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THE RELATION BETWEEN POLYAKOV'S AND FRADKIN'S PATH INTEGRALS FOR BOSONIC STRING *

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ABSTRACT

The relation between Polyakov's path integral and Fradkin's integral in extended phase space is analyzed on an example of a free closed bosonic string. It is shown in D = 26 that locally, in every Teichmüller sector, both methods provide the same result. Beyond D = 26 Fradkin's integral appears to be depending on the gauge fixing.

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I. Introduction.

Polyakov's functional integral for bosonic string was originally formulated within the euclidean framework [1]. It is constructed by analogy with euclidean path integral for Y-M theory and for point particle [2]. The construction is based on the riemannian geometrical approach to the functional measure considered previously in the context of Y-M theory [3]. Both in the case of point particle and the Y-M theory one can proceed from euclidean functional integral to the covariant one in Minkowski space-time (by Wick rotation).

It can be shown that these integrals can be obtained from the Fradkin phase space functional (with suitable choice of gauge fixing) by integration over momenta [4, 5]. As the Fradkin functional approach has clear physical interpretation and is unitary 'by definition' [5, 6] this provides in both cases the quantum mechanical interpretation of covariant path integral.

Our main aim in this paper is to show the equivalence of local structures of Polyakov's and Fradkin's functional integrals for bosonic string. Global problems such as allowed topologies of the world sheet in Minkowski space-time, boundary terms and the structure of the space of Lorentz metrics on Lorentz 2-dimensional manifolds will not be considered here. It means that we shall show this equivalence under an assumption that the whole space of metrics $\mathcal{M}$ can be obtained from the fixed one by the action of the semi-direct product of the group $\mathcal{D}$ of diffeomorphism and the group $\mathcal{W}$ of conformal deformations of the metrics, i.e. a gauge fixing $g_{ab}=\hat{g}_{ab}$ is correct.

Such assumption is justified only locally. If we try for example to compare the propagators evaluated in these two schemes the ' Teichmüller' parameters must be taken into account.

The equivalence of local structures, nevertheless, is the first and unavoidable stage in the proof of the full equivalence of covariant and phase-space functional quantization schemes. One can think about the 'local equivalence' as an equivalence of contributions of a fixed 'Teichmüller' sector to the propagator. The summation over these contributions is essentially a global problem.

2. Phase space path integral.

First we apply Fradkin's method of functional quantization [5,6] to the free closed bosonic string described by the BDHP action

$$S[g,x] \equiv \frac{1}{2}\int_M \sqrt{-g}\, d^2z\, g^{ab}\partial_a x^\mu \partial_b x^\mu \qquad (1)$$

where M is a cylinder with boundary, g_{ab} is a Lorentz metric on M such that M is space-like and x is a map from M into flat Minkowski space-time. In terms of coordinates $(\sigma,\tau)\in[0,2\pi]\times[0,t]$ we have periodic boundary condition in σ.

We start the Fradkin's construction from Legendre transformation:

$$\begin{aligned} (x^\mu, \dot{x}^\mu) &\longrightarrow (x^\mu, p^\mu = \sqrt{-g}\, g^{0b}\partial_b x^\mu) \\ (g_{ab}, \dot{g}_{ab}) &\longrightarrow (g_{ab}, p^{ab} = 0) \\ \mathcal{L} &\longrightarrow \mathcal{H} = \alpha\phi^1 + \beta\phi^2 + p^{ab}\dot{g}_{ab} \end{aligned} \qquad (2)$$

where

$$\alpha \equiv \frac{1}{\sqrt{-g}\,g^{00}} = -\frac{\sqrt{-g}}{g_{11}} \quad , \quad \beta \equiv -\frac{g^{01}}{g^{00}} = \frac{g_{01}}{g_{11}}$$

and

$$\phi^1 \equiv \frac{1}{2}(p^\mu p^\mu + \partial_1 x^\mu \partial_1 x^\mu)$$

$$\phi^2 \equiv p^\mu \partial_1 x^\mu$$

We have here three primary constraints p^{00}, p^{01}, p^{11} and two secondary constraints ϕ^1 and ϕ^2. They form the first class constraints algebra [7] with

$$\begin{aligned} \{\phi^1(\sigma),\phi^1(\sigma')\} &= (\phi^2(\sigma)+\phi^2(\sigma'))\partial_1\delta(\sigma-\sigma') \\ \{\phi^2(\sigma),\phi^2(\sigma')\} &= (\phi^2(\sigma)+\phi^2(\sigma'))\partial_1\delta(\sigma-\sigma') \\ \{\phi^1(\sigma),\phi^2(\sigma')\} &= (\phi^1(\sigma)+\phi^1(\sigma'))\partial_1\delta(\sigma-\sigma') \end{aligned} \qquad (3)$$

and all remaining Poisson brackets (PB) vanishing. As H is a linear combination of constraints one can assume that it vanishes strongly. The more convenient choice is ,however, $H_o = p^{ab}\dot{g}_{ab}$ (notice that $\dot{g}_{ab}$ has vanishing PB with all phase-space functions).

The extended phase space contains (D+3) + 5 bosonic variables $q_i = (x^\mu, g_{ab}, \lambda_{ab}, \lambda_1, \lambda_2)$ with conjugate momenta $p^i = (p^\mu, p^{ab}, \pi^{ab}, \pi^1, \pi^2)$ and 10 ghost variables $\eta_\alpha = (-i\mathcal{P}_{ab}, -i\mathcal{P}_1, -i\mathcal{P}_2, C_{ab}, C_1, C_2)$ with conjugate momenta (anti-ghosts) $\mathcal{P}^\alpha = (i\bar{C}^{ab}, i\bar{C}^1, i\bar{C}^2, \bar{\mathcal{P}}^{ab}, \bar{\mathcal{P}}^1, \bar{\mathcal{P}}^2)$.

The BRST operator is given by [5]

$$\Omega \equiv \int_0^{2\pi} d\sigma \left[-i(\mathcal{P}_{ab}\pi^{ab} + \mathcal{P}_1\pi^1 + \mathcal{P}_2\pi^2) + C_{ab}p^{ab} + C_1\phi^1 + C_2\phi^2 + \right. \qquad (4)$$
$$\left. -(C_1'C_1 + C_2'C_2)\bar{\mathcal{P}}^2 - (C_2'C_1 + C_1'C_2)\bar{\mathcal{P}}^1 \right]$$

It is easy to see that with our choice of hamiltonian $\{H_o,\Omega\}=0$ is satisfied.

The functional integral for the system is given by:

$$Z_F^D[C_i, C_f] = \int D\mu \; \exp i S_{eff} \qquad (5)$$

where

$$S_{eff} = \int_0^t d\tau \int_0^{2\pi} d\sigma \Big(\dot{x}^\mu p^\mu + \dot{\lambda}_{ab}\pi^{ab} + \dot{\lambda}_1 \pi^1 + \dot{\lambda}_2 \pi^2 + \dot{\eta}_\alpha \mathcal{P}^\alpha + \{\Psi, \Omega\} \Big) \tag{6}$$

and Ψ is the gauge fixing functional, i.e. an arbitrary functional on the extended phase space with ghost number -1*). The functional measure $\mathcal{D}\mu$ is defined as a formal product of Liouville measures on the extended phase space. The boundary condition on x^μ in parameter τ is such that the world sheets described by x's join two classical string configurations C_i, C_f in the D-dimensional Minkowski space-time. The variables C_{ab}, C_1, C_2 $\bar{C}^{ab}, \bar{C}^1, \bar{C}^2, \pi^{ab}, \pi^1, \pi^2$ vanish at $\tau = 0, t$. Other variables remain unrestricted. With this boundary condition functional integral (5) describes the propagator of the free closed string.

By Fradkin's theorem Z_F^D does not depend on the choice of Ψ. The choice of the relevant Ψ is, however, quite a complex problem [5]. Our proposal for Ψ is related to the gauge $g_{ab} = \hat{g}_{ab}$ in the covariant approach:

$$\Psi_\varepsilon = \int_0^{2\pi} d\sigma \Big(i\bar{C}^{ab}(\lambda_{ab} - g_{ab} + \hat{g}_{ab}) + \frac{1}{\varepsilon}\bar{\mathcal{P}}^{ab}\lambda_{ab} + \frac{i}{\varepsilon}\bar{C}^1(\lambda_1 - \alpha) + \bar{\mathcal{P}}^1\lambda_1 + \frac{i}{\varepsilon}\bar{C}^2(\lambda_2 - \beta) + \bar{\mathcal{P}}^2\lambda_2 \Big) \tag{7}$$

where ε is a positive parameter.

It must be stressed that since the quotient space of all Lorentz metrics on the cylinder by $\mathcal{D} \ltimes \mathcal{W}$ is nontrivial, the gauge fixing condition $g_{ab} = \hat{g}_{ab}$ is overcomplete and leads to the incorrect result. It can however, be justified as long as we are considering the local structure of the path integral. As was mentioned earlier the integral (5) with gauge fixing (7) can be thought of as a contribution of a fixed 'Teichmüller' sector to the string propagator.

*) We use the Henneaux notation and conventions [5].

Inserting (7) into (6) and changing the variables

$$\pi^1 \to \varepsilon\pi^1 \qquad \bar{C}^1 \to \varepsilon\bar{C}^1$$
$$\pi^2 \to \varepsilon\pi^2 \qquad \bar{C}^2 \to \varepsilon\bar{C}^2$$
$$\lambda_{ab} \to \varepsilon\lambda_{ab} \qquad \bar{\mathcal{P}}^{ab} \to \varepsilon\bar{\mathcal{P}}^{ab}$$

one obtains in the $\varepsilon \to 0$ limit the following effective action:

$$\begin{aligned} S_{eff} \underset{\varepsilon\to 0}{=} \int_0^t d\tau \int_0^{2\pi} d\sigma \Big(& \dot{x}^\mu p^\mu + \pi^{ab}(g_{ab} - \hat{g}_{ab}) - p^{ab}\lambda_{ab} + \\ & - \pi^1(\lambda_1 - \alpha) - \phi^1\lambda_1 + \\ & - \pi^2(\lambda_2 - \beta) - \phi^2\lambda_2 + \\ & - i\bar{\mathcal{P}}^{ab}\mathcal{P}_{ab} + (\bar{C}^{ab} + \dot{\bar{C}}^{ab})\mathcal{P}_{ab} + \\ & - i\bar{C}^{ab}C_{ab} - i(\bar{C}^1\partial^{ab}\alpha + \bar{C}^2\partial^{ab}\beta)C_{ab} + \\ & - i\bar{\mathcal{P}}^1\mathcal{P}_1 - \bar{\mathcal{P}}^1(\dot{C}_1 + \lambda_2' C_1 - \lambda_2 C_1' + \lambda_1' C_2 - \lambda_1 C_2') + \bar{C}^1\mathcal{P}_1 + \\ & - i\bar{\mathcal{P}}^2\mathcal{P}_2 - \bar{\mathcal{P}}^2(\dot{C}_2 + \lambda_1' C_1 - \lambda_1 C_1' + \lambda_2' C_2 - \lambda_2 C_2') + \bar{C}^2\mathcal{P}_1 \Big) \end{aligned} \tag{8}$$

The integrations over $p^{ab}, \lambda_{ab}, g_{ab}, \pi^{ab}, \pi^1, \pi^2, \lambda_1, \lambda_2, p^\mu$ $\bar{\mathcal{P}}^{ab}, \mathcal{P}_{ab}, \bar{C}^{ab}, C_{ab}, \bar{\mathcal{P}}^1, \mathcal{P}_1, \bar{\mathcal{P}}^2, \mathcal{P}_2$ yield:

$$Z_F^D[C_i, C_f] = \int (\det \hat{\alpha}\cdot \mathbb{1})^{-\frac{D}{2}} D[x^\mu]\, e^{\frac{i}{2}\iint d\sigma d\tau \sqrt{-g}\, g^{ab}\partial_a x^\mu \partial_b x^\mu} \times \int D[C] D[\bar{C}]\, e^{i\iint d\sigma d\tau\, \bar{C}\cdot \tilde{P}_g C} \tag{9}$$

where

$$C \equiv \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}, \quad \bar{C} \equiv (\bar{C}^1, \bar{C}^2), \quad \bar{A}\cdot B \equiv \bar{A}^a B_a$$

$$\widetilde{P}_{\hat{g}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\partial_0 - \begin{pmatrix} \hat{\beta} & \hat{\alpha} \\ \hat{\alpha} & \hat{\beta} \end{pmatrix}\partial_1 + \begin{pmatrix} \hat{\beta}' & \hat{\alpha}' \\ \hat{\alpha}' & \hat{\beta}' \end{pmatrix} \tag{10}$$

and

$$\hat{\alpha} = \frac{1}{\sqrt{-\hat{g}}\,\hat{g}^{00}} \quad , \quad \hat{\beta} = -\frac{\hat{g}^{01}}{\hat{g}^{00}} \quad .$$

The dependence on the initial C_i and final C_f configurations of the string can be extracted from the integral (9) by the shift $x^{\mu} \to x^{\mu}_{cl} + x^{\mu}_{0}$ and we finally get:

$$Z^D_F[C_i, C_f] = e^{iS[\hat{g}, x_{cl}]} \times Z^D_F[\hat{g}] \tag{11}$$

where

$$Z^D_F[\hat{g}_{ab}] = (\det \hat{\alpha}\cdot\mathbb{1})^{-\frac{D}{2}} \cdot \det \widetilde{P}_{\hat{g}} \cdot \det(\sqrt{-\hat{g}}\,\mathcal{L}_{\hat{g}})^{-\frac{D}{2}} \tag{12}$$

and $\mathcal{L}_g$ is the Laplace-Beltrami operator

$$\mathcal{L}_g \equiv \frac{1}{\sqrt{-g}}\,\partial_a g^{ab}\sqrt{-g}\,\partial_b$$

acting on the scalar functions.

3. Covariant path integral.

Polyakov's path integral for the bosonic string is defined by [8]

$$Z^D_P[C_i, C_f] = \int \frac{\mathcal{D}[g_{ab}]\,\mathcal{D}[x^{\mu}]}{\mathrm{Vol}\,\mathcal{D} \times \mathrm{Vol}\,\mathcal{W}}\, e^{iS[g,x]} \tag{13}$$

The space of metrics $\mathcal{M}$ and space of embeddings $\mathcal{E}$ are the same as in the previous case. Functional measures $\mathcal{D}[g_{ab}]$, $\mathcal{D}[x^{\mu}]$ are introduced as formal volume forms related to the riemannian structures:

$$G_{\mathcal{M},g}(\delta g_{ab}|\delta g'_{ab}) \equiv \frac{1}{2}\int_M \sqrt{-g}\,d^2z\,(g^{ac}g^{bd} + C g^{ab}g^{cd})\,\delta g_{ab}\,\delta g'_{cd} \tag{14.a}$$

$$G^g_{\mathcal{E},x}(\delta x^{\mu}|\delta x'^{\mu}) \equiv \int_M \sqrt{-g}\,d^2z\,\delta x^{\mu}\,\delta x'^{\mu} \tag{14.b}$$

on $\mathcal{M}$ and $\mathcal{E}$ respectively.

Under our assumption one can extract the dependence on the initial and final configurations from integral (13) exactly as in Fradkin's one. So it is sufficient to compare the functional $Z^D_F[g_{ab}]$ with the corresponding object in Polyakov's approach

$$Z^D_P = \int \mathcal{D}[g_{ab}]\,(\mathrm{Vol}\,\mathcal{D} \times \mathrm{Vol}\,\mathcal{W})^{-1} \cdot (\det \mathcal{L}_g)^{-\frac{D}{2}} \tag{15}$$

A standard way to evaluate this integral is to impose the conformal gauge fixing condition $g_{ab} = e^{\varphi}\hat{g}_{ab}$ for D-invariance (it is a correct gauge provided that $\mathcal{M} \approx \mathcal{D} \ltimes \mathcal{W}$). The Faddeev-Popov procedure results in

$$Z^D_P = \int \frac{\mathcal{D}[\varphi]}{\mathrm{Vol}\,\mathcal{W}}\,(\det P^{+}_{e^{\varphi}\hat{g}}\,P_{e^{\varphi}\hat{g}})^{\frac{1}{2}}\,(\det \mathcal{L}_{e^{\varphi}\hat{g}})^{-\frac{D}{2}} \tag{16}$$

where P_g denotes the Alvarez [9] operator acting on the space $\mathcal{V}$ of vector fields with values in the space $\mathcal{T}$ of traceless symmetric covariant tensors. It is defined by:

$$(P_g V)_{ab} \equiv (\nabla_a g_{bc} + \nabla_b g_{ac} - g_{ab}\nabla_c)V^c \tag{17}$$

where ∇_a is the covariant derivative related to g. The adjoint operator P_g is defined by:

$$G^g_{\mathcal{V}}(\delta V|P_g^+\delta T) = G^g_{\mathcal{T}}(P_g\delta V|\delta T)$$

where $\delta V\in\mathcal{V}$, $\delta T\in\mathcal{T}$ and

$$G^g_{\mathcal{V}}(\delta V|\delta V') \equiv \int_M \sqrt{-g}\, g_{ab}\,\delta V^a\delta V^{b\prime}$$
$$G^g_{\mathcal{T}}(\delta T|\delta T') \equiv \frac{1}{2}\int_M \sqrt{-g}\, d^2z\; g^{ab}g^{cd}\,\delta T_{ac}\delta T'_{bd}$$

are Riemannian structures on $\mathcal{V}$ and $\mathcal{T}$ respectively.

We define the $\mathcal{D}$-invariant functional

$$F^D[g_{ab}] \equiv (\det P_g^+P_g)^{\frac{1}{2}}(\det \mathcal{L}_g)^{-\frac{D}{2}} \tag{18}$$

It depends on the conformal factor of g by the local conformal anomaly evaluated by Polyakov [1]. With appropriate choice of renormalization constant one has [9]

$$F^D[e^{\varphi}g_{ab}] = e^{i\left(\frac{26-D}{2}\right)S_L[g,\varphi]}\,F^D[g_{ab}] \tag{19}$$

The insertion of (19) into (16) shows that the functional integral Z^D_P is well defined only for D=26. It vanishes in the euclidean sector if $D<26$ and strongly diverges if $D>26$.

In the critical dimension we have

$$Z^{26}_P[\hat{g}] = F^{26}[\hat{g}] \tag{20}$$

In order to compare (20) with (12) we rewrite $F^D[g]$ in a more convenient form. First we observe that

$$(\det P_g^+P_g)^{\frac{1}{2}} = \det P_g \tag{21}$$

provided that the determinant on the r.h.s. of (21) is evaluated in orthonormal bases of $\mathcal{V}$ and $\mathcal{T}$ with respect to the metrics $G^g_{\mathcal{V}}$ and $G^g_{\mathcal{T}}$ respectively.

We introduce the following external (distribution) basis $\{\delta T_{a,x}\}_{\substack{x\in M\\ a=0,1}}$ in the space $\mathcal{T}$:

$$\begin{aligned}(\delta T_{0,x})_{00}(z) &= (\alpha^2+\beta^2)(z)\delta(x-z)\\ (\delta T_{0,x})_{01}(z) &= \beta(z)\delta(x-z)\\ (\delta T_{0,x})_{11}(z) &= \delta(x-z)\end{aligned} \tag{22.a}$$

$$\begin{aligned}(\delta T_{1,x})_{00}(z) &= 2\alpha\beta(z)\delta(x-z)\\ (\delta T_{1,x})_{01}(z) &= \alpha(z)\delta(x-z)\\ (\delta T_{1,x})_{11}(z) &= 0\end{aligned} \tag{22.b}$$

For this basis we have:

$$G^g_{\mathcal{T}}(\delta T_{a,x}|\delta T_{b,y}) = \frac{\alpha^2}{\sqrt{-g}}\,\eta_{ab}\,\delta(x-y) \quad ,\ \eta_{ab}=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

To the orthonormal one we can proceed by the operator

$$(-g)^{\frac{1}{4}}\,\alpha^{-1}\cdot \mathbb{1} \tag{23}$$

The external basis $\{\delta V_{a,x}\}_{\substack{x\in M\\ a=0,1}}$ in $\mathcal{V}$ is chosen as follows:

$$\begin{aligned}(\delta V_{0,x})^0(z) &= \alpha^{-1}(z)\delta(x-z)\\ (\delta V_{0,x})^1(z) &= -\beta\alpha^{-1}(z)\,\delta(x-z)\end{aligned} \tag{24.a}$$

$$\begin{aligned}(\delta V_{1,x})^0(z) &= 0\\ (\delta V_{1,x})^1(z) &= \delta(x-z)\end{aligned} \tag{24.b}$$

and fulfills the orthogonality condition:

$$G^g_{\mathcal{V}}(\delta V_{a,x}|\delta V_{b,y}) = \frac{-g}{\alpha}\,\eta_{ab}\,\delta(x-y)$$

The transition operator to the orthonormal basis is

$$(-g)^{-\frac{1}{2}}\,\alpha^{\frac{1}{2}}\cdot 1\!\!1 \tag{25}$$

The straightforward but rather tedious calculations of the kernel $G^g_{\gamma}(\delta T_{a,x}|P_g\,\delta V_{a,y})$ of P_g in the bases (22) and (24) give the expression for P_g

$$\widetilde{P}_g = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\partial_0 - \begin{pmatrix}\beta & \alpha\\ \alpha & \beta\end{pmatrix}\partial_1 + \begin{pmatrix}\beta' & \alpha'\\ \alpha' & \beta'\end{pmatrix} \tag{26}$$

Therefore, taking into account (21) , (23) and (25) we find

$$\left(\det P_g^+ P_g\right)^{\frac{1}{2}} = \det \widetilde{P}_g \cdot \det(\sqrt{-g}\,\alpha)^{-1}\cdot 1\!\!1 \tag{27}$$

and

$$F^D[g_{ab}] = \det(\sqrt{-g}\,\alpha)^{-1} 1\!\!1 \,\det \widetilde{P}_g \left(\det \mathcal{L}_g\right)^{-\frac{D}{2}} \tag{28}$$

When we evaluate F^D in (28) for the metric $g^{00}g_{ab}$ the result is:

$$F^D[g^{00}g_{ab}] = \det \widetilde{P}_g \,(\det \alpha\cdot 1\!\!1)^{-\frac{D}{2}}\left(\det \sqrt{-g}\,\mathcal{L}_g\right)^{-\frac{D}{2}} \tag{29}$$

The r.h.s. of (29) is exactly equal to $Z^D_F[g_{ab}]$ as given by formula (12) . In consequence we have obtained

$$Z^D_F[g_{ab}] = F^D[g^{00}g_{ab}] \tag{30}$$

what is the crucial result of this letter.

To discuss the implications of rel. (30) we start with the critical dimension D=26. In this case the functional is both $\mathcal{D}$- and $\mathcal{W}$-invariant , therefore

$$\begin{aligned} Z^{26}_F[g_{ab}] &= F^{26}[g^{00}g_{ab}] = \\ &= F^{26}[g_{ab}] = Z^{26}_P[g_{ab}] \end{aligned} \tag{31}$$

So at the critical dimension Fradkin's and Polyakov's path integrals are equal and do not depend on the gauge fixing.

We proceed to the case $D\neq 26$. As has been mentioned earlier beyond the critical dimension Polyakov's integral is not well defined. But the Fradkin one still gives finite result (28). The point is that for $D\neq 26$ the Fradkin integral depends on the gauge fixing and therefore is inconsistent. In order to show this we consider two metrics defined by two positive functions γ, δ :

$$(h_{ab}) \equiv \begin{pmatrix}\gamma & 0\\ 0 & -\delta\end{pmatrix} \quad \text{and} \quad (\tilde{h}_{ab}) \equiv (h^{00}h_{ab}) = \begin{pmatrix}1 & 0\\ 0 & -\frac{\delta}{\gamma}\end{pmatrix}$$

Next we choose some function φ for which $S_L[\tilde{h},\varphi]\neq 0$. It is always possible to find such diffeomorphism f that

$$(\partial_a f^b) = \begin{pmatrix} e^{-\frac{1}{2}\varphi} & 0\\ 0 & \partial_1 f^1\end{pmatrix}$$

Suppose that Z^D_F is independent of g (as it follows from Fradkin's theorem [5,6]). Then from (28) and the $\mathcal{D}$-invariance of F^D we have

$$\begin{aligned} F^D[\tilde{h}] &= Z^D_F[h] = Z^D_F[(f^{-1})^* h] = \\ &= F^D[(f^{-1})^* e^{\varphi}\cdot\tilde{h}] = \\ &= F^D[e^{\varphi}\cdot\tilde{h}] = e^{i\frac{26-D}{2}S_L[\tilde{h},\varphi]}\,F^D[\tilde{h}] \end{aligned}$$

which is true only for D=26.

4. Conclusions.

It has been shown that the local structures of the phase space functional integral and the covariant Polyakov's one are identical in the critical dimension. It is not surprising. Fradkin's approach is conformally invariant in arbitrary D. On the other hand, Polyakov's one is invariant under diffeomorphisms. These two approaches can be compared only when both symmetries hold.

It follows from our basic relation (30) that the phase space functional integral (considered as a functional of the gauge fixing metric g) is invariant under diffeomorphisms in D=26. For $D\neq 26$ the frame anomaly appears. The formula (30) can be interpreted as the relation between the local conformal anomaly in the Polyakov $\mathcal{D}$-invariant approach and the 'frame' anomaly in the Fradkin $\mathcal{W}$-invariant one.

Let us notice that our result gives new insight into Fradkin's functional scheme of quantization. For $D\neq 26$ (i.e. when anomaly appears) phase space path integral is gauge dependent in contradiction to Fradkin's main theorem [5,6] . Anomalies are then related to the global structures of the extended phase space [10].

As is seen the cancellation of anomalies in critical dimension is crucial for the equivalence of the covariant and the phase space integrals. It seems to be interesting to apply this idea to finding the relation between de Witt [11] and Fradkin functional measures in quantum gravity [12].

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