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TOPOLOGICAL K-KOLMOGOROV GROUPS *

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ABSTRACT

In [2] and [3] the idea of the K-groups [1] and [4] is used to define K-Kolmogorov homology and cohomology (over pairs of coefficient groups) which are descriptions of certain modifications of the Kolmogorov groups given in [5], [10] and [12].

The present work is devoted to the study of the topological properties of the K-Kolmogorov groups which lie at the root of the group duality based essentially upon Pontrjagin's concept of group multiplication [13].

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I. K-KOLMOGOROV GROUPS

Let X be a locally compact Hausdorff space, B the collection of bounded sets of X, [7], A be a closed subset of X and $\ell: A \subset X$ the inclusion map. X^{j+1} will denote the $(j+1)$th Cartesian power of X and $\hat{x}_{(i)}$ denote the j-tuple $(x_0, \ldots, \hat{x}_i, \ldots, x_j)$ consists of the $(j+1)$-tuple $x = (x_0, \ldots, x_i, \ldots x_j) \in X^{j+1}$ with x_i omitted. Let K be a locally-finite simplicial complex and for any simplex $\tau \in K$ denote by $n(\tau)$ the integer $n + \dim\tau$, $n \geqslant 0$. Assume that (G, G') and (F, F') are pairs of discrete and compact groups, respectively, which are conjugated, i.e. G, F are dually paired and each of G', F' is the annihilator of the other, [13]. Denote by β the additive group of reals mod 1, [11].

The K-Kolmogorov homology groups over (G, G') are defined as follows, [2]:

Denote by $\bar{C}_n(X, G, G')$ (shall be abbreviated by $\bar{C}_n(X)$) the group of all collections $\bar{y}_n = \{\bar{y}_\tau\}$, where $\bar{y}_\tau : B^{n(\tau)+1} \longrightarrow G$ and $\tau \in K$, satisfying the first three conditions k1 - k3 of the following:

k1. for almost all τ, $\bar{y}_\tau(b) \in G'$ for all $b \in B^{n(\tau)+1}$;

k2. $\bar{y}_\tau$ is skew-symmetric, [12];

k3. $\bar{y}_\tau$ is additive, [12];

k4. if $b = (b_0, \ldots, b_{n(\tau)})$ and $\bigcap[b] = \bigcap_{i=0}^{n(\tau)} [b_i] = \phi$ then $\bar{y}_\tau(b) = 0$, where $[b_i]$ denotes the closure of b_i in X.

Denote by $C_n(X)$ the subgroup of $\bar{C}_n(X)$ consists of those elements that satisfy k1 - k4. The inclusion map ℓ induces a monomorphism $\ell_n : C_n(A) \to C_n(X)$ defined by $(\ell_n y_n)_\tau(b) = y_\tau(b \cap A)$, where $y_n = \{y_\tau\} \in C_n(A)$ and $b \cap A = (b_0 \cap A, \ldots, b_{n(\tau)} \cap A)$. We shall identify $C_n(A)$ with $\operatorname{Im} \ell_n$. Let $C_n(X,A) = C_n(X)/C_n(A)$ and $\bar{C}_n(X,A) = \bar{C}_n(X)/C_n(A)$; the former is called the group of n-dimensional K-Kolmogorov chains of (X,A) over (G,G').

The boundary homomorphism $\partial_n : C_n(X,A) \to C_{n-1}(X,A)$ is given by: if $h_n \in C_n(X,A)$ has a representative $y_n = \{y_\tau\} \in C_n(X)$, $b = (b_0, \ldots, b_{n(\tau)-1}) \in B^{n(\tau)}$ then

$$(\partial_n y_n)_\tau = \partial y_\tau + (-1)^{n(\tau)} y_{\partial\tau} \; ,$$

where $(\partial y_\tau)(b) = y_\tau(V,b) = y_\tau(V,b_0,\ldots,b_{n(\tau)-1})$,

$$y_{\partial\tau}(b) = \sum_\sigma [\tau:\sigma]\, y_\sigma(b),$$

V is an open bounded subset of X containing $\bigcup[b] = \bigcup_{i=0}^{n(\tau)-1}[b_i]$, $\sigma \in K$ with dimension $\dim\tau - 1$, and $[\tau:\sigma]$ is their incidence number, [14]. Note that the existence of such V follows from the compactness of $\bigcup_i [b]$, [8] and [9].

The homology group of the chain complex $C_{\neq}(X,A) = \{C_n(X,A), \partial_n\}$, [7], is called the K-Kolmogorov homology group of (X,A) over (G,G'); it is denoted by $\overset{k}{H}_n(X,A,G,G')$.

Denote by $\bar{C}^n(X,F,F')$ (which shall be abbreviated $\bar{C}^n(X)$) the group of all collections $\bar{z}^n = \{\bar{z}^\tau\}$, where $\bar{z}^\tau : X^{n(\tau)+1} \to F$, satisfying the following conditions:

$\overline{k1}$. for almost all $\tau \in K$, $\bar{z}^\tau(x) \in F'$ for all $x \in X^{n(\tau)+1}$;

$\overline{k2}$. $\bar{z}^\tau$ is skew-symmetric;

$\overline{k3}$. corresponding to $\bar{z}^n$ there is a finite system $D(\bar{z}^n)$ of pairwise disjoint bounded subsets of X such that:

(i) $\bar{z}^\tau(x) = \bar{z}^\tau(x')$ if x_i and x_i' belong to the same element of $D(\bar{z}^n)$, $0 \leq i \leq n(\tau)$,

(ii) $\bar{z}^\tau(x) = 0$ if any coordinate x_i of x does not belong to a member of $D(\bar{z}^n)$.

An elment $\bar{z}^n \in \bar{C}^n(X)$ is said to be locally zero on X if there is an open covering U of X on which $\bar{z}^n$ vanishes [14], i.e. for each $\tau \in K$, $\bar{z}^\tau(x) = 0$ when $x \in U_i^{n(\tau)+1}$ for some $U_i \in U$. Let $\bar{C}_0^n(X) = \{\bar{z}^n \in \bar{C}^n(X) : \bar{z}^n$ is locally zero on $X\}$ and $\bar{C}^n(X,A) = \{\bar{z}^n \in \bar{C}^n(X) : \bar{z}^n$ is locally zero on $A\}$. The group $C^n(X,A,)$ of n-dimensional K-Kolmogorov cochains of (X,A) over (F,F') is the factor group $\bar{C}^n(X,A)/\bar{C}_0^n(X)$. It is clear that $C^n(X) = \bar{C}^n(X)/\bar{C}_0^n(X)$.

<u>Lemma 1</u> If $\bar{z}^n \in \bar{C}^n(X,A)$ and $b = \{b_0,\ldots,b_{n(\tau)}\} \subset D(\bar{z}^n)$ such that $\bigcap[b] \cap A \neq \phi$ then $\bar{z}^\tau(x) = 0$ for each $\tau \in K$ and $x \in \prod_{i=0}^{n(\tau)} b_i$.

<u>Proof</u> Let $\bar{z}^n$ vanish on the open covering U of A, $a \in \bigcap_i [b] \cap A$, $U_a \in U$ contains a and $x' \in \prod_{i=0}^{n(\tau)} (b_i \cap U_a)$. Since the coordinates of x' belong to U_a, $\bar{z}^\tau(x') = 0$. Since x_i and x_i' are elements of the same member of $D(\bar{z}^n)$ $0 \leq i \leq n(\tau)$, then in view of $\overline{k3}(i)$, $\bar{z}^\tau(x) = \bar{z}^\tau(x') = 0$.

The coboundary homomorphism $\delta^n : C^n(X) \to C^{n+1}(X)$ is defined by: if $z^n \in C^n(X)$ with representative $\bar{z}^n \in \bar{C}^n(X)$, $x \in X^{n(\tau)+2}$ and V is an open bounded subset of X containing $\bigcup[D(\bar{z}^n)]$ then

$$(\delta^n \bar{z}^n)^\tau = \delta\bar{z}^\tau + (-1)^{n(\tau)+1}\,\bar{z}^{\delta\tau},$$

where

$$(\delta\bar{z}^\tau)(x) = \begin{cases} \sum_{i=0}^{n(\tau)+1} (-1)^i\, \bar{z}^\tau(\hat{x}_{(i)}), & \text{if } x \in V^{n(\tau)+2}, \\ 0 & \text{otherwise}, \end{cases}$$

$$\bar{z}^{\delta\tau}(x) = \sum_\sigma [\sigma:\tau]\, \bar{z}^\sigma(x), \quad \dim\sigma = \dim\tau + 1,$$

$D(\delta^n\bar{z}^n) = \{D(\bar{z}^n), V_1 - \bigcup[D(\bar{z}^n)]\}$ and V_1 is any open bounded subset of X containing $\bigcup[D(\bar{z}^n)]$.

The homology group of the cochain complex $C^{\neq}(X,A) = \{C^n(X,A), \delta^n\}$ is called the K-Kolmogorov cohomology group of (X,A) over (F,F') and denoted by $\overset{k}{H}{}^n(X,A;F,F')$.

II. TOPOLOGICAL K-KOLMOGOROV HOMOLOGY AND COHOMOLOGY GROUPS

This article is concerned to topologize the K-Kolmogorov cohomology groups and prove their duality with K-Kolmolorov homology groups.

2.1 In terms of the group multiplication of G and F, [11], we shall define and study the multiplication of some groups of article I.

<u>Definition 1</u> The multiplication of the two elements $\bar{y}_n = \{\bar{y}_\tau\} \in \bar{C}_n(X)$ and $\bar{z}^n = \{\bar{z}^\tau\} \in \bar{C}^n(X)$ is

$$\bar{y}_n \times \bar{z}^n = \sum_\tau \bar{y}_\tau \times \bar{z}^\tau,$$

where for $\tau \in K$,

$$\bar{y}_\tau \times \bar{z}^\tau = \sum_i \bar{y}_\tau(b_{i_0},\dots,b_{i_{n(\tau)}}) \cdot \bar{z}^\tau(x_{i_0},\dots,x_{i_{n(\tau)}}),$$

$$\{b_{i_0},\dots,b_{i_{n(\tau)}}\} \subset D(\bar{z}^n) \text{ and } x_{i_j} \in b_{i_j},\ 0 \le j \le n(\tau).$$

The finiteness of both $\sum_i$ and $\sum_\tau$ follows from $\overline{k3}$ and $k1$, $\overline{k1}$.

In view of $\overline{k3}$ (ii) the multiplication can be shown to be distributive in both variables.

Definition 2 The multiplication of $\bar{h}_n \in \bar{C}_n(X,A)$ and $\bar{z}^n \in \bar{C}^n(X,A)$ is $\bar{h}_n \times \bar{z}^n = \bar{y}_n \times \bar{z}^n$, where $\bar{y}_n \in \bar{C}_n(X)$ is a representative of $\bar{h}_n$.

The multiplication $\bar{h}_n \times \bar{z}^n$ is well-defined. Actually, if $\bar{y}'_n \in \bar{h}_n$ then, $\bar{y}_n - \bar{y}'_n \in C_n(A)$ and

$$\bar{y}_n \times \bar{z}^n - \bar{y}'_n \times \bar{z}^n = (\bar{y}_n - \bar{y}'_n) \times \bar{z}^n =$$

$$= \sum_\tau \sum_i (\bar{y}_\tau - \bar{y}'_\tau)(b_{i_0} \cap A,\dots,b_{i_{n(\tau)}} \cap A) \cdot \bar{z}^\tau(x_{i_0},\dots,x_{i_{n(\tau)}}).$$

In case $\bigcap_j [b_{i_j}] \cap A = \phi$ we have $\bigcap_j [b_{i_j} \cap A] = \phi$ and, by using $k4$, $(\bar{y}_\tau - \bar{y}'_\tau)(b_{i_0} \cap A,\dots,b_{i_{n(\tau)}} \cap A) = 0$. If $\bigcap_j [b_{i_j}] \cap A \ne \phi$ then, in view of Lemma 1, $\bar{z}^\tau(x_{i_0},\dots,x_{i_{n(\tau)}}) = 0$.

Lemma 2 The groups $\bar{C}_n(X,A)$ and $\bar{C}^n(X,A)$ are orthogonal.

Proof I. Let $\bar{z}^n$ be a nonzero element of $\bar{C}^n(X,A)$ and $D(\bar{z}^n) = \{b_0,\dots b_r\}$. This means that for some $\sigma \in K$ there is $x = (x_0,\dots,x_{n(\sigma)}) \in X^{n(\sigma)+1}$ such that $\bar{z}^\sigma(x) = f \ne 0$, where $f \in F$. According to $\overline{k3}$(i), the coordinates of x must belong to distinct elements of $D(\bar{z}^n)$. The duality of the groups G and F implies the existence of an element $g \in G$ such that $gf \ne 0$. Define $\bar{y}_\sigma : B^{n(\sigma)+1} \longrightarrow G$ by

$$\bar{y}_\sigma(b_0,\dots,b_{n(\sigma)}) = \varepsilon_\alpha \bar{y}_\sigma(b_{\alpha 0},\dots,b_{\alpha n(\sigma)}) = \begin{cases} g, & \text{if } x_i \in b_i,\ 0 \le i \le n(\sigma), \\ 0 & \text{otherwise} \end{cases}$$

where α is a permutation on the set $\{0,\dots,n(\sigma)\}$ and ε_α is its sign. Assume that $\bar{y}_n \in \bar{C}_n(X)$ is given by:

$$(\bar{y}_n)_\tau = \begin{cases} \bar{y}_\sigma, & \text{if } \tau = \sigma, \\ 0 & \text{otherwise.} \end{cases}$$

It is clear that $\bar{y}_n$ defines a nonzero element $\bar{h}_n$ of $\bar{C}_n(X,A)$ for which $\bar{h}_n \times \bar{z}^n = \bar{y}_\sigma \times \bar{z}^\sigma = gf \ne 0$.

II. If $\bar{h}_n$ is a nonzero element of $\bar{C}_n(X,A)$ with representative $\bar{y}_n$ then there is $\sigma \in K$ and $b = (b_0,\dots,b_{n(\sigma)}) \in B^{n(\sigma)+1}$ such that $\bar{y}_\sigma(b) = g \ne 0$. Assume that $\bigcap_i b_i = \phi$ and $gf \ne 0$ for some nonzero element f of F. Since $\bar{y}_n \notin C_n(A)$ one can consider that $\bigcap [b] \cap A = \phi$. Define the function $\bar{z}^\sigma : X^{n(\sigma)+1} \longrightarrow F$ and the element $\bar{z}^n \in \bar{C}^n(X)$ by:

$$\bar{z}^\sigma(x_0,\dots,x_{n(\sigma)}) = \varepsilon_\alpha \bar{z}^\sigma(x_{\alpha 0},\dots,x_{\alpha n(\sigma)}) =$$

$$= \begin{cases} f, & \text{if } x_i \in b_i,\ 0 \le i \le n(\sigma), \\ 0 & \text{otherwise,} \end{cases}$$

$$(\bar{z}^n)^\tau = \begin{cases} \bar{z}^\sigma, & \text{if } \tau = \sigma, \\ 0 & \text{otherwise,} \end{cases}$$

and $D(\bar{z}^n) = \{b_0,\dots b_{n(\sigma)}\}$. It is clear that $\bar{z}^n$ vanishes on the open covering $\{X - [b_i] : 0 \le i \le n(\sigma)\}$ of the subspace A. This means that $\bar{z}^n \in \bar{C}^n(X,A)$. Moreover, $\bar{h}_n \times \bar{z}^n = gf \ne 0$, which completes the proof.

Definition 3 The multiplication of $h_n \in C_n(X,A)$ and $c^n \in C^n(X,A)$ is $y_n \times \bar{z}^n$, where $y_n \in C_n(X)$, $y_n \in h_n$, $\bar{z}^n \in \bar{C}^n(X,A)$ and $\bar{z}^n \in c^n$.

It is not difficult to prove the correctness of this difinition. Actually, the independence of $h_n \times c^n$ on y_n follows from Lemma 2, since $C_n(X,A)$ is a subgroup of $\bar{C}_n(X,A)$. If $\bar{z}^n_1 \in c^n$ then

$$(\bar{z}^n - \bar{z}^n_1) \in \bar{C}^n_0(X) \quad \text{and}$$

$$y_n \times (\bar{z}^n - \bar{z}^n_1) = \sum_\tau \sum_i y_\tau(b_{i_0},\dots,b_{i_{n(\tau)}}) \cdot (\bar{z}^\tau - \bar{z}^\tau_1)(x_{i_0},\dots,x_{i_{n(\tau)}}).$$

In case $\bigcap_j [b_{i_j}] = \phi$ the condition k4 implies that $y_\tau(b_{i_0},\dots, b_{i_{n(\tau)}}) = 0$. If $\bigcap_j [b_{i_j}] \neq 0$ then, by putting $A = X$ in Lemma 1, we have $(z^\tau - \bar z_1^\tau)(x_{i_0},\dots, x_{i_{n(\tau)}}) = 0$.

Lemma 3 The boundary and coboundary homomorphisms of the complexes $C_{\neq}(X,A,)$ and $C^{\neq}(X,A)$ are conjugated.

Proof Let $h_n \in C_n(X,A)$ and $c^{n-1} \in C^{n-1}(X,A)$ with representatives y_n and $\bar z^{n-1}$, respectively. Thus

$$\begin{aligned}\partial_n h_n \times c^{n-1} &= \partial_n y_n \times \bar z^{n-1} \\ &= \sum_\tau \left(\partial y_\tau + (-1)^{n(\tau)} y_{\partial\tau}\right) \times \bar z^\tau \\ &= \sum_\tau \partial y_\tau \times \bar z^\tau + \sum_{\tau,\sigma} (-1)^{n(\tau)} (y_\sigma \times \bar z^\tau), \qquad \text{......} \quad (2.1)\end{aligned}$$

where $\dim\tau = \sigma + 1$ and $\delta\sigma = \sum_\tau [\tau:\sigma]\tau$, [7]. On the other hand we have,

$$\begin{aligned}h_n \times \delta^n c^{n-1} &= \sum_\sigma y_\sigma \times \left(\delta \bar z^\sigma + (-1)^{n(\sigma)+1} \bar z^{\delta\sigma}\right) \\ &= \sum_\sigma y_\sigma \times \delta \bar z^\sigma + \sum_{\sigma,\tau} (-1)^{n(\sigma)+1} [\tau:\sigma] (y_\sigma \times \bar z^\tau) \qquad \text{......} \quad (2.2)\end{aligned}$$

In view of k3 one can prove that, for each $\tau \in K$, $\partial y_\tau \times z^\tau = y_\tau \times \delta z^\tau$, [5]. Since $\sum_\tau$ and $\sum_\sigma$ of equalities (2.1) and (2.2) are running over all simplexes of K, it follows that $\sum_\tau \partial y_\tau \times z^\tau = \sum_\sigma y_\sigma \times \delta \bar z^\sigma$. Moreover, since $n(\sigma) + 1 = n(\tau)$, $[\tau:\sigma] = \pm 1$ one of τ or σ a face of other, and otherwise $[\tau:\sigma] = 0$, [11], the second terms $\sum_{\tau,\sigma}$ and $\sum_{\sigma,\tau}$ of (2.1) and (2.2) are equal.

Lemma 4 The groups $C_n(X,A)$ and $C^n(X,A)$ are orthogonal.

Proof In view of Lemma 3 it is only necessary to show that the group $C_n(X,A)$ coincides with the set :

$$\{\bar h_n \in \bar C_n(X,A) : \bar h_n \times \bar z_0^n = 0 \ , \ \forall\, \bar z_0^n \in \bar C_0^n(X)\} .$$

Actually, in the proof of the correctness of Definition 3, it is shown that if $h_n \in C_n(X,A)$ and $\bar z_0^n \in \bar C_0^n(X)$ then $h_n \times \bar z_0^n = 0$. Now, let $\bar h_n \in \bar C_n(X,A) - C_n(X,A)$ and $\bar y_n \in \bar C_n(X)$ be a representative of $\bar h_n$. This implies that there exist $\sigma \in K$ and $b = (b_0,\dots, b_{n(\tau)}) \in B^{n(\tau)+1}$ such that $\bigcap[b] = \phi$ and $\bar y_\sigma(b) = g \neq 0$. Assume that $f \in F$, $f \neq 0$ and $gf \neq 0$. Define $\bar z^n \in \bar C^n(X)$ by means of a function $\bar z^\sigma$ in a similar way to that method given in the proof of Lemma 2. It is easy to see that $\bar z^n$ vanishes on the open covering $\{X - [b_i]\}$ of X. Therefore, $\bar z^n \in \bar C_0^n(X)$ and $\bar h_n \times \bar z^n = gf \neq 0$.

2.2 By means of the multiplications and results given above, here we shall deal with the characters of groups.

Each element c^n of the group $C^n(X,A)$ may be considered as a character, [11], of the group $C_n(X,A)$ by assuming that $c^n(h_n) = h_n \times c^n$ for each $h_n \in C_n(X,A)$. In view of Lemma 4, it is easy to show that different elements of $C^n(X,A)$ define different characters of $C_n(X,A)$. Hence $C^n(X,A)$ can be identified with a subgroup of the character-group $C_n^*(X,A)$ of the group $C_n(X,A)$. The topology assigned to $C^n(X,A)$ as a subgroup of $C_n^*(X,A)$ turns it into a topological group $\tilde C^n(X,A)$.

Theorem 1 The group $\tilde C^n(X,A)$ is a dense subset of the group $C_n^*(X,A)$.

Proof Denote by $[\tilde C^n(X,A)]$ the closure of $\tilde C^n(X,A)$ in $C_n^*(X,A)$. Assume that the theorem is not true. Since $C_n(X,A)$ and $C_n^*(X,A)$ are orthogonal, [11], the annihilator $L_n(X,A)$ of $[\tilde C^n(X,A)]$ in $C_n(X,A)$ is a nonzero subgroup. If h_n is a nonzero in $L_n(X,A)$ then $h_n \times c^n = 0$ for all $c^n \in [\tilde C^n(X,A)]$, which contradicts with Lemma 4.

Lemma 5 The coboundary homomorphism $\delta^n : \tilde C^n(X,A) \longrightarrow \tilde C^{n+1}(X,A)$, induced by δ^n, is a continuous homomorphism.

Proof Let $U(E,N)$ be a nucleus of $\tilde C^{n+1}(X,A)$, [11], where E is a finite subset $\{y_{n+1}^{(i)} : 0 \leq i \leq q\}$ of $C_{n+1}(X,A)$ and N is a nucleus of $\mathfrak{B}$. Define a nucleus V of $\tilde C^n(X,A)$ by:

$$V = \{ z^n \in C^n(X,A) : \partial_{n+1} y_{n+1}^{(i)} \times z^n \in N, 0 \leq i \leq q \} .$$

In view of Lemma 3, we have

$$(\delta^n z^n)(y_{n+1}^{(i)}) = y_{n+1}^{(i)} \times \delta^n z^n = \partial_{n+1} y_{n+1}^{(i)} \times z^n \in N$$

and hence $\delta^n V \subset U$.

The continuous map δ^n has an extension, [6]: $[\delta^n] : [\check{C}^n(X,A)] \longrightarrow [\check{C}^{n+1}(X,A)]$. In the cochain complex $[\check{C}^{\neq}(X,A)] = \{\check{C}^n(X,A), [\delta^n]\}$, the kernel of $[\delta^n]$ is a closed subgroup of $[\check{C}^n(X,A)]$ and so it is compact, [8]. Also $\mathrm{Im}[\delta^n]$ is a closed subgroup of $\mathrm{Ker}[\delta^n]$ and thus their factor group is compact, [11].

Since $[\check{C}^n(X,A)]$, $C_n(X,A)$ are dually and $[\delta^n]$, ∂_n are conjugated, it follows that the complexes $[\check{C}^{\neq}(X,A)]$, $C_{\neq}(X,A)$ have dually homology groups, and the next main result follows.

<u>Theorem 2</u> Let (G,G') and (F,F') be conjugated pairs of discrete and compact groups, respectively. The K-Kolmogorov homology and cohomology group $\overset{k}{H}_n(X,A;\ G,G')$ and $\overset{k}{H}{}^n(X,A;\ G,G')$ are dually.

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