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QUANTIZATION OF GREEN-SCHWARZ SUPERSTRING *

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ABSTRACT

The problem of quantization of superstrings is traced back to the nilpotency of gauge generators of the first-generation ghosts. The quantization of such theories is performed. The novel feature of this quantization is the freedom in choosing the number of ghost generations as well as gauge conditions. As an example, we perform quantization of heterotic string in a gauge, which preserves space-time supersymmetry. The equations of motion are those of a free theory.

1. The covariant action of Green-Schwarz superstring [1] was quantized up to now in the light-cone gauge only. This quantization destroys the world-sheet global super-Poincaré invariance of space-time (x, θ), which is the main advantage of the "new formulation" of superstring theory [1].

The lack of a consistent quantization of a gauge theory in arbitrary gauge is very unusual and unsatisfactory. The aim of this paper is to find the underlying reason for this difficulty and to overcome it.

We start by analysing an important property of the local fermionic symmetry which is present in the superparticle actions [2,3,4] and in the superstring actions [1,5,6]: The generator of gauge symmetry of the first-generation ghosts in these theories is nilpotent on shell.

Consider the classical action of a gauge theory S_0 (φ) invariant under gauge transformations $\delta\varphi^i = R^i_{\mu_1}\xi^{\mu_1}$. If the vectors $R^i_{\mu_1}$ are linearly independent in the stationary point $\left.\frac{\partial_r S_0}{\partial\varphi^i}\right|_{\varphi_0} = 0$, then the theory is described by one generation of ghosts. If the generators $R^i_{\mu_1}$ have some zero modes $Z_1{}^{\mu_1}{}_{\mu_2}$, such that

$$R^i{}_{\mu_1} Z_1{}^{\mu_1}{}_{\mu_2}\Big|_{\varphi_0} = 0, \qquad (1)$$

or, using the formulation of Batalin and Vilkovisky (BV) [7]

$$R^i{}_{\mu_1} Z_1{}^{\mu_1}{}_{\mu_2} - 2\,\frac{\partial_r S_0}{\partial\varphi^j}\, B^{ji}_{\mu_2} (-1)^{\epsilon_i} = 0, \qquad (1')$$

ϵ_i being the Grassmann parity of φ^i, then the first-generation ghosts become gauge fields and the second-generation ghosts appear. If Z_1 has some zero modes $Z_1{}^{\mu_1}{}_{\mu_2} Z_2{}^{\mu_2}{}_{\mu_3}\big|_{\varphi_0} = 0$, the second-generation ghosts become gauge fields, the 3-d generation ghosts appear etc.

Local fermionic symmetry in theories [1-6] has the property that

$$Z_1{}^2\big|_{\varphi_0} = 0\,, \quad Z_1 = Z_2 = Z_3 = \cdots \qquad (2)$$

and this property of the gauge generator of the 1-st generation ghosts means that the sequence of ghosts for ghosts in theories, with the property (2) is infinite, or, in the terminology of [7], the theory is of infinite stage of reducibility.

The origin of eqs.(1,2) in theories [1-6] is the following. We start with the simplest case of Brink-Schwarz superparticle action [2], where the local fermionic symmetry means *)

$$\delta\theta = \not{p}\kappa\,, \quad \delta P_q = 0, \quad \not{p} \equiv \gamma^q P_q \qquad (3)$$

$$\delta x^q = \bar\theta\gamma^q\delta\theta\,, \quad \delta g = 4\bar\kappa\,\partial\theta \qquad (4)$$

It follows from (3), (4) that the gauge generators $R^i{}_{\mu_1}$ have a zero mode $Z_1 = \not{p}$, since

$$\tilde\delta\theta = P^2 C_2\,, \quad \tilde\delta P_q = 0, \quad \tilde\delta x^q = \bar\theta\gamma^q P^2 C_2\,, \quad \tilde\delta g = 4\,\bar C_2\,\not{p}\,\partial\theta \qquad (5)$$

*) We use notations of ref. [6] mainly.

Eqs.(5) define structure functions $B^{ji}_{\mu_2}$ in eq.(1'),

$$B^{g\theta^\alpha}_\beta = \delta^\alpha_\beta\,, \quad B^{g x^q}_\beta = -(\bar\theta\gamma^q)_\beta$$

The on-shell nilpotency of the gauge generator of the first-generation ghosts is now clear,

$$Z_1{}^2 = P^2 = -2\,\frac{\partial_r S_0}{\partial g}\,. \qquad (6)$$

In Siegel's approach [3] one introduces the conjugate momentum d for θ, and a gauge field ψ for fermionic symmetry with the transformation laws

$$\delta d = 0\,, \quad \delta\psi = \partial\kappa \qquad (7)$$

The action

$$S_0 = \int d\tau \left\{ P_m(\partial x^m - \bar\theta\gamma^m\partial\theta) - \tfrac{1}{2} g P^2 + \bar d\,\partial\theta - \bar d\,\not{p}\,\psi \right\} \qquad (8)$$

is invariant under (3), (4), (6). Eq.(7) means that the original fermionic symmetry has no zero mode, since eq.(1) is violated for $\varphi^i = \psi$ ($\partial(\not{p}C_2)$ does not vanish on shell). It can be shown, however, that Siegel's action (8) has a local fermionic symmetry of another type,

$$\delta\psi = \not{p}\eta\,, \quad \delta g = -2\bar d\eta\,. \qquad (9)$$

The generator of this symmetry has a zero mode $Z_1 = \not{p}$, satisfying eq.(6), since eq.(1) holds for all φ^i:

$$\tilde\delta\psi = P^2 C_2\,, \quad \tilde\delta g = -2\bar d\,\not{p}\,C_2\,.$$

The superparticle action with auxiliary fields d, f, closing the algebra of $R^i{}_{\mu_1}$, is given in [4]. It can be easily checked that the original fermionic symmetry, acting on all fields, has a zero mode $Z_1 = \not{p}$ with the on-shell nilpotency condition (6).

The Green-Schwarz N=2 superstring [1] has the local fermionic symmetry

$$\delta\theta^1 = \not{P}^+\kappa_+\,,\quad \delta\theta^2 = \not{P}^-\kappa_-\,,\quad \delta X^m = \bar{\theta}^A\gamma^m\delta\theta^A,\ A=1,2$$

$$\delta\gamma_{++} = 8\bar{\kappa}_+\partial_+\theta^1\,,\qquad \delta\gamma_{--} = 8\bar{\kappa}_-\partial_-\theta^2. \tag{10}$$

The zero modes of self-dual and anti-self-dual parts of the gauge generators are

$$Z_1^+ = \not{P}^+\,,\qquad Z_1^- = \not{P}^-$$

Eqs. defining $B_{\mu_2}^{+i}$ in (1') look as follows:

$$\tilde{\delta}\theta^1 = (P^+)^2 C_{2++}\qquad \tilde{\delta}\theta^2 = (P^-)^2 C_{2--}$$

$$\tilde{\delta}\gamma_{++} = 8\bar{C}_{2++}\not{P}^+\partial_+\theta^1\,,\quad \tilde{\delta}\gamma_{--} = 8\bar{C}_{2--}\not{P}^-\partial_-\theta^2.$$

The on-shell nilpotency eqs., generalizing eq.(6) are

$$(Z_1^+)^2 = (P^+)^2 = -2\frac{\partial S_0}{\partial\gamma_{++}}\,,\quad (Z_1^-)^2 = (P^-)^2 = -2\frac{\partial S_0}{\partial\gamma_{--}} \tag{11}$$

The heterotic string [8] in the Green-Schwarz formulation has the self-dual part of the fermionic symmetry (10) only, and Z_1^+ in eq.(11) is the on-shell nilpotent gauge generator for ghosts.

The most intricate is the form of eqs.(1),(2) in Siegel's formulation of superstrings [5]. Consider the Lagrangian for heterotic string in Green-Schwarz-Siegel formulation, as presented by Romans [6].

$$S_0 = \int d\tau d\sigma\Big\{P_m^\mu(\partial_\mu X^m - \bar{\theta}\gamma^m\partial_\mu\theta) - \varepsilon^{\mu\nu}\partial_\mu X^m\bar{\theta}\gamma^m\partial_\nu\theta - \tfrac{1}{2}\gamma_{\mu\nu}\eta^{mn}P_m^\mu P_n^\nu +$$
$$+\bar{d}^+\partial_+\theta - \bar{d}\not{P}^+\psi_{++} - \tfrac{1}{2}\bar{d}^+\gamma_{mnl}d^+\chi_{++}^{mnl} - \bar{d}^+\gamma_m D^+ d^+\Phi_{+++}^m + L'\Big\}, \tag{12}$$

L' represents gauge degrees of freedom. Four types of gauge symmetries of (12), called ABCD symmetries, are listed in [5,6]. All of them are irreducible. However some other important local fermionic and bosonic symmetries are present in (12). These symmetries are

$$\delta\psi_{\alpha++} = \eta_{\alpha+++}^{\gamma}\,d_\gamma^+\,,\quad \delta\chi_{++}^{[\alpha\beta]} = -2(P^+)^{[\alpha\gamma}\eta_{\gamma+++}^{\beta]}\,,\quad \chi_{++}^{[\alpha\beta]} \equiv \chi_{++}^{mnl}\gamma_{mnl}^{\alpha\beta} \tag{13}$$

The corresponding zero mode is $Z_1^+ = d^+$, the on shell nilpotency condition is

$$Z_1^+ Z_1^+ = d_\alpha^+ d_\beta^+ = -2\frac{\partial S_0}{\partial\chi_{++}^{(\alpha\beta)}} \tag{14}$$

and eqs.(1') take the form

$$\tilde{\delta}\psi_{\alpha++} = C_{2++++}^{\beta}\,d_\alpha^+ d_\beta^+\,,\quad \tilde{\delta}\chi^{[\alpha\beta]} = -2\not{P}^{+[\alpha\gamma}d_\gamma^+\,C_{2++++}^{\beta]}$$

There also exist some other local symmetries of (12), e.g. the one generalizing that of eq.(9) in superparticle case, and a new one of the form

$$\delta\chi_{++}^{mnl} = \bar{d}\gamma^{k[mn}d\,L^{l]k} + \bar{d}\Lambda^{[mnl]} \tag{15}$$

The reason why we have presented here the symmetry (13) is that it leads to nilpotent gauge generators for the corresponding ghosts, and, quite surprisingly, (15) with some part of (13) is an <u>exact</u> linearized symmetry of the superspace Lagrangian of ten-dimensional off-shell supergravity [9]. This last observation means that the investigation of <u>all</u> local symmetries of (12) and the correct quantization of Green-Schwarz-Siegel theory [1,5,6] might be the right way to a manifestly supersymmetric string field theory. The quantization performed in [6] does not take into account the local symmetries of (12), mentioned above. This is the origin of the misunderstanding with counting physical degrees of freedom in [6], see also [10].

Our statement may be formulated in another way, in terms of Siegel's algebra [5], which gives a supersymmetric generalization of Virasoro algebra. The generators $T_{a_1}(\sigma) = \{A = \tfrac{1}{2}P^2 + \Omega^\alpha D_\alpha\,,\ B^\alpha = (\not{P}^+ D)^\alpha\,,\ C_{[\alpha\beta]} = \tfrac{1}{2}D_\alpha D_\beta\,,\ \mathcal{D}_m = i\gamma_m^{\alpha\beta}D_\alpha D'_\beta\}$ are not linearly independent, i.e. they, together with zero modes, form an infinite stage algebra. The reducibility conditions look as follows:

$$T_{a_1}Z_1{}^{a_1}{}_{a_2} = 0\,,\quad Z_1{}^{a_1}{}_{a_2}Z_2{}^{a_2}{}_{a_3}\Big|_{T_{a_1}=0} = 0\,,\qquad \text{etc.}$$

In particular, the abovementioned symmetries have Hamiltonian partners, since

$$\frac{1}{2} B^{\alpha} D_{\beta} + C_{[\beta\gamma]} \mathcal{P}^{+\gamma\alpha} = 0 \tag{13'}$$

$$A D_{\alpha} - \frac{1}{2} \mathcal{P}^{+}_{\alpha\beta} B^{\beta} - 2\Omega^{\beta} C_{[\beta\alpha]} = 0 \tag{9'}$$

$$C_{[\alpha\beta]} Z_1^{[\alpha\beta]} = D\gamma_{mn\ell} D\, \delta \mathcal{J}^{mn\ell} = 0 \tag{15'}$$

Therefore the action (12) [5,6], constructed under the rule

$$\mathcal{L} = p_i \dot{q}^i - H_0(p,q) + \lambda^{a_1} T_{a_1}(p,q)$$

has additional gauge symmetries

$$\delta p_i = \delta q_i = 0 \quad , \quad \delta\lambda^{a_1} = Z_1{}^{a_1}{}_{a_2}(p,q)\, \xi^{a_2},$$

some of which are given in eqs.(13,15). The correct Hamiltonian treatment of the problem must take all these symmetries into account.

2. The main obstacle for quantization of theories with nilpotent generators is the fact that the most general covariant (Lagrangian) quantization of gauge theories, as performed in [7], can be applied for the theories with finite number of ghost generations.

Fortunately the generators R and Z in superstring-like theories [1-6] have some special properties which help us to perform the quantization:

A) The zero mode Z_1 is non-trivial only for the part of the gauge symmetry.*). The corresponding generators $R^i{}_\alpha$ do not contain differential operators (in particular this type of symmetry has no connection-type fields).

B) The nilpotent generator Z_1 does not contain differentional operators.

We will use the BV-formulation [7], extending it for the abovementioned theories with nilpotent generators (1, 2, A, B). In previous examples of quantization of reducible field theories, such as the theory of tensor fields of

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*) Z_1 is non-trivial only for local fermionic symmetry; generators of reparametrization symmetry have no zero modes.

high rank, the classical Lagrangian "knows" how many generations of ghosts are necessary for consistent covariant quantization. Information about this number is contained in properties of $R^i{}_{\mu_1}$, $Z_1{}^{\mu_1}{}_{\mu_2}$ etc. The BV-formulation for the case of some finite number of ghost generations describes the theory by the functional integral

$$\mathcal{Z} = \int \exp \frac{i}{\hbar} S(z)\, \delta(\mathcal{F}_A(z))\, J^{1/2} \prod_{\mathcal{A}} dz^{\mathcal{A}}, \tag{16}$$

where $z^{\mathcal{A}} = (\Phi^A, \Phi^*_A)$, A=1,...,N, $\mathcal{A}$ =1,...,2N (we neglect local measure of integration $\sim \delta^n(0)$, n=2 for strings). Φ^A include the minimal set Φ^A_{min}, consisting of classical fields φ^i and ghosts of all generations C_s, s=1,...,m. Φ^A includes also a set of s fields for every ghost of the same statistics - the antighost $\hat{C}_s$ and (s-1) extraghosts $C_s^{(1)}, \ldots, C_s^{(s-1)}$. Besides, for every $\hat{C}_s$, $C_s^{(1)} \ldots C_s^{(s-1)}$ the fields π_s, $\pi_s^{(1)}$ ... $\pi_s^{(s-1)}$ of opposite statistics are added (the Nakanishi-Latrup fields NL). To each field Φ^A an antifield Φ^*_A with opposite statistics is adjoined.

$S(\Phi, \Phi^*)$ satisfies the master equation

$$(S, S) = 0 \tag{17}$$

where the antibracket (,) is defined as follows:

$$(X, Y) = \frac{\partial_r X}{\partial \Phi^A} \frac{\partial_\ell Y}{\partial \Phi^*_A} - \frac{\partial_r X}{\partial \Phi^*_A} \frac{\partial_\ell Y}{\partial \Phi^A} \tag{18}$$

The function $\mathcal{F}_A(\Phi, \Phi^*)$ defining the class of surfaces in the phase space, satisfies equation

$$(\mathcal{F}_A, \mathcal{F}_B) = 0 \quad . \tag{19}$$

One can choose S and $\mathcal{F}_A$ in the form

$$S(\Phi, \Phi^*) = S_0 + \sum_{n=1}^{\infty} \Phi^*_{A_n} \cdots \Phi^*_{A_1} S^{A_1 \ldots A_n}(\Phi) \tag{20}$$

$$\mathcal{F}_A = \Phi^*_A - \frac{\partial \Psi}{\partial \Phi_A} \tag{21}$$

In this case eq.(19) is satisfied, the coefficients in (20) can be obtained from eq.(17), and the Jacobian of canonical transformations J equals 1. As it follows from (16), the maximal freedom in quantization of gauge theories with m generations of ghosts is connected with the possibility to perform canonical transformation of variables in (16) and, in particular, to choose the function Ψ defining the gauge condition in eq.(21).

We will show that the quantum theories of superparticles and superstrings [1-6] may be defined by the functional integral of the type (16) where, however, the number of fields z^{ab} (or the number of ghost generations) is arbitrary.

3. The procedure of quantization which we suggest consists of the following steps:

a) Impose some arbitrary algebraic constraints on the m-th generation ghosts of the type *)

$$\sigma^{a}{}_{\alpha} C^{\alpha}{}_{m} = 0 , \quad C_m{}^{\alpha} = \tilde{\sigma}^{\alpha}{}_{a} C^{a}{}_{m} , \quad \sigma^{q}{}_{\alpha} \tilde{\sigma}^{\alpha}{}_{b} = 0 \quad (22)$$

Eqs.(22) mean that ghosts of m-th generation are no longer gauge fields, i.e. the theory is reduced to the gauge theory with a finite (but arbitrary) number of ghost generations.

b) Use BV formulation [7] for this theory. We will show that the theory defined above i) is equivalent to the one quantized in the unitary gauge and ii) it does not depend on the number of generations of ghosts and on the way of truncation, i.e. on quantities $\sigma^{a}{}_{\alpha}$ (up to the terms $\sim \delta^{n}(0)$). Note that the usual gauge freedom in a truncated theory is present in accordance with [7].

To prove these statements we first impose constraints (22) on the 1-st generation ghosts, i.e. we take m=1 in (22). Then the quantization in accordance

*) For superparticles and superstrings $\alpha = 1,\dots,16$, $a = 1,\dots,8$. The possibility that σ^{a}_{α} represent 8 auxiliary 16-component spinors deserves further study.

with abovementioned rules means that the terms

$$\chi^{a}(\varphi^{i})\pi_{a} + \hat{C}_{a} \frac{\partial \chi^{a}}{\partial \varphi^{i}} R^{i}{}_{\alpha} \tilde{\sigma}^{\alpha}{}_{b} C^{b} \quad (23)$$

appear in quantum action. If the gauge function $\frac{\partial \chi^{a}}{\partial \varphi^{i}}$ does not contain differential operators (and $R^{i}{}_{\alpha}$ do not contain them according to A)), then it is clear that the introduction of ghosts is unnecessary. They are not propagating, and the integration over $\hat{c}$, c leads only to the change in the local measure of integration, which is not defined in the Lagrangian quantization anyway. Such a gauge is unitary. The dependence on $\tilde{\sigma}^{\alpha}{}_{b}$ also leads only to a modification of the local measure. However, if $\frac{\partial \chi^{a}}{\partial \varphi^{i}}$ does contain differential operators, the 1-st generation ghosts get kinetic terms and become propagating.

The second step in the proof is to show that the procedure of quantization suggested above leads to equivalent (up to the local measure) theories for k and $k-1$ generations of ghosts (see also Figs. 1 and 2). The conditions A), B) and a special form of the gauge condition ψ_k allow us to perform the integration in (16) over all fields of k-th generation. There are three kinds of integrations: i) the integration over $\hat{C}_k$, C_k gives a contribution to terms $\sim \delta^{n}(0)$, ii) the integration over k NL fields leads to constraints on k fields of ghosts, antighosts and extraghosts of (k-1)-th generation, i.e. on C_{k-1}, $\hat{C}_{k-1}$, $C^{(1)}_{k-1}, \dots, C^{(k-2)}_{k-1}$, iii) the integration over (k-1) extraghosts of k-th generation leads to constraints on (k-1) NL fields of (k-1)-th generation. Thus, up to the local measure, we are lead to the description of the theory as the theory with (k-1) generations of ghosts, in which all gauge conditions with the exception of $\sigma^{a}{}_{\alpha} C^{\alpha}_{k-1} = 0$ can be changed in a standard way by changing the function Ψ_{k-1}. There is no dependence on the way of truncation, i.e. on $\sigma^{a}{}_{\alpha}$, up to terms $\sim \delta^{n}(0)$.

It is rather interesting how the BV completeness condition (which guarantees the consistency of quantization and the correct counting of degrees of freedom) works in our case. The corresponding equation [7]

$$\operatorname{rank}_{\pm} \left. \frac{\partial_l \partial_r S_0}{\partial \varphi^i \partial \varphi^j} \right|_{\varphi_0} = n_{\pm} - \left\{ m_1 - \left[m_2 - \ldots - (m_{k+1} - m_k) \right] \right\} \qquad (24)$$

for the local fermionic symmetry discussed above in our case looks like

$$8 = 16 - \left\{ 16 - \left[16 - \ldots - (16 - 8) \right] \right\}. \qquad (25)$$

Eq.(25) shows the origin of 8 physical degrees of freedom of θ^{α} when the quantization is performed with arbitrary number of ghost generations, the constraints (22) being imposed only on the last ghost.

It is instructive to consider the general proof given above in more details for the case k=3 → k=2, i.e. consider eq.(22) for m=3. We choose the gauge condition Ψ_3 as follows:

$$\Psi_3 = \Psi_2 + \hat{C}_{3a}\, \omega^{a}{}_{\alpha}\, C_1^{\alpha} + \hat{C}_{2\alpha}\, \eta^{\alpha}{}_{a}\, C_3^{(1)a} + \hat{C}_{3\,a}^{(2)}\, \xi^{a}{}_{\alpha}\, C_2^{(1)\alpha},$$

Ψ_2 depends only on 1-st and 2-nd generation fields, ω, η, ξ do not contain differential operators. Dependence on $\hat{C}_{3a}$, C_3^{a} is of the form

$$\hat{C}_{3a}\, \omega^{a}{}_{\alpha}\, Z^{\alpha}{}_{\beta}\, \tilde{G}^{\beta}{}_{b}\, C_3^{b}$$

Due to the property B) of Z and due to our choice of $\omega^{a}{}_{\alpha}$, the integration over $\hat{C}_3$, C_3 gives the contribution to the local measure only. Integration over π_{3a}, $\pi_3^{(1)a}$, $\pi_{3\,a}^{(2)}$ gives $\omega^{a}{}_{\alpha} C_2^{\alpha} = \hat{C}_{2\alpha} \eta^{\alpha}{}_{a} = \xi^{a}{}_{\alpha} C_2^{(1)\alpha} = 0$, the one over $C_3^{(1)a}$, $C_{3\,a}^{(2)}$ gives $\xi^{a}{}_{\alpha} \pi_2^{(1)\alpha} = \pi_{2\alpha} \eta^{\alpha}{}_{a} = 0$, i.e. the constraints which must be satisfied by the second generation fields when this generation is the last one.

Thus, the problem of quantization of theories with nilpotent gauge generators with properties (1), (2), A), B) is solved.

4. As an example, we will perform here quantization of heterotic string theory [8] in Green-Schwarz formalism. The corresponding action is

$$S_0 = \int d\tau d\sigma \left\{ \tfrac{1}{2} \gamma^{\mu\nu} (\partial_\mu X^m - \bar{\theta} \gamma^m \partial_\mu \theta)(\partial_\nu X^n - \bar{\theta} \gamma^n \partial_\nu \theta)\, \eta_{mn} - \right.$$
$$\left. - \varepsilon^{\mu\nu} \partial_\mu X^m \bar{\theta} \gamma_m \partial_\nu \theta + \bar{\psi} \tau^- \partial_- \psi \right\}. \qquad (26)$$

For this theory the solution of master eq.(17) in the minimal sector $\Phi_{min} = \{ X^m, \theta^{\alpha}, \gamma^{\mu\nu}, C_1 = C_+, C_2 = C_{++}, \ldots C_n = C_{(n)+}, \ldots \}$ can be found before truncating the number of ghost generations. The existence of this solution means that we know the gauge algebra of infinite stage theory. The highest terms in Φ^*_{min} in (20) are bilinear in Φ^*_{min}, as in supergravity theories,

$$S(\Phi_{min}, \Phi^*_{min}) = S_0(\varphi) + \Phi^*_A S^A(\Phi) + \Phi^*_A \Phi^*_B S^{BA}(\Phi) =$$
$$= S_0 + \Phi^*_A \delta_{g.c.} \Phi^A(\Phi^A, \xi^A) - \left(\gamma^{*++} + \tfrac{1}{2} \tau^{*+} \tau^+\right) 8\, \partial_+ \bar{\theta} C_+ + \qquad (27)$$
$$+ 4 \bar{C}^{*+} C_+ \bar{C}_+ \partial^+ \theta - \left(\bar{C}^{*+} \gamma_m \partial^+ \theta - 2 \xi^{*+} P^+_m + X^*_m \gamma^{*++} \right) \bar{C}_+ \gamma^m C_+ +$$
$$+ \sum_{k=0}^{\infty} \left(\bar{C}^{*(k)+} \not{\partial}^+ C_{(k+1)+} + \gamma^{*++} \bar{C}^{*(k)+} C_{(k+2)+} \right) + 8 \xi^{*+} \gamma^{*++} \bar{C}_+ C_{++}.$$

where

$$P^m_\mu = \partial_\mu X^m - \bar{\theta} \gamma^m \partial_\mu \theta, \quad C^{*(0)} = \theta^* + X^*_m \theta \gamma^m \qquad (28)$$

The first term in (27) after S_0 is not presented here explicitely. It gives the world-sheet general coordinate transformations. θ and X^m transform as scalars, the metric tensor $\gamma_{\mu\nu} = \sqrt{g}\, g_{\mu\nu}$ transforms as a tensor of weight -1, the fermionic symmetry ghosts of n-th generation $C_{(n)+}$ transform as n-th rank self-dual tensors $C_{+\ldots+}$, ξ^{μ} being the general coordinate ghost. The deviation of the theory from the closed algebra case (i.e. from the linear dependence on Φ^* in (20)) is very soft: The terms $X^*_m \gamma^{*++} \bar{C}_+ \gamma^m C_+$ express the nonclosure of the algebra of classical gauge generators on X^m and γ_{++},

$$\frac{\partial_r R^i_{\mu_1} C^{\mu_1}_1}{\partial \varphi^j} R^j_{\nu_1} C^{\nu_1}_1 + R^i_{\mu_1} T^{\mu_1}_{\nu_1 \lambda_1} C^{\lambda_1}_1 C^{\nu_1}_1 - 2 \frac{\partial_r S_0}{\partial \varphi^j} E^{ji}_{\mu_1 \nu_1} C^{\nu_1}_1 C^{\mu_1}_1 (-1)^{\varepsilon_i} = 0$$
$$S^{ji} = E^{ji}_{\mu_1 \nu_1} C^{\nu_1}_1 C^{\mu_1}_1 \qquad S^{BA} = \left\{ S^{\gamma_{++} X^m} = \bar{C}_+ \gamma^m C_+ \right\} \qquad (29)$$

The term $\bar{C}^{*(0)} \gamma^{*++} C_{++}$ shows that classical gauge generators have zero modes only on shell, see eq.(1'),

$$S^{ji} = B^{ji}_{\mu_2} C_2^{\mu_2} \qquad S^{BA} = \{ S^{\theta \gamma_{++}} = C_{++} ,\ S^{x^m \gamma_{++}} = \bar{\theta} \gamma^m C_{++} \} \tag{30}$$

Besides, the new structure function $G^{j\mu_1}_{\nu_2 \nu_1}$ is generated by master equation (17), leading to term $8 \xi^* \gamma^{*++} \bar{C}_+ C_{++}$:

$$2 T^{\mu_1}_{\nu_1 \lambda_1} C_1^{\lambda_1} Z_1{}^{\nu_1}{}_{\nu_2} C_2^{\nu_2} + 2 \frac{\partial_r S_0}{\partial \varphi^j} G^{j\mu_1}_{\nu_2 \nu_1} C_1^{\nu_1} C_2^{\nu_2} (-1)^{\varepsilon_{\mu_1}} = 0 \tag{31}$$

$$S^{j\mu_1} = 2 G^{j\mu_1}_{\nu_2 \nu_1} C_1^{\nu_1} C_1^{\nu_2} \qquad S^{BA} = \{ S^{\gamma_{++} \xi_+} = 8 \bar{C}_+ C_{++} \} \tag{32}$$

Eqs.(1') and (31) were presented explicitely in [7]. The superstring theory (26) is the first non-trivial realization of these equations. The most unusual terms bilinear in Φ^* in (27) are $\sum_{k=1}^{\infty} \gamma^{*++} \bar{C}^{*(k)+} C_{(k+2)+}$. These terms show that the nilpotency condition holds only on shell,

$$Z_1^{k+2} C_{(k+2)+} + 2 \frac{\partial_r S_0}{\partial \gamma_{++}} C_{(k+2)+} = 0 \tag{33}$$

$$S^{BA} = \{ S^{C_{(k)+} \gamma_{++}} = C_{(k+2)+} \} \tag{34}$$

In fact, the solution of master equation for infinite number of ghosts (27) is not necessary for the quantization. Note also that before truncation (22), the completeness condition (24) is violated 8 ≠ 16 − {16 − ... . The correct procedure of quantization forces us to limit ourselves with some finite number of ghosts, the last one being constrained according to (22) so that the completeness condition (24), (25) becomes correct, and then find $S(\Phi_{min}, \Phi^*_{min})$ just for this limited number of fields. However, instead of solving the master equation for $S(\Phi_{min}, \Phi^*_{min})$ each time anew for each number of ghost generations, one may obtain a correct answer just by truncating the solution (27) for infinite number of ghosts. Besides, solution (27) gives us the possibility to analyze the general properties of quantization connected with the nonclosure of the algebra.

The first type of nonclosure terms (29,32) which lead in principle to the 4-ghost coupling, is easily eliminated in gauges where there is no dependence of Ψ

on x^m, ξ_+, since for such gauges $X^*_m = \xi^{*+} = 0$. The second type of nonclosure terms (30) and (34) leads to the 3-ghost coupling, when the number of ghosts generations is not less than 2.

Now we will use the simplest possibility to perform the quantization of (26) without violating global supersymmetry. We will impose on the 1-st generation ghosts the constraints (22) in the form

$$\not{n} C_1 = 0 \qquad C_1 = \not{n} \not{m} C_1$$

where

$$\not{n} = n_k \gamma^k , \quad \not{m} = m_k \gamma^k , \quad n^2 = 0 , \quad m^2 = 0 , \quad m \cdot n = \frac{1}{2}$$

Now the theory is quantized as an irreducible theory with one generation of ghosts [7]. We take the gauge fermion Ψ_1 in the form

$$\Psi_1 = \hat{\xi}^{\rho\sigma} P^{\mu\nu}_{\rho\sigma} \gamma_{\mu\nu} + \hat{C}_1^+ \not{m} \not{n} \partial_+ \theta ,$$

where

$$P^{\mu\nu}_{\rho\sigma} = \frac{1}{2} (\delta^\mu_\rho \delta^\nu_\sigma + \delta^\nu_\rho \delta^\mu_\sigma - \eta^{\mu\nu} \eta_{\rho\sigma})$$

For $\Phi_{min} = \{ X_k , \theta^\alpha , \gamma_{\mu\nu} , \not{n} \not{m} C_+ \}$ we get

$$S(\Phi, \Phi^*) = S(\Phi_{min}, \Phi^*_{min}) + \hat{\xi}^*_{\mu\nu} P^{\mu\nu}_{\rho\sigma} \pi^{\rho\sigma} + \hat{\bar{C}}^*_+ \not{n} \not{m} \pi^+ , \tag{35}$$

where $S(\Phi_{min}, \Phi^*_{min}) = S_0(\varphi) + \Phi^*_A \delta_{g.c.} \Phi^A(\Phi^A, \xi^\alpha) -$

$$-(\gamma^{*++} + \tfrac{1}{2} \tau^{*+} \tau^+) 8 \partial_+ \bar{\theta} \not{n} \not{m} C_+ + 4 \bar{C}^{*+} \not{n} \not{m} C_+ \bar{C}_+ \not{m} \not{n} \partial^+ \theta - \tag{36}$$

$$-(\bar{C}^*_+ \gamma_k \partial^+ \theta - 2 \xi^{*+} P^+_k + X^*_k \gamma^{*++}) \bar{C}_+ \not{m} \not{n} \gamma^k \not{n} \not{m} C_+ + (\bar{\theta}^* + X^*_k \bar{\theta} \gamma^k) \not{p}^+ \not{n} \not{m} C_+ .$$

Taking into account that $\Phi^*_A = \frac{\partial \Psi_1}{\partial \Phi_A}$ leads to

$$X^*_k = C^*_+ = \xi^*_\mu = 0 , \qquad \theta^* = \hat{C}_1^+ \not{m} \not{n} \partial_+ ,$$

$$\gamma^{*\mu\nu} = \hat{\xi}^{\rho\sigma} P^{\mu\nu}_{\rho\sigma} , \quad \hat{\xi}^*_{\mu\nu} = P^{\rho\sigma}_{\mu\nu} \gamma_{\rho\sigma} , \quad \hat{C}^*_+ = \not{m} \not{n} \partial_+ \theta$$

we get the quantum action

$$S(\Phi,\Phi^*)\Big|_{\Phi^*=\frac{\partial\Psi_1}{\partial\Phi}} = S_0 + \gamma_{\mu\nu} P^{\mu\nu}_{\rho\sigma}\pi^{\rho\sigma} + \partial_+\bar\theta \not{n}\not{m}\,\pi^+ + \tag{37}$$
$$+\hat\xi^{\rho\sigma}P^{\mu\nu}_{\rho\sigma}(\nabla_\mu\xi_\nu - \gamma_{\mu\nu}\nabla_\rho\xi^\rho) - \hat\xi^{++}8\partial_+\bar\theta\not{n}\not{m}\,c_+ +$$
$$+\hat{c}^+\not{m}\not{n}\,\partial_+(\partial_\mu\theta\,\xi^\mu) + \not{p}^+\not{n}\not{m}\,c_+\}.$$

After integration over $\pi^{\rho\sigma}$ and the use of the condition $\gamma_{\mu\nu}=\eta_{\mu\nu}$ we get

$$S = S_0 + \partial_+\bar\theta\not{n}\not{m}\,\tilde\pi^+ + \hat\xi^{--}\partial_-\xi_- + \tilde{\hat\xi}^{++}\partial_+\xi_+ - \tag{38}$$
$$-\partial_+\hat{c}^+\not{m}\not{n}\,\not{p}^+\not{n}\not{m}\,c_+ ,$$

where

$$\tilde\pi^+ = \pi^+ + 8\hat\xi^{++}c_+ - \partial_\mu(\hat{c}^+\xi^\mu) + \hat{c}^+\partial_+\xi_-$$

$$\tilde{\hat\xi}^{++} = \hat\xi^{++} + \hat{c}^+\not{m}\not{n}\,\partial^+\theta .$$

It is important that this action has global supersymmetry $\delta\theta=\varepsilon$, $\delta X^k = \bar\varepsilon\gamma^k\theta$. Its BRST invariance follows from the fact that $S(\Phi,\Phi^*)\big|_{\Phi^*=\frac{\partial\Psi}{\partial\Phi}}$ in (35-37) is invariant under the transformations

$$\delta\Phi^A = (\Phi^A, S)\Lambda\Big|_{\Phi^*=\frac{\partial\Psi}{\partial\Phi}} = \frac{\partial_r S(\Phi,\Phi^*)}{\partial\Phi^*}\Big|_{\Phi^*=\frac{\partial\Psi}{\partial\Phi}} \tag{39}$$

where Λ is a constant fermi-parameter [7].

Equations of motion following from the action (38) are

$$\partial_+\partial_- X^k = \partial_-\xi_- = \partial_+\xi_+ = \partial_-\hat\xi^{--} = \partial_+\tilde{\hat\xi}^{++} = 0 \tag{40}$$

$$\partial_+\theta = \not{n}\not{m}\,\partial_+\tilde\pi^+ = \not{n}\not{m}\,\partial_+\hat{c}^+ = \not{n}\not{m}\,\partial_+ c_+ = 0 \tag{41}$$

$$\gamma^-\partial_-\psi = 0 \tag{42}$$

When deriving eqs.(40), (41) we have taken into account that the matrics $\not{n}\not{p}^+\not{n}$ are non-degenerate. Eqs.(40) are simply equations of the covariantly quantized bosonic string, including ghosts and antighosts for reparametrization invariance. Eqs.(41) give (instead of 8 physical degrees of freedom of θ in the unitary

gauge) 16 θ^α, 2·8 commuting F-P ghosts and 8 anticommuting N-K ghosts *), i.e. 8 = 16 - 8 - 8 + 8. Eq.(42) defines the gauge degrees of freedom of the heterotic string.

Many other possibilities to quantize the action (26) are now open, in particular BRST as well as anti-BRST invariant gauges may be found.

Thus we have proposed a method of first quantization of superstrings in arbitrary gauges. Further investigations are necessary to find the most suitable gauges and to check the status of anomalies in the BRST quantization. The consistent first quantization of superstrings is necessary for the second quantization of this theory as well as for development of Polyakov's method of integration over surfaces with the purpose of analysis of 1-loop string diagrams and their divergences in a manifestly supersymmetric way.

We have already discussed the quantization problem for superstring theories in connection with the properties of gauge generators in [12].

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*) The relation between NL fields (Lagrange multiplyers) and N-K ghosts in the BRST-construction of irreducible theories is clarified in [11].

REFERENCES

[1] M.B. Green and J.H. Schwarz, Phys. Lett. 136B, (1984) 367; Nucl. Phys. B243, (1984) 285.

[2] L. Brink and J.H. Schwarz, Phys. Lett. 100B, (1981) 310.

[3] W. Siegel, Class. Quantum Grav. 2, (1985) L 95.

[4] P.K. Townsend, in "Superunification and Extra Dimensions", Ed. R. D'Auria and P. Fré (World Scientific, Singapore, 1986).

[5] W. Siegel, Nucl. Phys. B263, (1985) 93.

[6] L.J. Romans, Nucl. Phys. B281, (1987) 639.

[7] I.A. Batalin and G.A. Vilkovisky, Phys. Rev. D28, (1983) 2567.

[8] D.J. Gross, J.A. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 54, (1985) 502; Nucl. Phys. B256, (1985) 253.

[9] P. Howe, H. Nicolai and A. van Proeyen, Phys. Lett. 112B, (1982) 446.

[10] Th.J. Allen, Preprint CALT-68-1373, 1986; S. Randjbar-Daemi, Abdus Salam and J.A. Strathdee, Mod. Phys. Lett. A (1987) 145.

[11] I.A. Batalin and R.E. Kallosh, Nucl. Phys. B222, (1983) 139.

[12] R.E. Kallosh, Pis'ma JETPh, 45, (1987) 365.

Fig. 1. All fields of (m-1) generations ghosts theory. In all horisontal lines except the last one the fields π, C have $\alpha = 1,\ldots,16$ components, and in the last line only $a = 1,\ldots,8$ components.

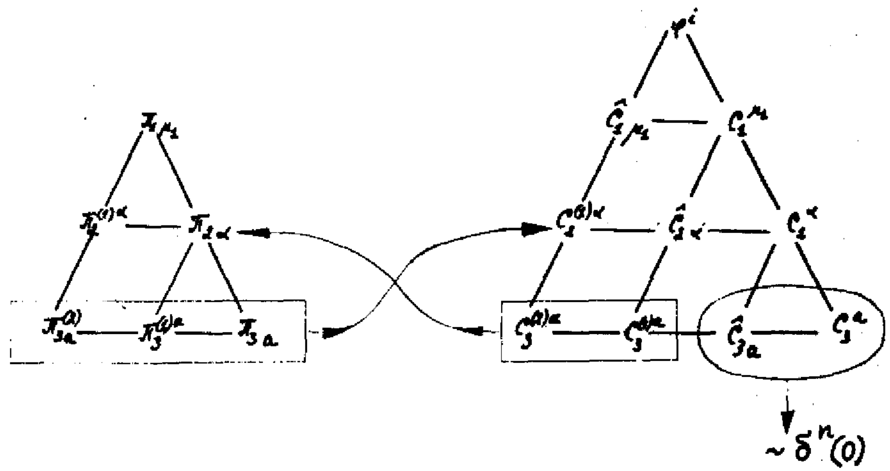

Fig. 2. All fields of m generations ghosts theory. The integration over 8-th component fields of the last horisontal line leads to constraints on all fields of the previous line and they become 8-th-component instead of 16-th-component, i.e. the Fig.1 is reproduced.