RATIONAL MAPPING (RAM) OF IN-CORE DATA

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ABSTRACT

The paper describes and demonstrates a unique processing of in-core flux detector data, such that the detailed in-core power distribution can be derived with great accuracy by combining a specially "smoothed-out" set of in-core data with neutron diffusion theory.

RAM is designed in such a way that erratic detector signals are recognized very efficiently and can be eliminated from the experimental data set: this is achieved by modal expansion of the difference between theoretical fluxes and experimental fluxes at the detector sites.

Sensitivity studies have shown that RAM is quite stable, does not absorb the "wild" detector errors in the mapping procedure and results in mapped fluxes with errors about three times smaller than would be obtained by direct interpolation of detector readings.

I. INTRODUCTION

It is economically desirable to operate nuclear power reactors at or close to the designed power level. In achieving this end, it is of vital importance that "point" information available from the diagnostic tools be processed to give the most accurate representation of the operating state of the reactor. One possible approach in accomplishing this is to express a flux distribution in terms of a finite number of modes, weighted by modal amplitudes determined to minimise the least squares error between a set of actual detector readings and the synthesized flux values at the detector points. Another method consists in generating a mapped flux distribution on the basis of coupling interpolated in-core measurements, having used a bilinear function of spatial coordinates to represent the neutron flux, with diffusion theory calculations.[1]

In section II a novel procedure is presented that embraces the significant features of both of the above mentioned approaches in a manner giving rise to a unique scheme, the RAtional Mapping (RAM) of in-core data. In section III brief mention is made of the DEtector Wildness (DEW) criterion designed to discover the presence of erroneous measurements. Numerical examples examining the efficacy of the DEW criterion, in the context of the RAM procedure, is presented in section IV, and section V is devoted to a few concluding remarks.

II. THEORY

Reactor simulation for the purposes of obtaining flux and thus power distributions is achieved by solving a multi-group three-dimensional steady-state neutron diffusion equation. Using standard numerical techniques gives the resulting vector equation

$$\mathbf{A}\ \underline{\psi} = \frac{1}{\lambda}\mathbf{F}\ \underline{\psi}$$

which expresses the neutron balance, and where $\mathbf{A}$ and $\mathbf{F}$ are removal (including leakage) and multiplication matrices, respectively, $\underline{\psi}$ is the flux vector and λ the eigenvalue. The spatial dependence is embodied in the matrix formulation; for example, for G groups and N spatial meshpoints the vector $\underline{\psi}$ has NG components.

RAtional Mapping (RAM) of in-core data is a simulation procedure designed to provide flux distributions with the distinctive feature of continuous coupling of measurements and diffusion theory. RAM procsses in-core flux detector signals in two stages.

(a) A "SLOW" map is obtained off-line via a diffusion theory calculation incorporating the detector measurements in a unique manner. The SLOW map, which is generated periodically, perhaps at the end of each refuelling cycle, includes fluctuations due to irradiation, refuelling ripples, other local effects and an estimate of detector errors.

(b) An essentially instantaneous, on-line, "FAST" map is obtained via a modal expansion of the flux update, that is, the difference between the current measurements and the most recently computed SLOW map.

Normally the set of full-power modes used satisfy the diffusion equation representative of a time-averaged reactor; however, in particular cases when severely distorted flux shapes are expected, different sets of modes may be necessary. For example, during start-up transients, different sets for different adjuster configurations may be required.(*) Normally, however, experience shows that only the fundamental mode (all-positive, corresponding to the highest eigenvalue) requires updating, whereas the time-averaged higher harmonics are normally adequate.

(*)The terminology used here is typical of a CANDU (CANada Deuterium Uranium) heavy water reactor. In a CANDU a number of absorber rods (called "adjusters")are normally inserted in the core; besides providing power flattening, the adjusters represent a potential excess reactivity which is used for example to override Xe^{135} buildup during startup after a short shutdown. In other reactors, other absorbers (generally "control rods") contribute to determining the flux modes.

In cases where core properties are sufficiently altered, for example, due to insertion or removal of reactivity devices(s), it is evident that the FAST map, with the use of full-power modes, may fail to account for local effects. These can be corrected for by using a well tested semi-empirical model.[2] An alternative is to generate a SLOW map using a material distribution that accounts for the above perturbation, and thereafter continue to monitor the reactor with FAST.

SLOW is a five-step iterative calculation coupling diffusion theory and measurements, as presented below. The symbols in the following equations will be explained in the ensuing discussion of the procedure.

$$N^{(i)} = \sum_p \mu m_p \Big/ \sum_p \phi_p(i) \tag{1}$$

$$\delta_p^{(i)} = \mu m_p - N^{(i)}\phi_p^{(i)} \tag{2}$$

$$\underline{\delta}^{(i)} = M\ \underline{a}^{(i)} + \underline{r}^{(i)} \tag{3a}$$

$$\underline{a}^{(i)} = (M^T M)^{-1} M^T \underline{\delta}^{(i)} \tag{3b}$$

$$A\ \underline{\phi}^{(i+1)} = F\ [N^{(i)}\underline{\phi}^{(i)} + \tilde{M}\ \underline{a}^{(i)}] \tag{4a}$$

$$A\ \underline{\phi}^{(i+1)} = F\ [N^{(i)}\underline{\phi}^{(i)} + \underline{\tilde{\delta}}^{(i)}] \tag{4b}$$

$$\text{If} \left| \frac{\phi_k^{(i+1)}}{\phi_k^{(i)}} - 1 \right|_{max} < \varepsilon \text{ and } \left| \frac{N^{(i)}}{N^{(i-1)}} - 1 \right| < \varepsilon \tag{5}$$

then terminate and store mapped flux, otherwise update iteration index, i, and go to equation (1).

The raw detector reading, m_p, at each measurement point, p, is converted to a measured flux, μm_p, where μ is the measurement-to-flux opeator. The overall normalisation, $N^{(i)}$, between the measured fluxes and the current flux iterate, $\phi_p^{(i)}$, is computed by equaton (1). The correction flux, $\delta_p^{(i)}$, at each measurement point is calculated in equation (2). This is then expanded in terms of representative modes in equation (3a), where $M_{\ell k}$ is the detector modal matrix element ℓk representing mode k flux at detector site ℓ and $\underline{r}^{(i)}$ is the residue vector. The modal amplitude vector, $\underline{a}^{(i)}$, is determined by equation (3b) to minimise the square of the residue vector. The mapped correction flux, $\underline{\tilde{\delta}}^{(i)} = \tilde{M}\ \underline{a}^{(i)}$, at all mesh points is calculated using the modal amplitude vector, $\underline{a}^{(i)}$, and the total modal matrix, $\tilde{M}$. The mapped flux, $(N^{(i)}\underline{\psi}^{(i)} + \underline{\tilde{\delta}}^{(i)})$, is then used in equation (4b), presupposing a critical system, to compute the new flux iterate, $\underline{\psi}^{(i+1)}$. The iterative process is continued to convergence of the flux iterate and the normalisaton constant. The resulting SLOW map is the mapped flux.

Note the following distinctions:

M, $\underline{\delta}$	refer to the detector locations only
$\tilde{M}$, $\underline{\tilde{\delta}}$	refer to all the mesh points used in the flux discretization
$N\underline{\psi} + \underline{\tilde{\delta}}$	is called "mapped flux" and is assumed to be the best representation of the flux consistent with the measurements
$\underline{\psi}$	is a "flux iterate".

Note that the eigenvalue simulation is replaced by an iterative source calculation in which the source term is forced to agree as closely as possible with measured data. Moreover, the use of a finite-difference code in equation (4b) permits the inclusion of all predictable local effects from the outset; therefore, the correction flux, $\underline{\delta}^{(i)}$, should be primarily free of local effects and as a result expandable with time-averaged harmonics.

FAST entails using the most recent SLOW map, $\underline{\psi}^S$,

$$\underline{\psi}^S = \text{converged } (N\underline{\psi} + \underline{\tilde{\delta}}) \text{ of last SLOW map,}$$

and the on-line flux detector measurements, μm_p, to modally expand the flux update $(\mu m_p - N\underline{\psi}^S)$, where N is the normalisaton constant as computed in equation (1). The resulting FAST map is,

$$\underline{\phi} = N\underline{\psi}^S + \tilde{M}\ \underline{a} \quad , \qquad (6)$$

where $\underline{a}$ is the modal amplitude vector, as computed in equation (3b). In other words, the SLOW procedure is reduced to the non-iterative sequence of steps in equations (1) to (3b) with the most recent SLOW map, $\underline{\phi}^S$, as the reference flux.

Note that the FAST map is obtained by modal expansion of the flux update, relative to the most recently computed SLOW map, and not by expanding the flux detector measurements directly. This procedure is followed to preserve all the local effects contained within the SLOW map. Moreover, the "smooth" component, $\tilde{\mathbf{M}}\ \underline{a}$, should include macroscopic tilts due to factors not included in the diffusion calculation.

III. SKETCH OF DETECTOR WILDNESS (DEW) CRITERION

The DEtector Wildness (DEW) criterion is essentially a three-stage procedure[3]. The first stage entails ascertaining possibly erroneous measurements given a current SLOW map and certain computable statistics. The next step involves using the preceding information to generate "new" SLOW maps and certain comparative statistics. The final stage requires the use of a particular linear relation in the above comparative statistics, based on a certain statistical test and test run experience, to evaluate DEW. Let T_Q, T_E be two appropriate "threshold" values:

$$\text{if } T_Q \leq \text{DEW} < T_E$$

then the potential erroneous detector is pronounced "quasi-erratic" or "quasi-wild";

$$\text{if DEW} \geq T_E$$

then the detector is classified as "erratic" or "wild". In both cases, the detectors in question are dropped from the measurement set.

IV. NUMERICAL EXAMPLES

Sample calculations were performed to examine the accuracy of the SLOW map procedure and the efficacy of DEW. The analysis was based on the CANDU Darlington nuclear rector model with time-averged lattice properties, all adjuster rods inserted and nominal level in the liquid zone controllers. The reactor model consists of three regions, namely, the inner-core, outer-core and the reflector. The measurement set was comprised of 52 thermal flux detectors situated interstitially consistent with the actual penetrations available for neutron flux monitoring rods: actually, all the "detector readings" used in this study are originated from simulated fluxes. The flowchart of the RAM system, as implemented for this study, is shown in Figure 1. The SLOW map is computed by SOCHE, a modified two energy group static 3-dimensional diffusion equation code.

The examples considered are subdivided into three groups:

Group A (4 cases) involves at most <u>one</u> wild detector

Group B (4 cases) involves always <u>two</u> wild detectors

Group C (3 cases) involves always one quasi-wild detector and it provides a sensitiviy test of the DEW criterion.

In group A the four cases studied entail generating SLOW maps using simulated flux measurements with the following features: random error of up to ± 2 percent in case A1; in cases A2, A3 and A4, in addition to random errors, one wild detector measurement is introduced in each case with errors of -10, +10 and -10 percent respectively, and are located near an adjuster rod, a liquid zone controller and the reflector respectively.

The pertinent data for the cases considered above are presented in Table 1. The results indicate that SLOW has a tendency to "smooth-out" or reduce the incorportion of detector errors in the mapped flux distribution. Specifically the variance in the error in the SLOW map fluxes at detector sites (compared to the actual) is smaller by more than a factor of four compared to the variance in error in the detector signals. This means that a direct interpolation of detector readings will have approximately twice the error in SLOW maps. The number of iterations to achieve convergence, having used as guess flux the distribution satisfying the time-average lattice properties, is small in all cases; thus, making SLOW a feasible alternative for local effects corrections.

The DEW criterion is shown to satisfactorily ferret out the presence of single wild detectors. It is interesting to note that the magnitude of DEW for a wild detector in the central core (4.06 and 3.82) compared to one located near the reflector (3.46) indicate relative wildness of each measurement as viewed by the mapping procedure. The more conspicuous nature of wild detectors in the central core is probably due to the fact that most of the flux detectors are situated here compared to very few near the reflector. Also, the flux shape, generally, gets "smoother" moving from the central core towards the reflector(*), and as a result there is some chance of the erroneous result being absorbed somewhat into the mapped flux.

In group B (Table 2) four cases were examined each with two wild detectors situated "strategically" to see if these "activate" unrepresentative modes thereby yielding meaningless maps. In case B1 the two detectors, each with +10 percent error, are symmetric, approximately, about the x-plane; in case B2 the detectors, one with +10 and other -10 percent error, are symmetric, approximately, about the x and y-planes; in case B3 the detectors, both with +10 percent error, are locted side by side; and in case B4 the detectors, one with +10 and other -10 percent error, are situated as in case B3. In all cases the criterion detects the presence of two erroneous detectors. It is interesting to note the magnitude of DEW (2.93) for one of the wild detectors, (30,16,6) in case B3, almost falling withing the confines of quasi-wild mesurements. This is probably a result of the detector being located close to the reflector. Cases with several wild detectors have not been considered as this is inadmissible once a system like SLOW, is available that permits the timely detection (and removal) of all defective detectors.

(*)In a typical CANDU reactor the flux shape in the central core is somewhat rippled due to the presence of adjuster rods.

Finally in group C (Table 3) three cases were considered to essay the efficacy of DEW in detecting quasi-erratic measurements. The three cases consist of a single quasi-wild detector of errors -4, +6 and -8 percent, respectively, situated at random. DEW fails to perceive the error in the first case but is able to detect the presence of erroneous measurements in the last two; and, interestingly enough, the detector with -8 percent, in case C3 is appropriately classified as wild.

V. CONCLUDING REMARKS

The favourable results obtained indicate that the RAM procedure, including the DEW criterion, can offer a very detailed, on-line power distribution, quasi-continuous and detailed burnup bookkeeping, and information on erratic detectors. The psychological advantages, from the perspective of the controller, of having accurate representation of the state of the core are obvious. From the viewpoint of the protective system, RAM would be the ideal implement in providing easily updatable on-line calibration of triplicated Neutron Over-Power (NOP) detectors.

The general nature of the RAM procedure makes it applicable for other nuclear power reactor systems, such as the Pressurised Water Reactors. Also, the DEW criterion can be easily adopted with changes in certain parameters subject to the reactor system in use.

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TABLE 1

Statistics of Cases Examined in Group A SLOW Map Sensitivity to Single Erratic Detector*

	Case A1	Case A2	Case A3	Case A4
Variance in simulated detector measurements‡, σ^2	2.0204	4.1770	3.7500	4.1391
No. of interations for convergence ($\varepsilon = 10^{-4}$)	16	17	17	17
Variance in mapped flux (at detector sites)‡‡, s^2	0.4829	0.8393	1.0105	1.4874
No. of potentially wild detectors**	--	1	1	1
Back mapped error at (location) (detector reading-mapped flux)**	--	-8.33 at (15,12,9)	6.46 at (19,8,11)	-5.64 a (20,25,6)
Measure of detector wildness, DEW***	--	4.06	3.82	3.46

Case A1: Detectors with random errors of up to ± 2 percent.

Case A2: A wild detector, with -10 percent error at (15,12,9), in addition to random errors.

Case A3: A wild detector, with +10 percent error at (19,8,11), in addition to random errors.

Case A4: A wild detector, with -10 percent error at (20,25,6), in addition to random errors.

* The reactor model has 38 (radial horizontal) x 32 (radial vertical) x 16 (axial) meshpoints.
Variance in units of 10^{-4} and errors in percent.

‡ Refers to the distribution: $\frac{\text{Detector Reading-"Exact" Flux}}{\text{"Exact" Flux}}$.

‡‡ Refers to the distribution: $\frac{\text{Mapped Flux-"Exact" Flux}}{\text{"Exact" Flux}}$.

** Determined using the criterion developed.

*** If DEW $\geq$ 2.9, the potentially wild detector is pronounced erratic.

TABLE 2

Statistics of Cases Examined in Group B
SLOW Map Sensitivity to Two Erratic
Detectors*

	Case B1	Case B2	Case B3	Case B4
Variance in simulated detector measurements‡, σ^2	5.5340	5.9371	5.4583	5.8504
No. of interations for convergence ($\varepsilon = 10^{-4}$)	16	15	17	17
Variance in mapped flux (at detector sites)‡‡, s^2	1.0786	0.8738	1.3911	0.7748
No. of potentially wild detectors**	2	2	2	2
Back mapped error at (location) (detector reading-mapped flux)**	7.73 at (15,12,6)	8.98 at (24,12,6)	8.13 at (24,16,6)	9.60 at (24,16,6)
	7.55 at (24,12,6)	-8.87 at (15,21,6)	6.07 at (30,16,6)	-8.62 at (30,16,6)
Measure of detector wildness, DEW***	3.28 at (15,12,6)	3.60 at (24,12,6)	3.42 at (24,16,6)	3.72 at (24,16,6)
	3.24 at (24,12,6)	3.62 at (15,21,6)	2.93 at (30,16,6)	3.61 at (30,16,6)

Case B1: Two wild detectors, each with +10 percent error at (15,12,6) and at (24 12,6), in addition to random errors.

Case B2: Two wild detectors, with +10 percent error at (24,12,6), and -10 percent error at (15,21,6), in addition to random errors.

Case B3: Two wild detectors, each with +10 percent error at (24,16,6) and at (30,16,6), in addition to random errors.

Case B4: Two wild detectors, with +10 percent error at (24,16,6) and -10 percent error at (30,16,6), in addition to random errors.

* The reactor model has 38 (radial horizontal) x 32 (radial vertical) x 16 (axial) meshpoints.
Variance in units of 10^{-4} and errors in percent.

‡ Refers to the distribution: $\frac{\text{Detector Reading-"Exact" Flux}}{\text{"Exact" Flux}}$.

‡‡ Refers to the distribution: $\frac{\text{Mapped Flux-"Exact" Flux}}{\text{"Exact" Flux}}$.

** Determined using the criterion developed.

***If DEW > 2.9, the potentially wild detector is pronounced erratic.

TABLE 3
Statistics of Cases Examined in Group C
SLOW Map Sensitivity to Quasi-Erratic Detectors*

	Case C1	Case C2	Case C3
Variance in simulated detector measurements ‡, σ^2	2.3783	2.4933	3.4318
No. of interations for convergence ($\varepsilon = 10^{-4}$)	17	17	15
Variance in mapped flux (at detector sites)‡‡, s^2	0.7497	0.7164	0.5383
No. of potentially wild detectors**	--	1	1
Back mapped error at (location) (detector reading-mapped flux)**	--	4.2427 at (24,12,8)	-7.3316 at (15,21,9)
Measure of detector wildness, DEW***	--	2.75	3.83

Case C1: Quasi-wild detector, with -4 percent error at (19,19,8), in addition to random errors.

Case C2: Quasi-wild detector, with +6 percent error at (24,12,8), in addition to random errors.

Case C3: Quasi-wild detector, with -8 percent error at (15,21,9), in addition to random errors.

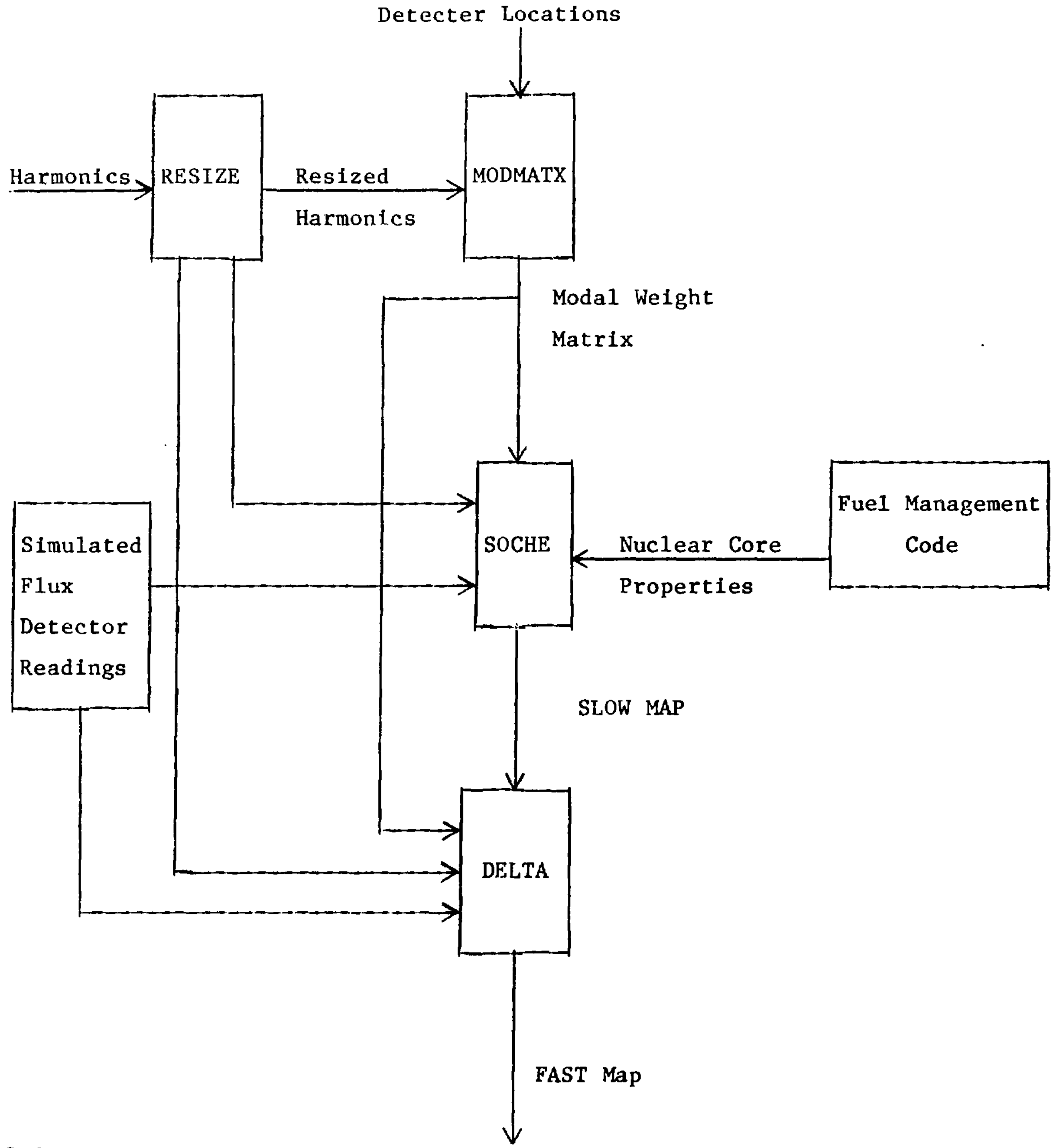
* The reactor model has 38 (radial horizontal) x 32 (radial vertical) x 16 (axial) meshpoints.
Variance in units of 10^{-4} and errors in percent.

‡ Refers to the distribution: $\dfrac{\text{Detector Reading-"Exact" Flux}}{\text{"Exact" Flux}}$.

‡‡ Refers to the distribution: $\dfrac{\text{Mapped Flux-"Exact" Flux}}{\text{"Exact" Flux}}$.

** Determined using the criterion developed.

*** If $2.7 \leq \text{DEW} < 2.9$, the potentially wild detector is pronounced quasi-wild; if $\text{DEW} \geq 2.9$, it is declared erratic.



<u>Code</u>

RESIZE : The mesh spacings of the eigenfunctions are "resized" to those used in the model.

MODMATX: Evaluates the modal weight matrix, $(M^TM)^{-1}M^T$.

SOCHE : CHEBY, static 3-dimensional diffusion code in two energy groups, modified to incorporate the SLOW procedure.

DELTA : Evaluates the on-line FAST map.