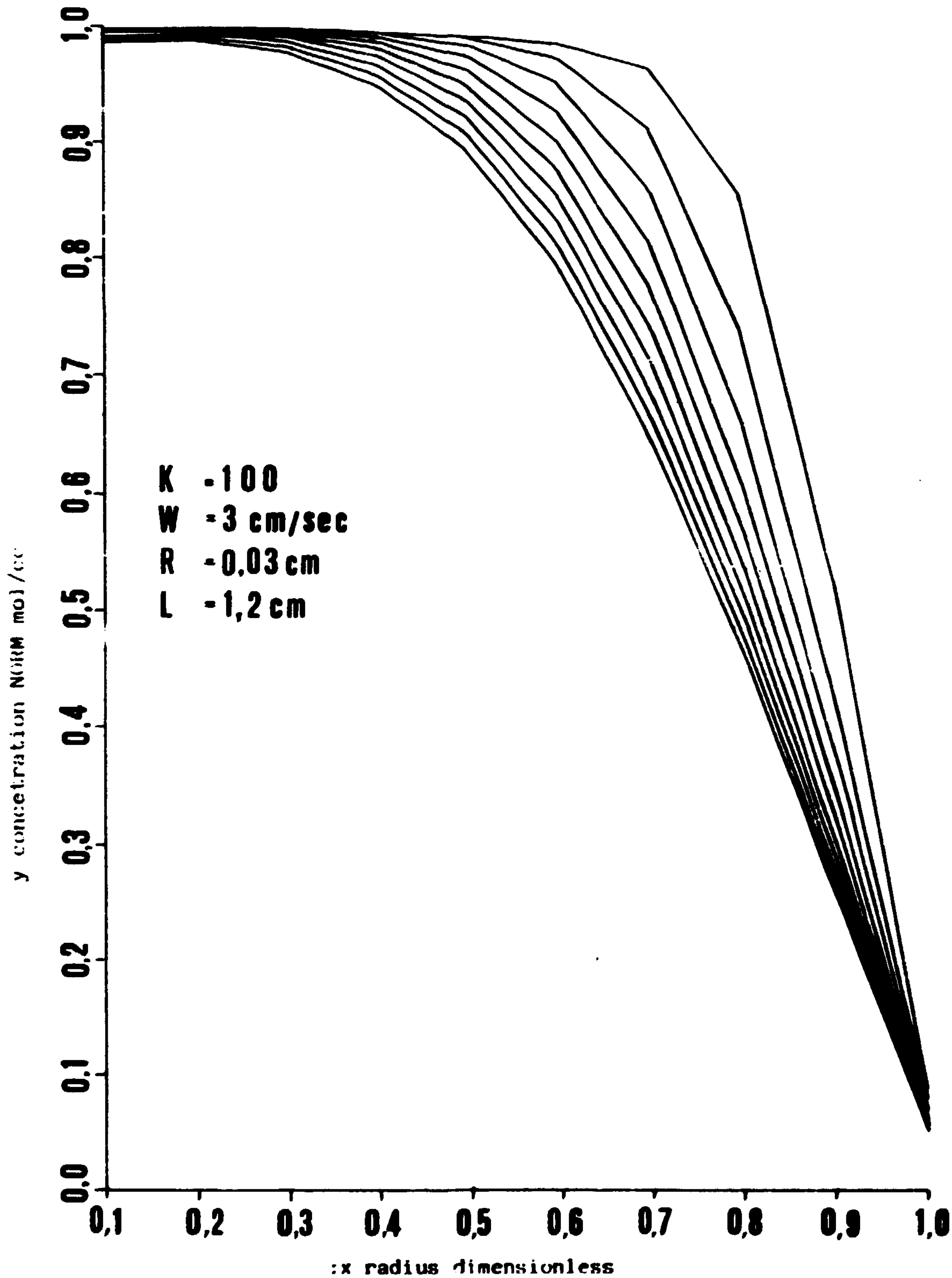

Fig. 4: Concetration Gradient for the SLM Permeable Material in the Feed vs. It's Position from the Center Line of a Hollow Fiber. Each line (from top to bottom) is for a travel distance of 1 mm. further into the fiber; the first line is 1 mm. from the beginning of the fiber, which is 1.2 cm. long.
Important variable values are listed on the figure.

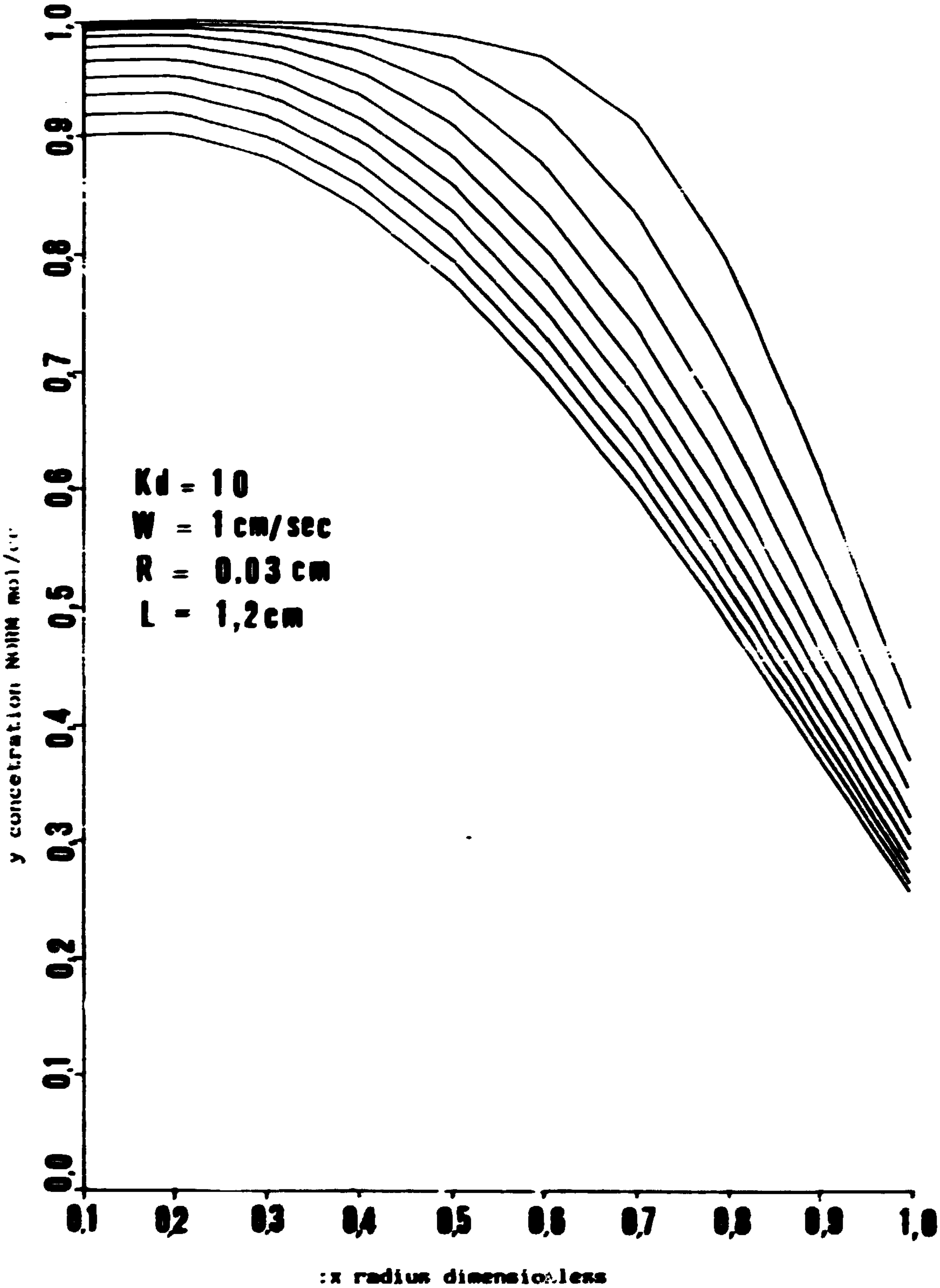

Fig. 5: Concetration Gradient for the SLM Permeable Material in the Feed vs. It's Position from the Center Line of a Hollow Fiber. Each line (from top to bottom) is for a travel distance of 1 mm. further into the fiber; the first line is 1 mm. from the beginning of the fiber, which is 1.2 cm. long.
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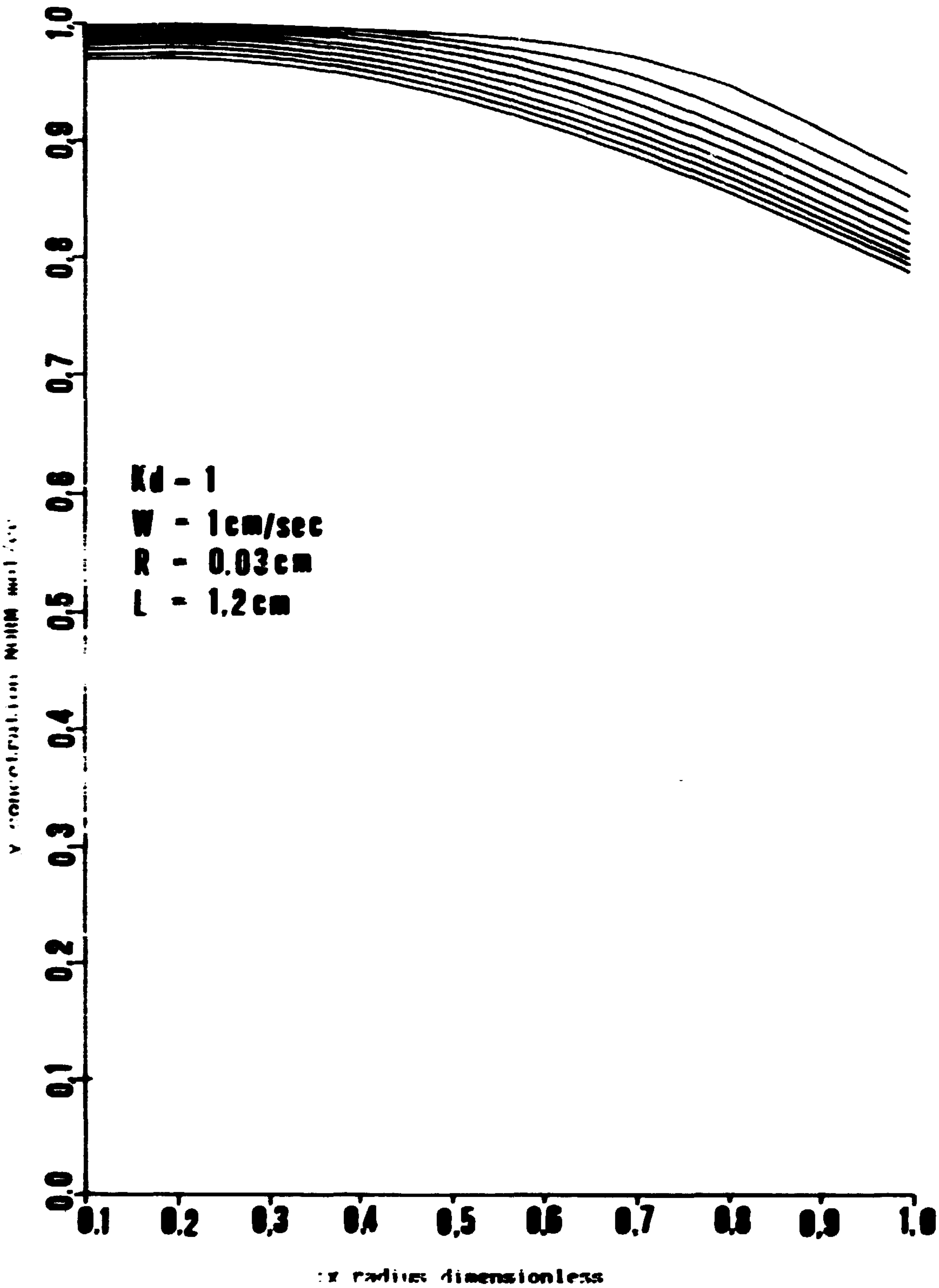

Fig. 6: Concetration Gradient for the SLM Permeable Material in the Feed vs. It's Position from the Center Line of a Hollow Fiber. Each line (froma top to bottom) is for a travel distance of 1 mm. firther into the fiber; the first line is 1 mm. from the beginning of the fiber, which is 1.2 cm. long.
Importan variable values are listed on the figure.

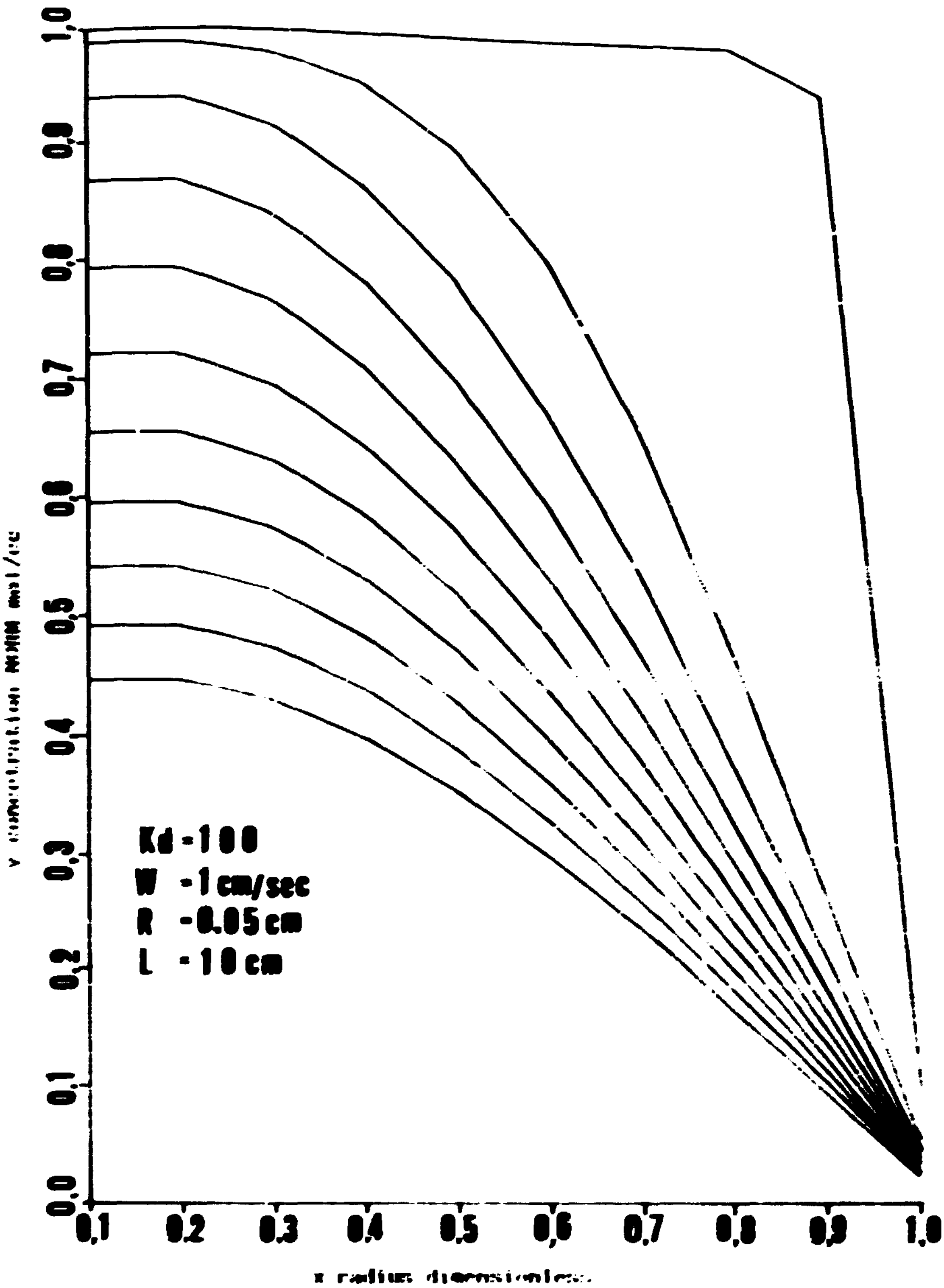

Fig. 7: Concetration Gradient for the SLM Permeable Material in the Feed *vs.* It's Position from the Center Line of a Hollow Fiber. Each line (from top to bottom) is for a travel distance of 1 cm. further into the fiber; the first line is 1/2 R from the beginning of the fiber, which is 10 cm. long.
Important Variable values are listed on the figure.

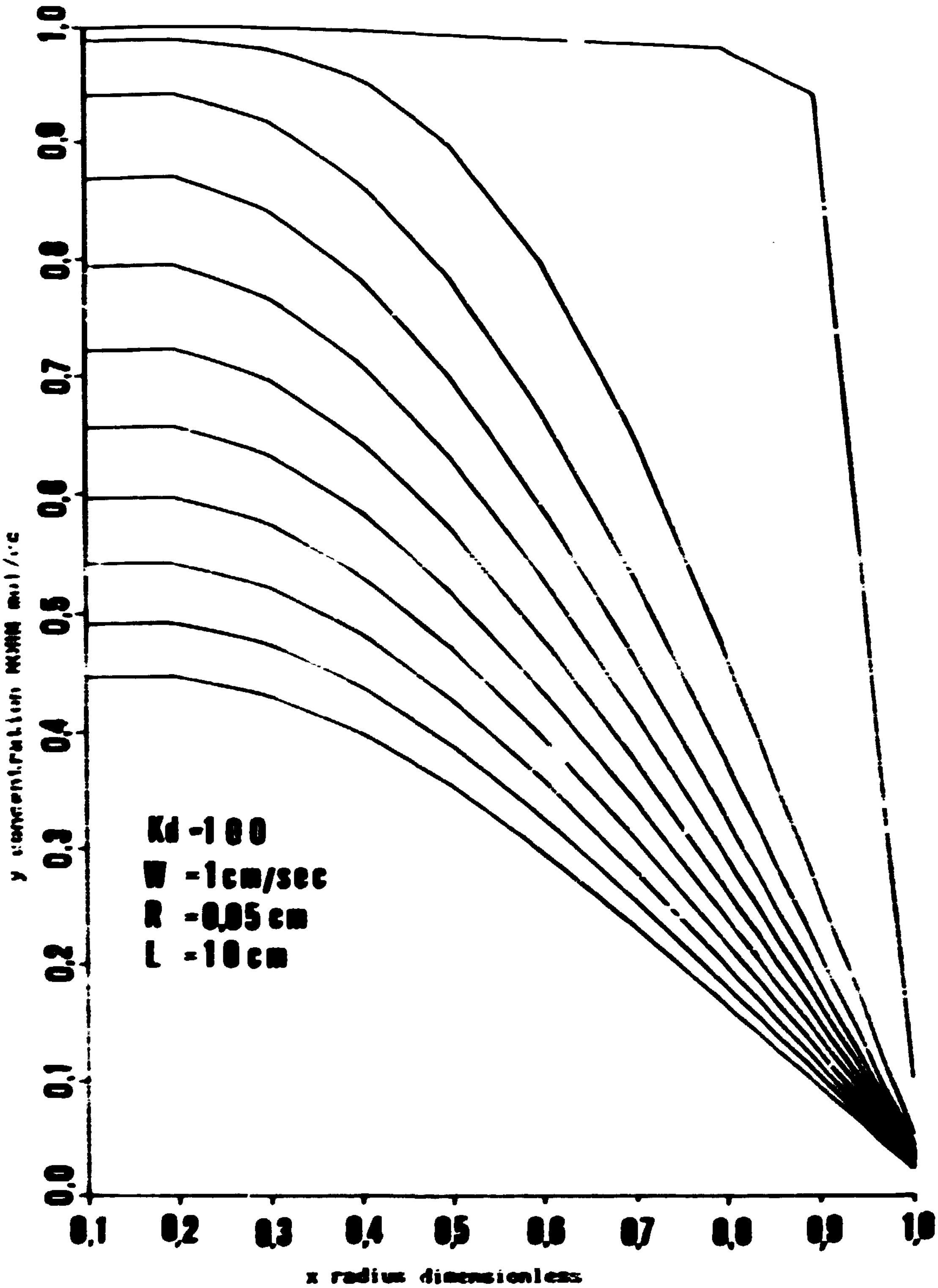

Fig. 8: ConcetrationGradient for the SLM Permeable Material in the Feed vs. It's Position from the Center Line of a Hollow Fiber. Each line (from top to bottom) is for a travel distance of 1 cm. further into the fiber; the first line is 1/2 R from the beginning of the fiber, which is 10 cm. long.
Important variable values are listed on the figure.

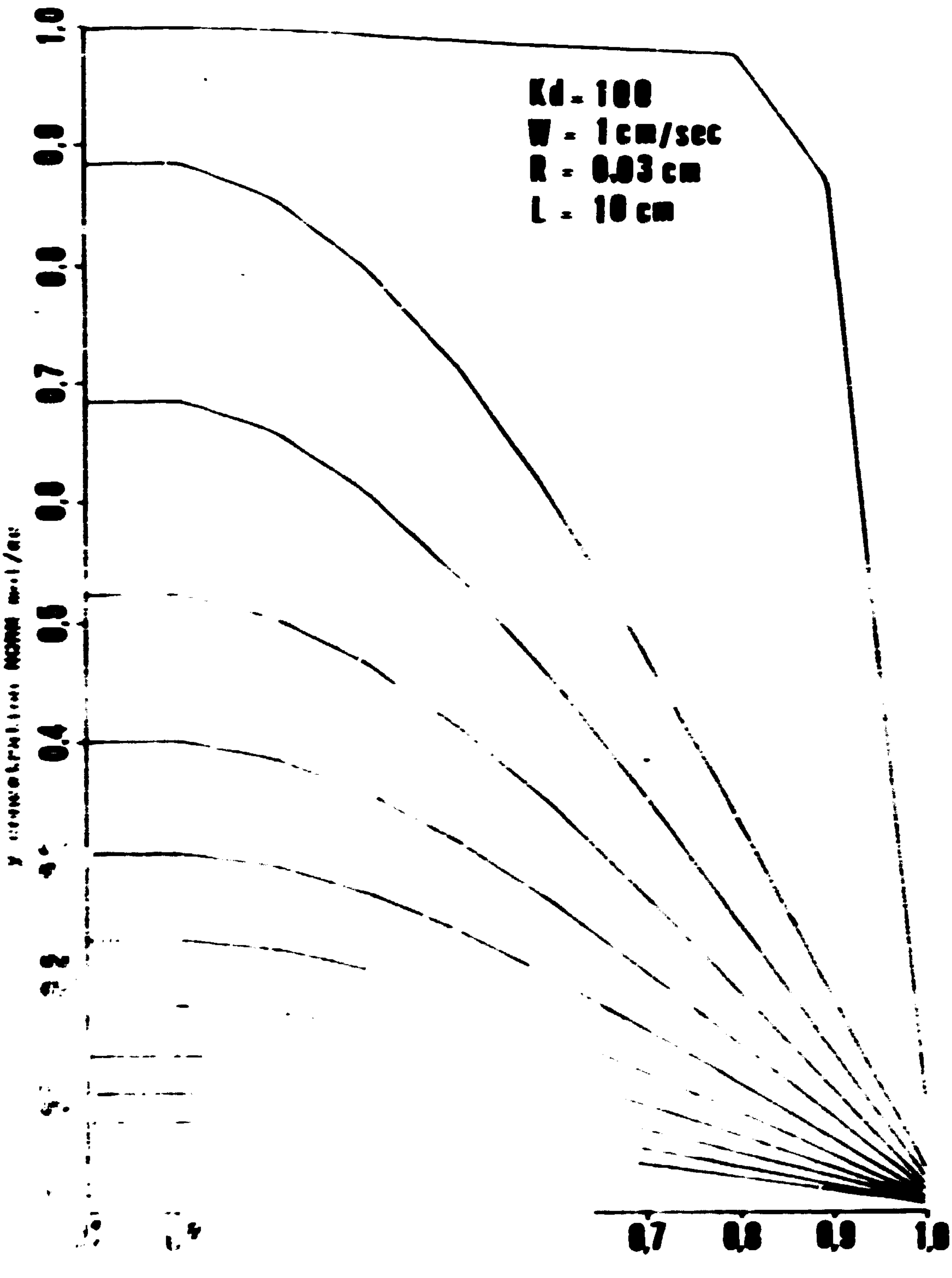

Fig. 9: ... able Material in the Feed vs. It's Position ... Each Line (from topto bottom) is for a ... of 1 cm. further ... the fiber; the first line is 1/7 R from the ... of ... fiber, which is 10 cm. long. ... values are listed on the figure.

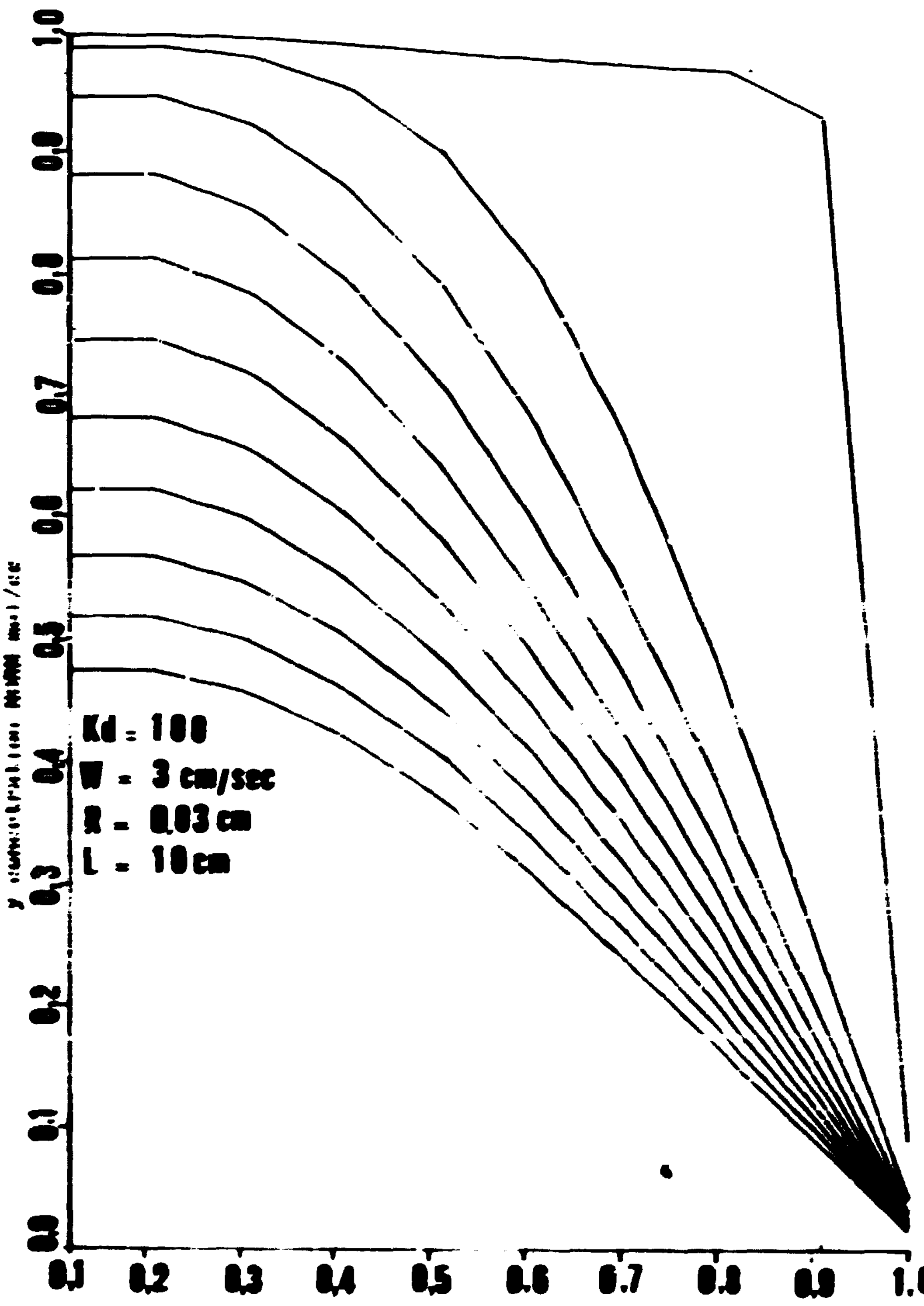

Fig. 10: Concetration Gradient for the SLM Permeable Material in the Feed vs. It's Position from the Center Line of a Hollow Fiber. Each line (from top to bottom) is for a tra distance of 1 cm. further into the fiber; the first line is 1/2 R from the beginning of the fiber, which is 10 cm. long.
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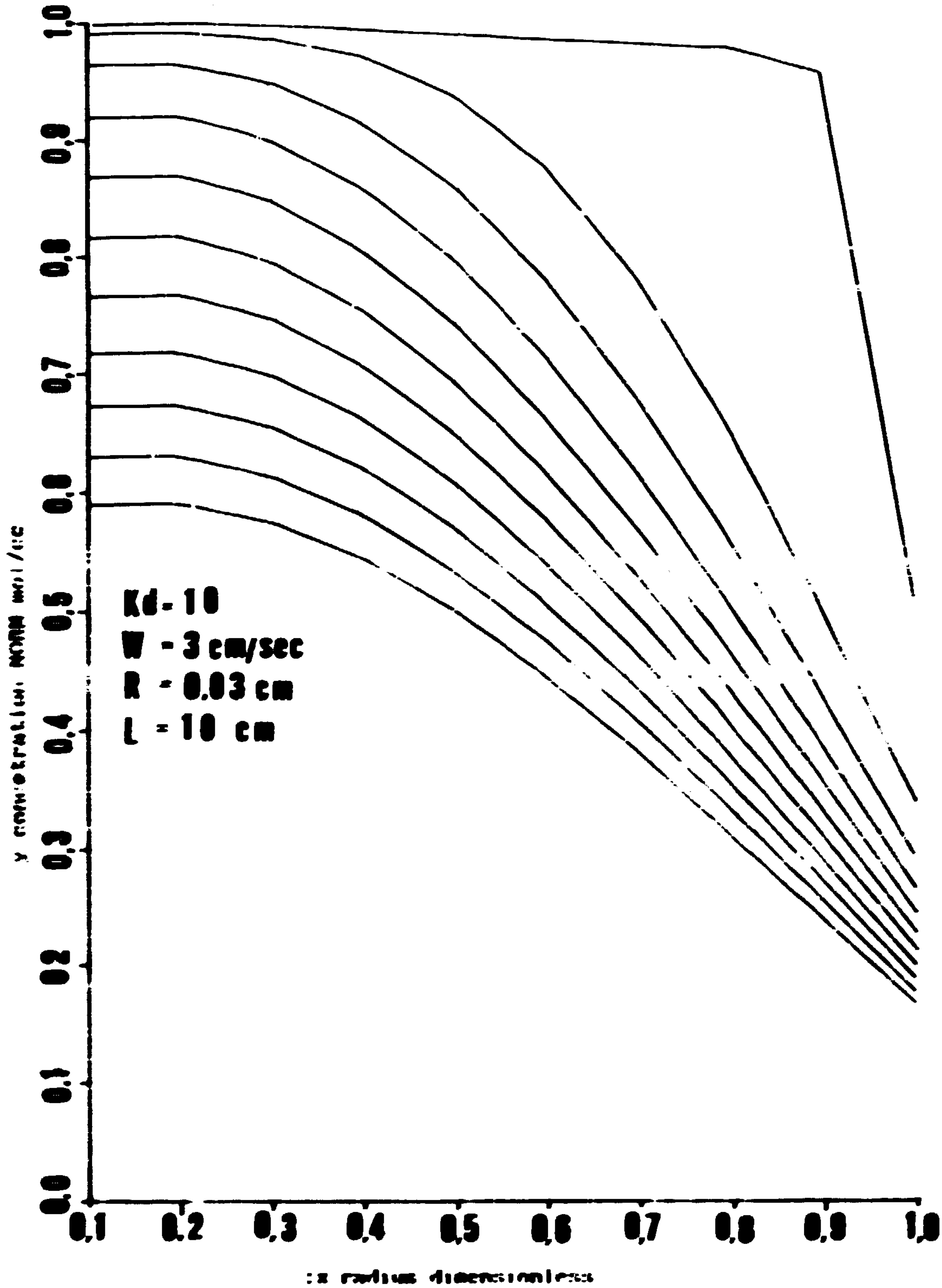

Fig. 11: Cancetration Gradient for the SLM Permeable Material in the Feed *vs*. It's Position from the Center Line of a Hollow Fiber. Each line (from top to bottom)is for a travel distance of 1 cm. further into the fiber; the first line is 1/2 R from the beginning of the fiber, which is 10 cm. long.
Important variable values are listed on the figure.

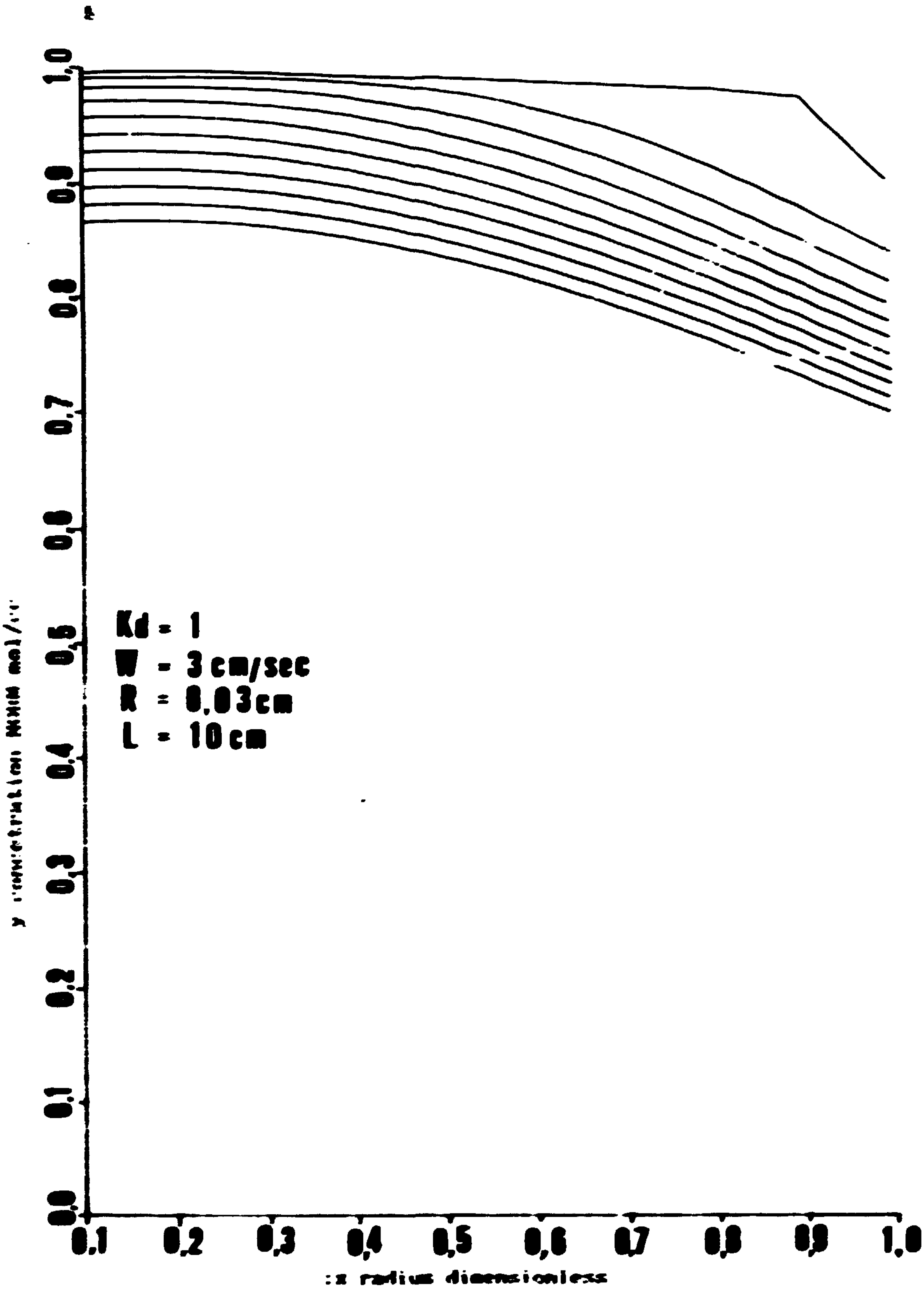

Fig. 17: Concetration Gradient for the SLM Permeable Material in the Feed vs. It's Position from the Center Line of a Hollow Fiber. Each line (from top to bottom) is for a travel distance of 1 cm. further into the fiber; the first line is 1/2 R from the beginning of the fiber, which is 10 cm. long.
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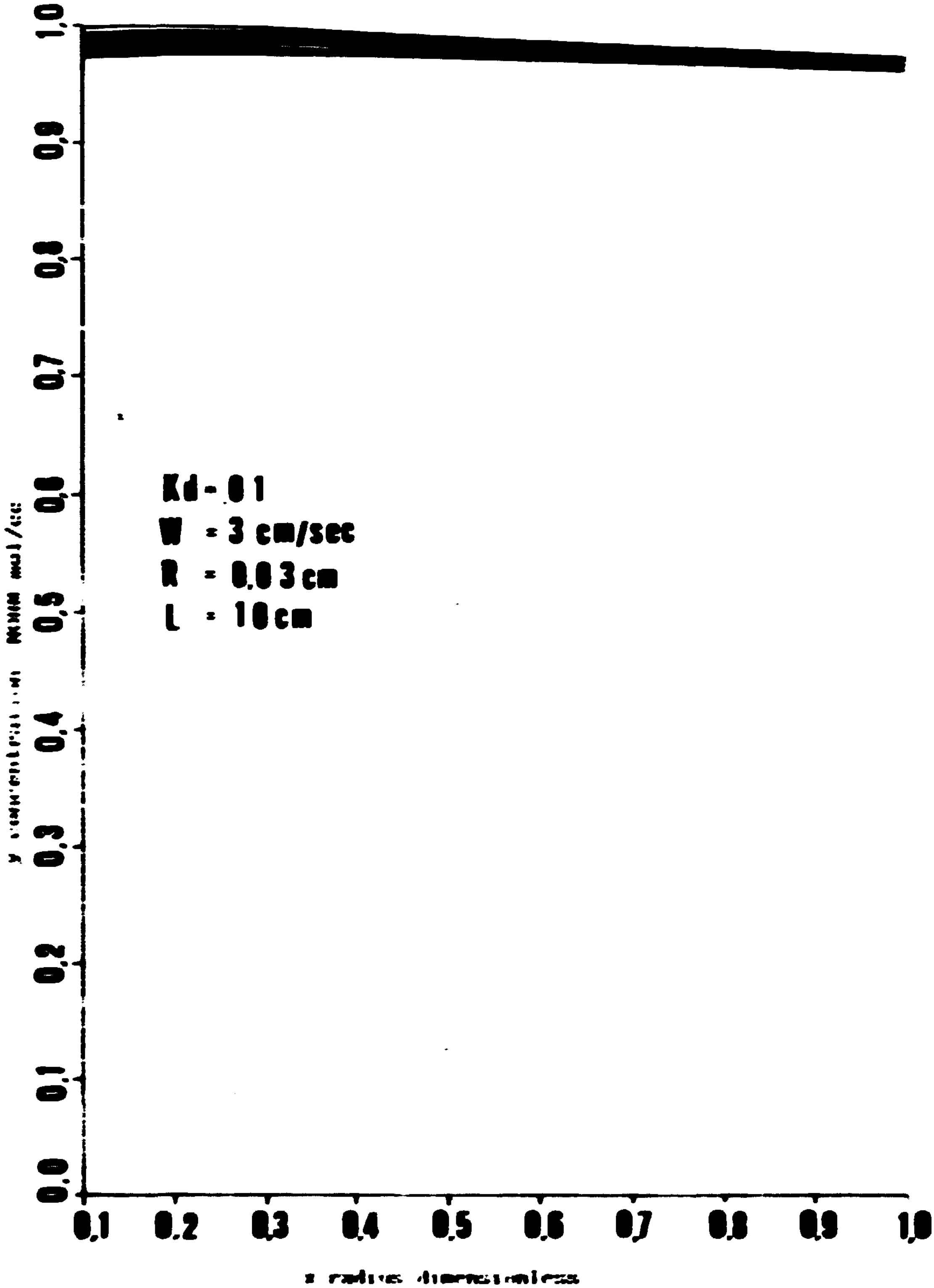

Fig. 11: Concentration Gradient for the SLM Permeable Material in the Feed vs. It's Position from the center Line of Hollow Fiber. Each line (from top to bottom) is for a travel distance of 1 cm. further into the fiber; the first is 1/2 R from the beginning of the fiber, which is 10 cm. long.
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