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Abstract: The production probability of π-mesonic atom in high-energy nuclear collisions is estimated by a coalescence model. The production cross section is calculated for p+Ne and Ne+Ne systems at 2.1 GeV/A and 5.0 GeV/A beam energy. It is shown that nuclear fragments with larger charge numbers have the advantage in the formation of π-mesonic atoms. The cross section is proportional to Z^3 and of the order of magnitude of 1~10 μb in all the above cases. The production cross sections of K-mesonic atoms are also estimated.

1. Introduction

In high energy nuclear collisions, many kinds of particles and nuclide are produced. Previously we used a coalescence model[1,2] to estimate the hypernucleus production probability in high energy nuclear collisions and got the conclusion that hypernuclei could be produced with fairly large cross sections, of the order of magnitude of μb in p+Ne and Ne+Ne collisions at 2-5 GeV/A[3].

As for the branching ratios of the possible elementary processes, those of pion-production q_π are much larger than that of Λ-particle, q_Λ, that is, $q_\pi/q_\Lambda \sim 10^2$ at 5 MeV/A.

This fact suggests us the unnegligible production probability of π-mesonic atoms in high energy nuclear collisions. The purpose of this paper is to estimate the π-mesonic atom production probability in nuclear collisions by using the coalescence model. Numerical calculations are performed in the cases of p+Ne, Ne+Ne at 2.1 and 5 GeV/A. The model is also extended to the case of K-mesonic atom production.

2. Production cross section of π-mesonic atom

The details of the calculation method are described in the previous papers[2,3] and only a summary is given here.

According to the coalescence model[1,2], the production cross section of π-mesonic atom is expressed by the product of the π^- meson and nuclear fragment production cross sections as follows:

$$\frac{\gamma}{\sigma_r}\,\frac{d^3\sigma^{(\pi F)}}{dk_c^3} = \left(\frac{m_\pi+m_F}{m_\pi m_F}\right)^3 S_{\pi F}\cdot\left(\frac{\gamma}{\sigma_r}\cdot\frac{d^3\sigma^{(\pi)}}{dk_c^3}\right)\cdot\left(\frac{\gamma}{\sigma_r}\cdot\frac{d^3\sigma^{(F)}}{dk_c^3}\right), \quad (1)$$

where k_c is the momentum per particle, $m_\pi = M_\pi/M_n$, $m_F = M_F/M_n$, $\gamma = [1+(k_c/M_n)^2]^{1/2}$, M_n the nucleon mass, M_π the pion mass, M_F the fragment mass, and σ_r is the reaction cross section. $S_{\pi F}$ is the coalescence factor of a pion and a fragment, which is given as the overlap between the relative wave function $\psi(r)$ of a pion bound with a nuclear fragment F and the spatial distribution of the source matter $D^{(i)}(r)$ $(i=\pi,F)$,

$$S_{\pi F} = W(\pi F)(2\pi)^3\int dr|\psi(r)|^2\xi_{\pi F}(r) \ , \qquad (2)$$

where

$$\xi_{\pi F}(r) = \int dr' D^{(\pi)}(r+r')D^{(F)}(r') \ .$$

If sufficient data of the pion and F-fragment production cross sections are available, we can calculate the right-hand side of Eq.(1) with theoretically given values of $S_{\pi F}$.

In this paper, we estimate those values using models which can reproduce existing data well.

3. Coalescence factor

In a π-mesonic atom, a negative pion is bound with a fragment nucleus F through the Coulomb interaction.

We must solve a relativistic Schrödinger equation to obtain the wave function $\psi(r)$ in Eq.(2), but we approximate it with the wave functions of a Hydrogen-like atom, $\psi^H_{nlm}(r\theta\phi)$.

The spatial extent of the source matter $D^{(i)}(r)$ is assumed to be of a gaussian form as,

$$D^{(i)}(r) = (\sqrt{\pi}\beta_i)^{-3}\exp[-(r/\beta_i)^2] \quad (i = \pi \text{ or } F) \ . \qquad (3)$$

Then, the expression of $\xi_{\pi F}(r)$ in Eq.(2) are given as,

$$\xi_{\pi F}(r) = (\sqrt{\pi}\beta_{\pi F})^{-3}\exp[-(r/\beta_{\pi F})^2] , \tag{4}$$

where $\beta_{\pi F} = (\beta_\pi{}^2+\beta_F{}^2)^{1/2}$.

As wave functions of highly excited states of a Hydrogen-like atom are much pushed-out outside of the range of $\xi_{\pi F}(r)$, only the following ground and first excited states (n=1,2) are taken into account,

$$\psi^H_{100} = 2(\frac{Z}{a_\pi})^{3/2}\exp(-\frac{Z}{a_\pi}r)\cdot Y_{00}(\theta) ,$$

$$\psi^H_{200} = \frac{1}{2\sqrt{2}}(\frac{Z}{a_\pi})^{3/2}(2-\frac{Z}{a_\pi})\exp(-\frac{Z}{a_\pi}r)\cdot Y_{00}(\theta) , \tag{5}$$

$$\psi^H_{21m} = \frac{1}{2\sqrt{6}}(\frac{Z}{a_\pi})^{3/2}\cdot(\frac{Z}{a_\pi}r)\exp(-\frac{Z}{2a_\pi}r)\cdot Y_{1m}(\theta,\phi) ,$$

where Z and a_π are the charge of the fragment and the Bohr radius of a pion bound in the fragment. The values of $S_{\pi F}$'s are numerically calculated. It is, however, useful to make an approximate estimation of the main part to see the gross feature of the coalescence factor.

When the range of $\xi_{\pi F}$, $\beta_{\pi F}$, is smaller than that of wave functions, a_π/Z, the contributions of 1s- and 2s-states are approximated as

$$S_{\pi F} \cong (2\pi)^3(|\psi^H_{100}(0)|^2+|\psi^H_{200}(0)|^2)\int dr\xi_{\pi F}(r)$$
$$= 9\pi^2(\frac{Z}{a_\pi})^3 . \tag{6}$$

Grossly saying, the coalescence factor shows a cubic dependence on the charge of a fragment Z. Therefore nuclear fragments with larger charge numbers have the advantage in the formation of mesonic atoms.

The coalescence factor for a K^--atom, S_{KF}, can be evaluated by the substitution of a_π and $\beta_{\pi F}$ with a_K and β_{KF}.

4. Inclusive Cross Section of π^- Particles

Inclusive cross section of negative pions are evaluated by the statistical phase space model[4,5].

This model gives the inclusive cross section as a sum of contributions of sub-processes in which M nucleons in the projectile A interact with N nucleons in the target B;

$$E\frac{d^3\sigma(\pi)}{dp^3} = \sum_{MN}\sigma_{AB}(M,N)\,\mathcal{G}_{MN}(p) \quad . \tag{7}$$

Here $\sigma_{AB}(M,N)$ specifies the cross section for the sub-process and $\mathcal{G}_{MN}(p)$ denotes the momentum distribution of the particle emitted in the subprocess. The momentum distribution is given by the relative value of related phase space volume and branching ratios of particle production processes. As the possible type of particle production processes we consider only that of the π production,

$$N + N \rightarrow N + N + \alpha\pi \qquad (\alpha = 0,1,2,3) \quad . \tag{8}$$

Other processes, such as

$$N + N \rightarrow N + Y + K + \alpha\pi \qquad (Y = \Lambda \text{ or } \Sigma) \; , \tag{9}$$

are ignored in evaluating the pion production cross reaction, because inclusion of such processes gives only a small perturbative effect on the π-production cross section. The effect of the nuclear Fermi motion on the $\mathcal{G}_{MN}(p)$ is taken into account by the procedure of Ref.4. In evaluating the K^--particle production cross section, we also take into account the following process,

$$N + N \to N + N + K + \bar{K} . \tag{10}$$

The calculated π^- cross sections in the case of Ne+Ne collision at 2.1 GeV/A are shown in Fig.1a and compared with the experiment[6]. The theoretical calculation can reproduce the slope of the experimental curve, but the absolute values of the cross section are three-times larger than the experimental values. We then used theoretical values of cross sections multiplied by the factor 1/3 as π^- production cross section in Eq.(1). Fig.1b shows the experimental K^- cross section for 0° angle in the $^{28}Si+^{28}Si$ system at 2.1 GeV/A[7] and the calculated cross section. There is overall agreement with the experimental data.

5. Nuclear fragment cross section

Nuclear fragments are produced mainly at peripheral collisions, whose characteristic feature appears in the factorization of production cross sections into two parts, isotope production ratios and momentum distributions of fragments[8].

The projectile breaks up into fragments through various channels and threshold Q-values $Q_{gg}^{(F)}$ of the break-up channels are related to the isotopes yields by the formula $\exp(Q_{gg}^{(F)}/T)$. Here T is the temperature of the projectile exited by the collision. In the projectile rest frame (or in the target rest frame for target fragmentation), the momentum distribution of a fragment has a form of a gaussian type, $\exp(-p^2/2\mu_F T)$ in the statistical model, where μ_F is the reduced mass of the fragment, $M_F(M_P-M_F)/M_P$, and M_P is the projectile mass.

Then, in the rest frame of the projectile (or in the target rest frame) the cross section is given in the nonrelativistic approximation by

$$\frac{d^3\sigma^{(F)}}{dp^3} = \frac{\sigma_c}{4\pi}\cdot p \exp\left(\frac{Q_{gg}^{(F)}}{T} - \frac{p^2}{2\mu_F T} \right)/\left[2\mu_F T^2 \sum_i \mu_i \exp\left(\frac{Q_{gg}^{(i)}}{T}\right)\right] , \qquad (11)$$

where $\sigma_c = \pi r_0^2(A_P^{1/3}+A_T^{1/3}-\delta)$ with $\delta = 1.6$.

6. Results and Discussion

For the numerical calculations, the sets of parameters are taken to be same as used in Ref.3. We took the value of $\beta_{\pi F} = 5.39$ fm ($\beta_\pi = \beta_F = 3.81$ fm)[3].

Calculated results of π-mesonic atom production cross sections are displayed in Figs. 2 and 3 for p+Ne and Ne+Ne collisions at 2.1 GeV/A and 5 GeV/A. The production cross sections show a gradual increase with the increase of the atomic number Z as suggested by the dependence on Z approximately given in Eq.(6).

It is very interesting to compare the above results with the production cross sections of hypernuclei calculated also with the coalescence model in Ref.3. For example, the nuclear fragment ^{16}O, which has large isotope production ratio, coalesces with Λ- and π^--particles to form $^{17}_{\Lambda}O$ and π-^{16}O with cross sections $\sigma_{\Lambda F}$ = 2.5 μb and $\sigma_{\pi F}$ = 65 μb in the Ne+Ne collisions at 5 GeV/A. The ratio, $\sigma_{\pi F}/\sigma_{\Lambda F}$, is greater than 10. Checking other cases, we see that the formation probability of π-mesonic atoms are, in average, ten times as large as those of hypernuclei.

The production cross sections of K^--atoms are plotted in Fig.4. The cross sections are about 10^3~10^4 factor smaller than the π^--atom production cross sections, reflecting the difference of the magnitude of the π^- and K^- production cross sections.

In the above treatment we ignored the effects of the strong interaction of the meson with the nucleus and the time dependence of the coalescence process.

The strong interaction of the mseon with the nucleus is usually represented in terms of an optical potential and may change the mesonic wave functions $\psi_{n\ell}(r)$ from the hydrogen-like one due to a point charge. We evaluate $S_{mF}(n\ell)$ ($m = \pi^-$ or K^-) of Eq.(2) with $\psi_{n\ell}$ obtained by using the program code Y3/TC/GA01[9], in which various effects such as strong interaction, finite size and vacuum polarization are taken into account. The optical potential parameters are taken from Ref.10 for π^- and from Ref. 11 for K^-. The ratios of thus obtained $S^S_{mF}(n\ell)$ to $S^H_{mF}(n\ell)$ from the hydrogen-like $\psi^H_{n\ell}$ are illustrated in Table 1 for the ^{12}C case. With these values, we see that the mesonic atom production cross sections should be reduced by a factor of 0.7 and 0.1 for π^- and K^-, respectively.

The effect of temporal distributions for the coalescence of particles produced in collisions has been discusse by Sato et al.[12] and also Ko et al.[13,14] We estimated the effect as a distortion of the particle distributions $D^{(1)}$ due to the time-dependence of particle emissions and got the value of the reduction factor of $S_{\Lambda F}$, 0.5-0.7[4] for hypernucleus formation. In the case of mesonic atoms, the range of the mesonic wave functions is much wider than that of the particle distributions and we can expect only small reduction of the coalescense factors S_{mF} ($m = \pi$ or K).

In conclusion, our calculated π^--atom productin cross sections are in qualitative agreement with theoretical values estimated by using other models[13-15].

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Table 1 The ratios of $S_{mF}(n\ell)$ with and without strong interaction effect for the ^{12}C case.

$n\ell$	$S^S_{mF}(n\ell)/S^H_{mF}(n\ell)$	
	π^-	K^-
1s	0.67	0.11
2p	1.01	0.68
3d	0.94	0.78

Figure Captions

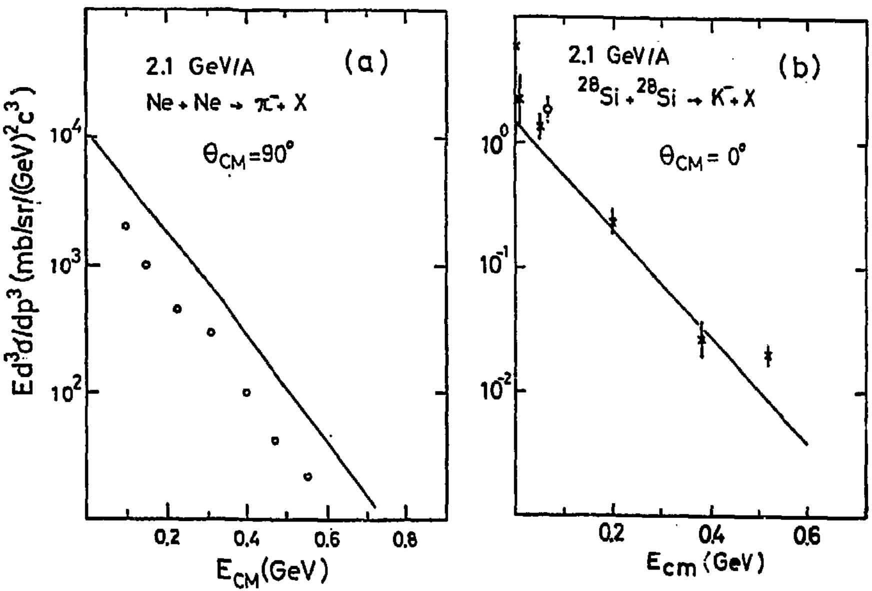
Fig.1. a) π^--energy spectrum at 90° in the c.m.s. resulting from Ne+Ne collisions at 2.1 GeV/A. Data are from Ref.6.

b) K^--energy spectrum at 0° in the c.m.s. resulting from $^{28}Si+^{28}Si$ at 2.1 GeV/A. Data are from Ref.7. The solid lines represent the predictions.

Fig.2. The production cross sections of π^--mesonic atoms in P+Ne collisions at 2.1 and 5.0 Gev/A, as a function of nuclear fragment mass number A_F. The number of the inside of brackets indicates an atomic number.

Fig.3. The production cross sections of π^--mesonic atoms in Ne+Ne collisions at 2.1 and 5.0 GeV/A.

Fig.4. The production cross sections of K^--mesonic atoms in P+Ne and Ne+Ne collisions at 5.0 GeV/A.



2.1 GeV/A
(a)
Ne + Ne → π⁻ + X
θ_CM = 90°
Ed³σ/dp³ (mb/sr/(GeV)²c³)
10⁴
10³
10²
0.2
0.4
0.6
0.8
E_CM (GeV)
2.1 GeV/A
(b)
²⁸Si + ²⁸Si → K⁻ + X
θ_CM = 0°
10⁰
10⁻¹
10⁻²
0.2
0.4
0.6
E_cm (GeV)

Fig. 1

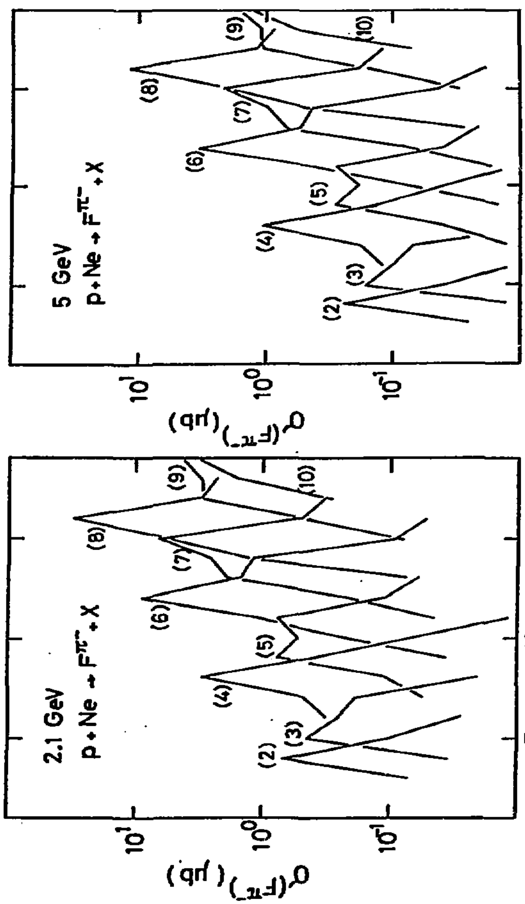
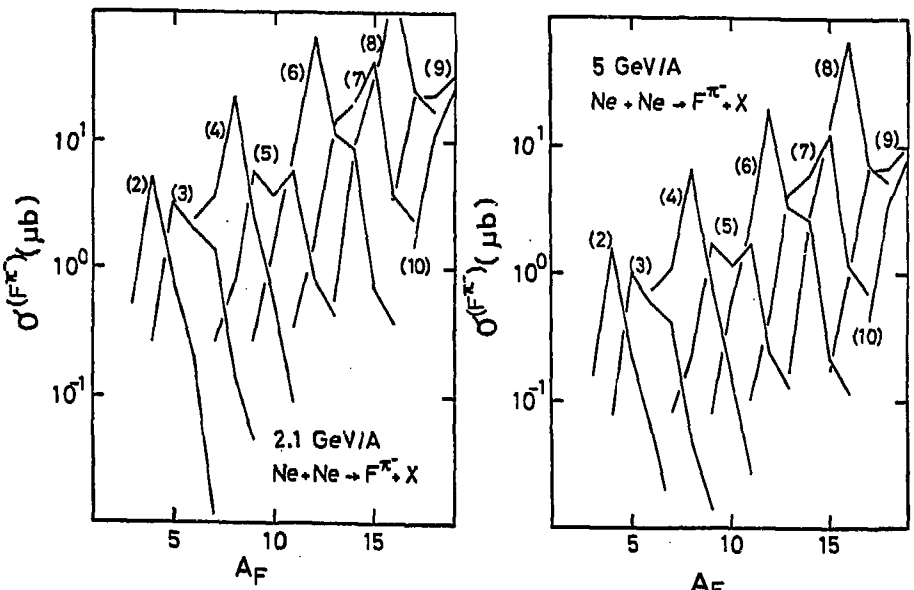


Fig. 2



2.1 GeV/A
Ne+Ne → F$^{\pi^-}$+ X
5 GeV/A
Ne + Ne → F$^{\pi^-}$+ X
σ(F$^{\pi^-}$)(μb)
10^1
10^0
10^{-1}
A$_F$
5
10
15
(2)
(3)
(4)
(5)
(6)
(7)
(8)
(9)
(10)

Fig. 3

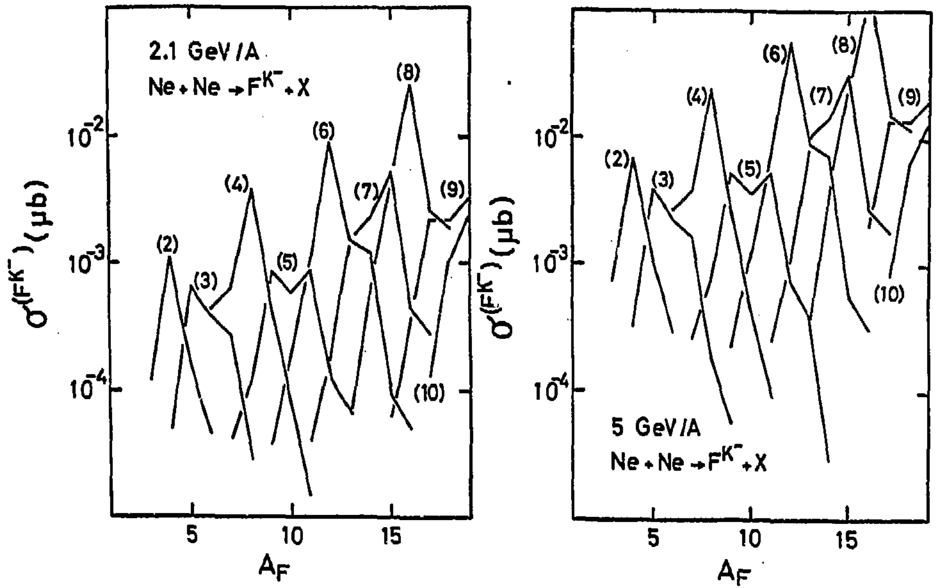

2.1 GeV/A
Ne + Ne → F^K⁻ + X
(2)
(3)
(4)
(5)
(6)
(7)
(8)
(9)
(10)
10^{-2}
10^{-3}
10^{-4}
$\sigma^{(F^{K^-})}$ (μb)
5
10
15
A_F
5 GeV/A
Ne + Ne → F^K⁻ + X

Fig. 4