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STOCHASTIC QUANTUM GRAVITY

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1. INTRODUCTION

From an experimental point of view, diffusion processes, quantum physics, and space-time structure seem still as unrelated to each other as they were in 1905 when Einstein wrote three classical papers on these subjects. The situation is slightly different in theoretical physics, where various interconnections have been sought, and sometimes found, during the past few decades. We have in mind the laborious attempts to define a quantum theory of gravity, the interpretation of quantum mechanics in terms of diffusion processes (called stochastic mechanics) (Nelson, 1969), and the discovery of thermodynamical properties of certain classical gravitational fields, as exemplified by the Hawking effect (Hawking, 1975). Most physicists will agree that according to present knowledge of these three endeavours the last one can claim to have its feet on the firmest ground. Regarding the quantization of gravity, the superstring models promise a finite theory whose classical precursor is already radically different from Einstein's General Relativity, the latter emerging only as a low energy phenomenon. Even if this turns out to be correct, there will nevertheless be a certain domain of validity of quantized General Relativity as an effective field theory. The limit of this domain will be defined by the Planck energy, which, alas, would preclude tackling the most interesting problems such as how Nature circumvents the classical cosmological and black hole singularities. Despite these possible limitations, we shall in the following apply the stochastic quantization method of Parisi and Wu (1981) to Einstein gravity. This is indeed an instance where all three of the concepts mentioned in the beginning meet together, albeit in a rather abstract fashion. Our motivation for this investigation is the same that led to the original application of the method to ordinary gauge theories and hence need not be elaborated as this has been done by other speakers of this meeting. Suffice it to say that a covariant quantization method that can dispose of gauge fixing is particularly attractive in the case of gravitation and that there was the hope that this method would also shed light on some structural elements of the classical theory that so far had gone unnoticed. We feel that this expectation has been validated.

We shall begin ~~the technical part of these lectures~~ with a naive application of the Parisi-Wu scheme to linearized gravity. This will lead into trouble as one peculiarity of the full theory, the indefiniteness of the Euclidean action, shows up already at this level. After discussing some proposals to overcome this problem, Minkowski space stochastic quantization will be introduced. This will still not result in an acceptable quantum theory of linearized gravity, as the Feynman propagator turns out to be non-causal. This defect will be remedied only after a careful analysis of general covariance in stochastic quantization has been performed. The analysis requires the notion of a metric on the manifold of metrics, and a natural candidate for this is singled out. With this a consistent stochastic quantization of Einstein gravity becomes possible. It is even possible, at least perturbatively, to return to the Euclidean regime.

2. THE PARISI-WU ANSATZ AND LINEARIZED EUCLIDEAN GRAVITY

The general Parisi-Wu ansatz for a Euclidean quantum field $\Phi(x)$ consists in defining a Markovian stochastic process $\Phi(x,s)$ by the Langevin equation

$$\frac{\partial\Phi(x,s)}{\partial s} = -\frac{\delta S_E[\Phi]}{\delta\Phi(x,s)} + \xi(x,s) \tag{2.1}$$

where $S_E[\Phi]$ is the Euclidean action and ξ is a Gaussian white noise with correlation

$$\langle\xi(x,s)\xi(x',s')\rangle = 2M\delta^{(4)}(x-x')\delta(s-s') \tag{2.2}$$

(we have suppressed any indices that Φ, ξ and the matrix M, which reduces to unity in the case of a scalar field, may bear). The stochastic averages of observables (in the case of many-point functions taken at equal fictitious time s) are expected to reproduce the expectation values of Euclidean quantum field theory in the limit $s \to \infty$.

In this section we apply the method to the case of Euclidean linearized gravity, where Φ is given in terms of the space-time metric

$$g_{ab} = \delta_{ab} + 2\kappa^{1/2}h_{ab} \tag{2.3}$$

as

$$\Phi = h_{ab} \tag{2.4}$$

κ being 8π times Newton's constant. The Euclidean Einstein-Hilbert action

$$S_E[g_{ab}] = -\frac{1}{2\kappa}\int d^4x\, g^{1/2}R(g_{ab}) \tag{2.5}$$

with $g = \det(g_{ab})$ and R the curvature scalar implies the following linearized action for h_{ab}:

$$S_E^{(0)}[h_{ab}] = \frac{1}{2}\int d^4x\, h_{ab} V_{abcd} h_{cd} \tag{2.6}$$

where the kinetic operator V_{abcd} is given in momentum space by

$$V_{abcd} = k^2(1_{abcd} - \delta_{ab}\delta_{cd}) + (k_a k_b \delta_{cd} + \delta_{ab} k_c k_d) - (k_a k_{(c|}\delta_{b|d)} + k_b k_{(c|}\delta_{a|d)}) \tag{2.7}$$

with

$$1_{abcd} \equiv \frac{1}{2}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) \tag{2.8}.$$

V is a self-adjoint operator on the space of symmetric tensor fields:

$$V_{abcd} = V_{(ab)(cd)} = V_{cdab}. \tag{2.9}$$

The action (2.6) is invariant under the abelian gauge transformations (a remnant of the general coordinate invariance of (2.5))

$$h_{ab} \rightarrow h_{ab} + k_a\Lambda_b + k_b\Lambda_a. \tag{2.10}$$

In the following we shall adopt a more compact notation by introducing a complete orthogonal set of spin projection operators (van Nieuwenhuizen, 1973). We first define

$$L_{ab} = k_a k_b / k^2, \tag{2.11}$$

$$T_{ab} = \delta_{ab} - L_{ab}. \tag{2.12}.$$

Then the operators

$$P^2_{abcd} = \frac{1}{2}(T_{ac}T_{bd} + T_{ad}T_{bc}) - \frac{1}{3}T_{ab}T_{cd} \tag{2.13}$$

$$P^1_{abcd} = \frac{1}{2}(T_{ac}L_{bd} + T_{ad}L_{bc} + T_{bc}L_{ad} + T_{bd}L_{ac}) \tag{2.14}$$

$$P^0_{abcd} = L_{ab}L_{cd} \tag{2.15}$$

$$P^{0'}_{abcd} = \frac{1}{3}T_{ab}T_{cd} \tag{2.16}$$

fulfil

$$\sum_A P^A = 1, \qquad P^A P^B = \delta^{AB} P^B. \tag{2.17}$$

Written in terms of P^A, V becomes

$$V = k^2(P^2 - 2P^{0'}) \tag{2.18}$$

which shows that the action (2.6) is not bounded from below. Thus the well-known indefiniteness of (2.5) (Gibbons, Hawking, Perry, 1976) manifests itself already at the linearized level. The indefiniteness can be traced back to the conformal degrees of freedom of the metric. If we decompose the field h according to (2.17) as

$$h = h^{(2)} + h^{(1)} + h^{(0)} + h^{(0')} \tag{2.19}$$

then $h^{(2)}$ contains pure (massive) spin 2, $h^{(1)}$ and $h^{(0)}$ are pure gauge contributions of spin 1 and 0, respectively,

$$h^{(1)}_{ab} = k_a \Lambda^T_b + k_b \Lambda^T_a \tag{2.20}$$

$$h^{(0)}_{ab} = k_a k_b \Lambda \tag{2.21}$$

and $h^{(0')}$ is the spin-0 conformal part:

$$h^{(0')}_{ab} = T_{ab}\Lambda. \tag{2.22}$$

The latter gives a negative contribution to the action (the fact that P^1 and P^0 do not appear in V at all is an expression of the gauge invariance). It will also contribute to gauge invariant quantities, which can be constructed from the linearized Riemann tensor

$$R_{abcd} = 4k_{[a}h_{b][c}k_{d]}. \tag{2.23}$$

Let us now write down a Langevin equation (2.1) for the field (2.4) with the action (2.5):

$$\frac{\partial h_{ab}}{\partial s} = -V_{abcd}h_{cd} + \xi_{ab}. \tag{2.24}$$

This seems obvious, but note that we have chosen to transvect the second pair of indices with the Euclidean metric δ_{ab} in order to construct an endomorphism out of the quadratic form V. We will be forced to modify this in a later section. Likewise the obvious choice for the correlation function

$$\langle \xi_{ab}(x,s)\xi_{cd}(x',s')\rangle = 2 1_{abcd}\delta^{(4)}(x-x')\delta(s-s') \tag{2.25}$$

will not be our final one.

The Langevin equation (2.24) can be solved by means of the heat kernel

$$H = e^{-sV} = e^{-k^2 s}P^2 + e^{2k^2 s}P^{0'} + P^1 + P^0. \tag{2.26}$$

If we assume $h(s=0) = 0$, then

$$h_{ab}(k,s) = \int_0^s H_{abcd}(k, s-s')\xi_{cd}(k,s')ds'. \tag{2.27}$$

The 2-point correlation function

$$\langle h_{ab}(k,s)h_{cd}(k',s')\rangle = 2\cdot(2\pi)^4\delta^{(4)}(k+k')\int_0^{\min(s,s')} d\sigma H_{abij}(k,s-\sigma)H_{cdij}(k',s'-\sigma) \tag{2.28}$$

implies the equal time correlation

$$\langle h_{ab}(k,s)h_{cd}(k',s)\rangle = 2(2\pi)^4\delta^{(4)}(k+k')\int_0^s d\sigma H_{abcd}(k,2s-2\sigma) \tag{2.29}$$

$$= (2\pi)^2\delta^{(4)}(k+k')[k^{-2}(P^2-\frac{1}{2}P^{0'})+2(P^1+P^0)s-k^{-2}(e^{-2k^2s}P^2-\frac{1}{2}e^{4k^2s}P^{0'})]_{abcd}. \tag{2.30}$$

The term linear in s is due to the random walk along the gauge orbits implied by (2.24) and familiar from stochastic Yang-Mills theory. It does not contribute to gauge invariant expectation values. The last term in (2.30), however, is of a type not encountered in non-gravitational theories. It is a consequence of the indefiniteness of (2.6) which implies an antidamping for the conformal modes in the Langevin equation. This behaviour is also reflected in the Feynman propagator K implied by

$$\lim_{s\to\infty}\langle h_{ab}(k,s)h_{cd}(k',s)\rangle = (2\pi)^4\delta^{(4)}(k+k')K_{abcd}(k). \tag{2.31}$$

The propagator consists of three parts: The finite part is an analog of the Landau gauge propagator in gauge theories and perfectly acceptable (for its detailed form see (Hüffel and Rumpf, 1985)). The quadratically divergent part (linear in s) is a pure gauge without physical consequences. The exponentially divergent part, however, involves physical modes and has to be rejected. Several modifications of the above stochastic quantization method have been proposed in order to get rid of this term.

One proposal, due to Fukai and Okano (1984), is to replace the kinetic operator V by

$$V' = k^2(P^0+2(\gamma-1)P^{0'}), \qquad \gamma > 1, \tag{2.32}$$

and let $\gamma \to 0$ after expectation values have been calculated. This yields the correct propagator, but it is hard to see how this prescription can be generalized to the nonlinear theory.

A second proposal is the so-called stabilization method for bottomless actions (Greensite and Halpern, 1984). It is based on the Fokker-Planck equation for the probability functional $P[\Phi(x),s]$ defined by the relation

$$\langle F[\Phi(x,s)]\rangle = \int d[\Phi]F[\Phi(x)]P[\Phi(x),s] \tag{2.33}$$

for the stochastic averages of arbitrary functionals F. Equation (2.33) implies

$$\frac{\partial P}{\partial s} = \int d^4x[\frac{\delta^2}{\delta\Phi^2(x)}+\frac{\delta}{\delta\Phi(x)}\frac{\delta S}{\delta\Phi(x)}]P \tag{2.34}$$

(the Fokker-Planck equation). Upon the substitution

$$\tilde{P} = P\,e^{S[\Phi]/2} \tag{2.35}$$

(2.34) becomes

$$\frac{\partial\tilde{P}}{\partial s} = -H\tilde{P}[\Phi,s] \tag{2.36}$$

with the Fokker-Planck Hamiltonian

$$H = \int d^4x R^\dagger(x) R(x) \geq 0 \tag{2.37}$$

$$R(x) = \frac{\delta}{\delta\Phi(x)} + \frac{1}{2}\frac{\delta S}{\delta\Phi(x)}. \tag{2.38}$$

If $e^{-S/2}$ is normalizable and there exists a "mass gap" for the Fokker-Planck Hamiltonian, then

$$\tilde{P} \xrightarrow{s\to\infty} e^{-S/2} \tag{2.39}$$

If $e^{-S/2}$ is not normalizable (as in the gravity case), then the ground state of H is different and its lowest eigenvalue greater than zero. The rule proposed by Greensite and Halpern (and also taken up by Haba, 1985) is to take this ground state to define the equilibrium distribution for Φ via (2.35). This method is of universal applicability, but unfortunately it does not produce the correct physics, as it yields (in momentum space) the propagator $|K|$ instead of K.

A third proposal, Minkowski space stochastic quantization (Hüffel and Rumpf, 1984), will be the subject of the next section.

3. MINKOWSKI SPACE STOCHASTIC QUANTIZATION

The indefiniteness of the action poses no problem in Minkowskian field theories, the path integral being ill-defined there for a different reason. One might therefore hope to get rid of the indefiniteness problem by performing in the Langevin equation the same substitution, $S_E[\Phi] \rightarrow -iS[\Phi]$, ($S$ being the Minkowskian action) that distinguishes the Minkowskian from the Euclidean path integral. Consider then the modified Langevin equation

$$\frac{\partial\Phi(x,s)}{\partial s} = i\frac{\delta S}{\delta\Phi(x,s)} + \xi(x,s) \tag{3.1}$$

where the correlation function for ξ differs from that of the Euclidean case by the replacement of δ_{ab} by the Minkowski metric η_{ab}. This implies that ξ will be complex, in general. Even if ξ remains real (as in the scalar case), Φ becomes a complex process owing to the factor i appearing in (3.1). This doubling of the number of degrees of freedom is necessary, as the two-point functions of Minkowskian field theories are complex (even for the neutral scalar field). But it is not at all clear how these theories should arise from (3.1), as the drift term of the modified Langevin equation is not of the restoring force type. Therefore no equilibrium limit exists in the usual sense. However it can be shown, at least perturbatively, that the equilibrium limit of the correlation functions does exist in the sense of tempered distributions of the arguments.

Let us consider as the simplest example the free massless scalar field. Here also $\xi(x,s)$ is scalar with

$$\langle\xi(x,s)\xi(x',s')\rangle = 2\delta(s-s')\delta^{(4)}(x-x'). \tag{3.2}$$

The solution of (3.1), subject to the initial condition $\Phi(k,0) = 0$, is given by

$$\Phi(k,s) = \int_0^s d\sigma e^{ik^2(s-\sigma)}\xi(\sigma). \tag{3.3}$$

Hence the correlation function for Φ is

$$\begin{aligned}\langle\Phi(k,s)\Phi(k',s')\rangle &= \int_0^{min(s,s')} d\sigma\, e^{ik^2(s+s'-2\sigma)}\cdot 2(2\pi)^4\delta^{(4)}(k+k') \\ &= i(2\pi)^4\delta^{(4)}(k+k')\,\tfrac{1}{k^2}(e^{ik^2|s-s'|} - e^{ik^2(s+s')})\end{aligned} \tag{3.4}$$

The limit of this expression for $s = s' \to \infty$ exists, if we interpret it in the sense of tempered distributions and use the well-known relations

$$\lim_{s\to\infty} e^{ixs} = 0 \tag{3.5}$$

$$\lim_{s\to\infty} P(\frac{1}{x})e^{ixs} = i\pi\delta(x) \tag{3.6}$$

where P denotes the principal value. In this way we obtain

$$\lim_{s\to\infty}\langle\Phi(k,s)\Phi(k',s)\rangle = i(2\pi)^4\delta^{(4)}(k+k')\frac{1}{k^2+i0} \tag{3.7}$$

i.e. the ordinary Feynman propagator.

An alternative procedure to obtain the same equilibrium limit is to add a negative imaginary mass term $-i\epsilon\Phi^2/2$ to the Lagrangian for Φ. Then the correlation function converges in the usual sense for $s \to \infty$ and yields (3.7) with $k^2 + i\epsilon$ in the denominator. In the end one lets $\epsilon \to 0$. By this method one can prove the perturbative equivalence of Minkowski space stochastic quantization with standard quantization for non-gauge theories (Hüffel and Rumpf, 1984).

There exists also a non-perturbative argument for the equivalence with standard quantization due to Nakazato and Yamanaka (1986) and based on considerations on complex probabilities by Parisi (1983): Write

$$\Phi(x,s) = \Phi_R(x,s) + i\Phi_I(x,s) \tag{3.8}$$

with both Φ_R and Φ_I real. Similarly we decompose ξ as

$$\xi(x,s) = \xi_R(x,s) + i\xi_I(x,s). \tag{3.9}$$

Then sufficient conditions for (3.2) to hold are

$$\langle\xi_R(x,s)\xi_R(x',s')\rangle = 2\alpha\delta^{(4)}(x-x')\delta(s-s') \tag{3.10}$$

$$\langle\xi_I(x,s)\xi_I(x',s')\rangle = 2\beta\delta^{(4)}(x-x')\delta(s-s') \tag{3.11}$$

$$\alpha - \beta = 1 \tag{3.12}$$

and all other correlations vanishing. The Fokker-Planck probability for the complex process Φ is a functional $P[\Phi_R, \Phi_I; s]$ and obeys

$$\frac{\partial}{\partial s} P[\Phi_R, \Phi_I; s] = \mathcal{H} P[\Phi_r, \Phi_I; s] \tag{3.13}$$

with

$$\mathcal{H} = \int d^4x [\alpha \frac{\delta^2}{\delta \Phi_R^2(x)} + \beta \frac{\delta^2}{\delta \Phi_I^2(x)} + \frac{\delta}{\delta \Phi_R(x)} (\frac{\delta S}{\delta \Phi(x)})_I - \frac{\delta}{\delta \Phi_I(x)} (\frac{\delta S}{\delta \Phi(x)})_R]. \tag{3.14}$$

Now the expectation value of an analytic functional $F[\Phi(x, s)]$ can be written as

$$\begin{aligned} \langle F[\Phi(x,s)] \rangle &= \int d[\Phi_R] d[\Phi_I] F[\Phi_R + i\Phi_I] P[\Phi_R, \Phi_I; s] \\ &\equiv \int d[\Phi_R] F[\Phi_R] P_{eff}[\Phi_R, s] \end{aligned} \tag{3.15}$$

where

$$P_{eff}[\Phi_R, s] = \int d[\Phi_I] e^{-i \int d^4x \Phi_I(x) [\delta / \delta \Phi_R(x)]} P[\Phi_R, \Phi_I, s] \tag{3.16}$$

is complex and obeys the effective Fokker-Planck equation

$$\frac{\partial}{\partial s} P_{eff}[\Phi_R; s] = \mathcal{H}_{eff} P_{eff}[\Phi_R; s], \tag{3.17}$$

$$\mathcal{H}_{eff} = \int d^4x \frac{\delta}{\delta \Phi_R} (\frac{\delta}{\delta \Phi_R} - i \frac{\delta S[\Phi_R]}{\delta \Phi_R}). \tag{3.18}$$

The effective Fokker-Planck equation has a stationary solution proportional to $e^{iS[\Phi_R]}$. The solution becomes a true equilibrium distribution if the action S is replaced by

$$\bar{S}[\Phi] = S[\Phi] + i\frac{\epsilon}{2} \int d^4x \Phi^2(x). \tag{3.19}$$

For then it can be shown that the eigenvalues of $\mathcal{H}_{eff}$ have the real part

$$\Re\lambda = \epsilon n, \qquad n = 0, 1, 2, \cdots. \tag{3.20}$$

With the same modification of the action as in (3.19), also numerical simulations indicate the existence of the equilibrium limit in Minkowski space for $\epsilon > 0$ (Callaway et al., 1985).

It is a remarkable fact that Minkowski space stochastic quantization distinguishes a pair of quantum states and thus proposes a solution to a well-known problem of quantum field theory in curved space-time, namely how to generalize the notion of vacuum state in Minkowski space. In stochastic quantization the generalization comes about as follows. Consider a linear (non-gauge) field Φ on a curved space-time manifold M. Its field equation is of the form

$$\frac{\delta S[\Phi]}{\delta \Phi} = V\Phi = 0 \tag{3.21}$$

where V is a self-adjoint linear differential operator defined on a suitable domain $D \subset L^2(\mathcal{M})$, the latter being the space of square-integrable functions φ on M obeying

$$\int_M d^4x(-g)^{1/2}|\varphi|^2 < \infty. \tag{3.22}$$

(Examples show that even for non-scalar fields a natural positive definite scalar product of the type appearing on the left-hand side of (3.22) exists (Rumpf, 1987).) The problem of generalizing the definition of vacuum may be stated equivalently as the problem of defining the Feynman propagator in curved space-time, since the propagator can be identified with the Schwinger average

$$K(x,x') = -i\frac{\langle\text{out}|T(\Phi(x)\Phi^*(x'))|\text{in}\rangle}{\langle\text{out}|\text{in}\rangle}. \tag{3.23}$$

This provides an implicit definition of the two states $|\text{out}\rangle$ and $|\text{in}\rangle$ generalizing the Minkowski vacuum. According to stochastic quantization,

$$K(x,x') = -i\lim_{s\to\infty}\langle\Phi(x,s)\Phi^*(x',s)\rangle \tag{3.24}$$

$$= -i\int_0^\infty ds e^{iVs} = (V + i0)^{-1} \tag{3.25}$$

i.e. $K(x,x')$ is the integral kernel of the operator (3.25). This result holds for an arbitrary initial distribution of fields $\Phi(x,0)$ subject to the regularity condition

$$\Phi(x,0) \in D_{a.c.} \tag{3.26}$$

where $D_{a.c.}$ is the subdomain of D introduced above that corresponds to the absolutely continuous part of the spectrum of V (i.e. normalizable eigenfunctions of V have to be excluded). The "resolvent property" (3.25) of K implies that the states $|\text{out}\rangle$ and $|\text{in}\rangle$ are Fock space vacua, at least if there are no singularities in the external field and the space-time topology is that of $\mathbf{R}^4$. These states provide a reasonable description of the creation of scalar and Dirac particles in a variety of external electromagnetic and gravitational fields (Rumpf, 1979 and 1983). We mention that $|\text{in}\rangle$ and $|\text{out}\rangle$ can be replaced by arbitrary quantum states also in the framework of stochastic quantization, if the regularity condition on the initial data is relaxed.

We are now in a position to apply Minkowski space stochastic quantization to linearized gravity. The analogs of the Langevin equation (2.24) and the correlation function (2.25) in the case of Lorentzian signature of the metric are

$$\frac{\partial h_{ab}}{\partial s} = i\eta_{ac}\eta_{bd}\frac{\delta S[h]}{\delta h_{cd}} + \xi_{ab} \tag{3.27}$$

$$\langle\xi_{ab}(x,s)\xi_{cd}(x',s')\rangle = (\eta_{ac}\eta_{bd} + \eta_{bc}\eta_{ad})\delta^{(4)}(x-x')\delta(s-s'). \tag{3.28}$$

The heat kernel (2.26) corresponds to the Schrödinger kernel

$$H^M(k,s) = e^{ik^2s}P^2 + e^{-2ik^2s}P^{0'} + P^1 + P^0 \tag{3.29}$$

which implies the following correlation function for the process $h_{ab}(k,s)$ defined by (3.27):

$$\langle h_{ab}(k,s)h_{cd}(k',s)\rangle = i(2\pi)^4\delta^{(4)}(k+k')\{\frac{1}{k^2}[1-e^{2ik^2s})P^2-$$

$$-\frac{1}{2}(1-e^{-4ik^2s})P^{0'}] - 2i(P^0+P^1)s\} \tag{3.30}$$

$$\stackrel{s\to\infty}{\to} i(2\pi)^4\delta^{(4)}(k+k')[\frac{P^2}{k^2+i0}-\frac{1}{2}\frac{P^{0'}}{k^2-i0}-i\infty^2(P^2+P^0)]. \tag{3.31}$$

(Note that only the principal part of $1/k^2$ enters in (3.30).) The Feynman propagator defined by (3.31) is free from the exponential divergence encountered in the Euclidean case and shows only the quadratically divergent pure gauge term which has no physical consequences. The gauge-independent part of the propagator reads

$$K_{abcd}(g.ind.) = \frac{1}{k^2+i0}(\frac{1}{2}\eta_{ac}\eta_{bd}+\frac{1}{2}\eta_{bc}\eta_{ad}-\frac{1}{3}\eta_{ab}\eta_{cd}) - \frac{1}{k^2-i0}\frac{1}{6}\eta_{ab}\eta_{cd} \tag{3.32}$$

The last term exhibits a non-causal pole structure which affects gauge-invariant expectation values. The Minkowski space quantization defined by (3.27) and (3.28) therefore does not reproduce the standard linearized gravity theory. A further modification will be required to obtain full equivalence. This will be discussed in detail in the next section.

In the remainder of this section we disregard the non-causal propagation and discuss the stochastic gauge fixing of linearized gravity, as the general features of it will survive the modifications of the next section. With this qualification it will turn out that the finite part of the above propagator is identical to the propagator in the Landau gauge with respect to a certain covariant gauge-fixing in the standard formalism. We recall that in the standard formalism one adds a gauge-fixing term to the classical Lagrangian. In the case of linearized gravity there is in fact a 2-parameter family of covariant and quadratic gauge-fixing terms, namely

$$\mathcal{L}_g^{(\lambda,\alpha)} = \alpha^{-1}C_a^{(\lambda)}C_b^{(\lambda)}\eta^{ab} \tag{3.33}$$

$$C_a^{(\lambda)} = \partial_c h_a{}^c - \lambda\partial_a h_c{}^c. \tag{3.34}$$

The modified action implies a modified kinetic operator $V^{(\lambda,\alpha)}$ and the corresponding propagator

$$K_{abcd}^{(\lambda,\alpha)} = [V^{(\lambda,\alpha)}]^{-1}_{abcd}. \tag{3.35}$$

The finite part of the propagator in (3.31) is identical to (3.35) for $\alpha = \lambda = 0$. Different $(0,\alpha)$ gauges can be obtained from (3.27) by choosing appropriate initial distributions of h instead of $h(s=0) = 0$. More general gauges, in particular the Feynman-de Donder gauge ($\lambda = 1/2, \alpha = 1$) with

$$K_{abcd}^{(1/2,1)} = \frac{1}{2k^2}(\eta_{ac}\eta_{bd}+\eta_{ad}\eta_{bc}-\eta_{ab}\eta_{cd}) = K_{abcd}^{(\lambda,\alpha)}(g.ind.) \tag{3.36}$$

can be reproduced in stochastic quantization by stochastic gauge-fixing (Zwanziger 1981).

In the gravity case, stochastic gauge fixing introduces a new process f_{ab} by the s-dependent gauge transformation

$$f_{ab} = h_{ab} + i(k_a \Lambda_b(k,s) + k_b \Lambda_a(k,s)). \tag{3.37}$$

This obeys in virtue of (3.27)

$$\dot{f}_{ab} = iV_{abcd} f_{cd} + i(k_a \dot{\Lambda}_b + k_b \dot{\Lambda}_a) + \xi_{ab}. \tag{3.38}$$

The drift term in this Langevin equation contains an additional (imaginary) "restoring force" along the gauge orbits. Therefore f will yield the same gauge-invariant expectation values as h, but may be free from the random walk behaviour of h. In order to find the conditions for this to happen, we consider the most general covariant gauge transformation that is linear in f:

$$\dot{\Lambda}_a^{(\alpha,\beta,\gamma)}(f) = \alpha^{-1}(k_b f_{ab} + \frac{\beta}{2} k_a f_{cc} + \frac{\gamma}{2} k_a k_b k_c k^{-2} f_{bc}). \tag{3.39}$$

It implies via (3.38)

$$\dot{f}_{ab} = iW_{abcd}^{(\alpha,\beta,\gamma)} f_{cd} + \xi_{ab} \tag{3.40}$$

where the operator W is self-adjoint only if $\beta = 0$ (otherwise the gauge-fixing force is non-holonomic already at the linearized level, in contrast to ordinary gauge theories). Consequently also

$$K^{(\alpha,\beta,\gamma)} = (W^{(\alpha,\beta,\gamma)} + i0)^{-1} \tag{3.41}$$

holds only if $\beta = 0$. It can be shown (Hüffel and Rumpf, 1985) that the transformations (3.39) can produce all the (λ, α)-gauges (3.33), (3.34) and even more. Specifically the connection between the various gauge parameters is

$$\beta = \frac{2}{\lambda - 1}(4\lambda^2 - 2\lambda + 1 - \alpha) \tag{3.42}$$

$$\gamma = \frac{2\lambda(4\lambda - \alpha - 1)}{1 - \lambda}. \tag{3.43}$$

As a peculiarity we mention that the ordinary gauges $(0,1)$ and $(1/2,1)$ correspond to the same stochastic gauge in the sense that

$$K^{(1,0,0)} = \begin{cases} K^{(1/2,1)} & \text{if } \beta = -\gamma/2 \to 0 \\ K^{(0,1)} & \text{otherwise} \end{cases} . \tag{3.44}$$

The reason for this is that the transformation $h_{ab} \to h_{ab} - \delta_{ab} h_{cc}/2$ is an involution and an isometry with respect to the metric $\eta_{ac}\eta_{bd} + \eta_{ad}\eta_{bc}$ on the space of symmetric tensors. Therefore the Langevin equation (3.27) gives equivalent results, if h is transformed this way. This is not true for the general transformation

$$\tilde{h}_{ab} = h_{ab} - \lambda \eta_{ab} h^c{}_c \qquad (\lambda \neq 1/4) \tag{3.45}$$

when $\lambda \neq 1/2$. Obviously, then, the outcome of the quantization prescription (3.27) depends on the choice of the field variable, and this poses the problem of the correct choice of that variable. This problem will be addressed in full generality in the next section.

4. GENERAL COVARIANCE IN STOCHASTIC QUANTIZATION

The problem raised at the end of the preceding section will be solved if we can formulate stochastic quantization in such a manner that it is independent of the choice of the field variable. We thus propose to generalize the Langevin equation (2.1) so that it becomes covariant with respect to general field redefinitions $\Phi^A \to \Phi'^A[\Phi]$ (we adopt the DeWitt (1965) notation of denoting the space-time argument x and any discrete indices of Φ collectively by A). In this section we shall return to the Euclidean regime, as the covariance and indefiniteness problems are quite separate issues. The following ansatz for the Langevin equation and the noise correlation fulfils the requirement of general covariance in field configuration space:

$$\dot{\Phi}^A(s) = -G^{AB}[\Phi]\frac{\delta S_E[\Phi]}{\delta\Phi^B(s)} + \xi^A(s) \tag{4.1}$$

$$\langle\xi^A(s)\xi^B(s')\rangle = 2\langle G^{AB}[\Phi]\rangle\delta(s-s'). \tag{4.2}$$

Here $G^{AB}[\Phi]$ is the inverse of a field metric $G_{AB}[\Phi]$. Its introduction is necessary to make (4.1) an equation among field vectors. The noise $\xi^A(s)$ is defined only implicitly by (4.2) because of the appearance of a stochastic average on the right hand side. A noise satisfying (4.2) can be constructed explicitly, however, upon the introduction of a deterministic reference metric $G^{(0)MN}$ and a vielbein (actually an "infinity-bein") $E_M{}^A[\Phi]$ fulfilling

$$G^{AA'}[\Phi] = E_M{}^A[\Phi]E_{M'}{}^{A'}[\Phi]G^{(0)MM'}. \tag{4.3}$$

The $\xi^A(s)$ can be defined in terms of a Gaussian reference noise $\xi^{(0)M}(s)$ with correlation

$$\langle\xi^{(0)M}(s)\xi^{(0)M'}(s')\rangle = 2G^{(0)MM'}\delta(s-s') \tag{4.4}$$

as

$$\xi^A(s) = E_M{}^A[\Phi]\xi^{(0)M}(s). \tag{4.5}$$

The noise $\xi^A(s)$ has the desired property (4.2), if $\Phi(s)$ is independent of $\xi^{(0)M}(s)$, i.e. if we adopt Ito's calculus:

$$\xi^A(s)ds = E_M{}^A[\Phi(s)]dW^{(0)M}(s) = \lim_{\Delta s\downarrow 0} E_M{}^A[\Phi(s)][W^{(0)M}(s+\Delta s) - W^{(0)M}(s)] \tag{4.6}$$

where $\xi^{(0)M}(s) = dW^{(0)M}/ds$ in the distributional sense ($W^{(0)}$ is a generalized Wiener process). Since in Ito's calculus $\dot{\Phi}^A$ does not transform like a field vector but rather according to

$$\dot{\Phi}'^A = \frac{\delta\Phi'^A}{\delta\Phi^B}\dot{\Phi}^B + G^{BC}\frac{\delta^2\Phi'^A}{\delta\Phi^B\delta\Phi^C}, \tag{4.7}$$

we have to modify (4.1) slightly and obtain the following final form of the manifestly covariant Langevin equation:

$$d\Phi^A - \Delta_G \Phi^A ds = -G^{AB} \frac{\delta S_E[\Phi]}{\delta \Phi^B} ds + E_M{}^A[\Phi(s)] dW^{(0)M}(s). \tag{4.8}$$

Here Δ_G is the Laplace-Beltrami operator for the field metric G_{AB}, and the left-hand side of (4.8) is indeed a field vector.

We remark that an equivalent definition of $\Phi^A(s)$ is implied by the textbook definition of Brownian notion on Riemannian manifolds (Ikeda and Watanabe 1981):

$$d\Phi^A = -G^{AB}[\Phi] \frac{\delta S_E[\Phi]}{\delta \Phi^B} ds + E_M{}^A[\Phi(s)] \circ dW^{(0)M}(s) \tag{4.9}$$

$$dE_M{}^A[\Phi(s)] = -\Gamma^A{}_{BC}[\Phi(s)] E_M{}^C[\Phi(s)] \circ d\Phi^B(s). \tag{4.10}$$

The Stratonovich product $\circ$ appearing in these equations is defined by

$$A(s) \circ dB(s) = \lim_{\Delta s \downarrow 0} \frac{1}{2}[A(s) + A(s + \Delta s)][B(s + \Delta s) - B(s)] \tag{4.11}$$

and $\Gamma^A{}_{BC}$ is the Levi-Civita connection of G_{AB}. Although $\dot{\Phi}^A$ is a field vector in the Stratonovich calculus, the Ito version of the Langevin equation is more practical for numerical simulations, as $\Phi(s)$ is non-anticipating and $E_M{}^A$ is unconstrained.

The generally covariant Fokker-Planck equation corresponding to (4.8) is

$$\frac{\partial Q}{\partial s} = \nabla^B \left(\frac{\delta Q}{\delta \Phi^B} + \frac{\delta S_E}{\delta \Phi^B} \right) Q \tag{4.12}$$

where Q is related to the Fokker-Planck probability P by

$$Q[\Phi, s] = |G|^{-1/2} P[\Phi, s] \tag{4.13}$$

$$G \equiv \det(G_{AB}) \tag{4.14}$$

and ∇_B is the covariant derivative corresponding to the Levi-Civita connection. The Fokker-Planck equation (4.12) has the stationary solution

$$P_{eq}[\Phi] \propto |G|^{1/2} e^{-S_E[\Phi]}. \tag{4.15}$$

Thus the desired equilibrium limit is formally implied by (4.8).

5. NONLINEAR GRAVITY

The generalized stochastic quantization scheme developed in the last two sections is applicable to the full Einstein gravity theory. We shall stick for the moment to the Euclidean version of the theory, since it will turn out to allow a sensible stochastic perturbation theory despite of the breakdown of the naive approach of section 2.

Our field variable for concrete calculations will be the standard parametrization of Riemannian metrics in terms of the covariant metric tensor field,

$$\Phi^A = g_{\alpha\beta}(x). \tag{5.1}$$

The use of the covariant Langevin equation (4.8) will ensure that the results will hold also for any other choice of Φ^A.

The choice of the field metric is more crucial, since it determines the path integral measure, as can be seen from (4.15) and (2.33). There are, however, two natural requirements that greatly restrict the set of possible field metrics G_{AB}. The first requirement is that $G_{AB}[\Phi]$ be local in Φ, and the second that the actions of diffeomorphisms on $g_{\alpha\beta}(x)$ be isometries with respect to G_{AB}. It has been known for a long time (DeWitt 1962) that the field metrics obeying these requirements form a 1-parameter family:

$$G_{AA'} \equiv G^{\alpha\beta,\alpha'\beta'}(x,x') = \frac{C}{2} g^{1/2}(g^{\alpha\alpha'}g^{\beta\beta'} + g^{\alpha\beta'}g^{\beta\alpha'} + \lambda g^{\alpha\beta}g^{\alpha'\beta'})\delta^{(4)}(x-x'). \tag{5.2}$$

The relevant parameter is λ, which has to be different from $-1/2$ for the field metric to be non-singular. The constant C is not important and will be chosen

$$C = (4\kappa)^{-1} \tag{5.3}$$

in the following.

Substituting the Einstein action (2.5) into the covariant Langevin equation (4.8) we obtain

$$\dot{g}_{\alpha\beta} + 18\kappa \frac{\lambda+1}{2\lambda+1}\delta^{(4)}(0)g^{-1/2}g_{\alpha\beta} = -2R_{\alpha\beta} + \frac{\lambda+1}{2\lambda+1}g_{\alpha\beta}R + \xi_{\alpha\beta}. \tag{5.4}$$

The divergent term on the left hand side stems from $-\Delta_G g_{ab}$ and vanishes for $\lambda = -1$. For the moment we shall leave the choice of λ open, however, as in 4 space-time dimensions the determinant G is independent of $g_{\alpha\beta}$ for all values of λ and therefore the formal equilibrium limit (4.15) does not depend on λ. This formal equilibrium limit is characterized by the partition function

$$Z = \int \mathcal{D}[g]e^{-S_E[g]} \tag{5.5}$$

with the measure

$$D[g] = \prod_x \prod_{\alpha\leq\beta} dg_{\alpha\beta}(x). \tag{5.6}$$

The measure is identical to that proposed by DeWitt (1962) and Fujikawa and Yasuda (1984). (Other covariant measures of the type $\prod g^r(x)\prod dg_{\alpha\beta}(x)$ can be obtained from stochastic quantization only if the term $-\Delta_G\Phi$ is dropped from the Langevin equation, i.e. field covariance is given up.)

Next we consider the linear approximation to (5.4). Splitting the metric according to (2.3), we obtain the linearized Langevin equation

$$\dot{h}_{ab} = -B_{ab}{}^{cd} + \tilde{\xi}^{(0)}_{ab} \tag{5.7}$$

$$B_{ab}{}^{cd} = G^0_{ab,ij}V^{ijcd}. \tag{5.8}$$

The kinetic operator was defined in (2.6), (2.7), (2.18), and the inverse reference metric of the linear approximation is given by

$$G^0_{ab,cd} = \frac{1}{2}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc} - \frac{\lambda}{2\lambda+1}\delta_{ab}\delta_{cd}). \tag{5.9}$$

The linearized *noise* correlation is

$$\langle \tilde{\xi}^{(0)}_{ab}(x,s)\tilde{\xi}^{(0)}_{cd}(x',s')\rangle = 2G^0_{ab,cd}\delta^{(4)}(x-x')\delta(s-s'). \tag{5.10}$$

The most remarkable property of the operator B is that it is positive for $-2 < \lambda < -1/2$. Therefore in this range of λ the stochastic perturbation theory for the gauge-invariant expectation values of Euclidean gravity is well-defined (though for $\lambda \neq -1$ the divergent term in (5.4) necessitates to perturb in the field variable of type $g^r g_{\alpha\beta}$ rather than in $g_{\alpha\beta}$).

There are several reasons to consider $\lambda = -1$ as the natural parameter value for the field metric. The strongest reason is that this special field metric is distinguished dynamically. The metric that governs the linearized Einstein dynamics can be read off the gauge independent part of the Feynman propagator

$$K_{abcd}(g.ind.) = k^{-2}G^0_{ab,cd}(\lambda = -1). \tag{5.11}$$

Another remarkable fact is that only for $\lambda = -1$ the operator B is a projection:

$$P_{grav} = B(\lambda = -1). \tag{5.12}$$

At this stage it may appear somewhat mysterious that we have got rid of the indefiniteness problem by the choice of a certain field metric. In fact the indefiniteness is still there, but it resides now in the field metric, which is pseudo-Riemannian for $\lambda < -1/2$. In this case the correlation function (5.10) implies that the probabilistic interpretation of the noise $\tilde{\xi}^{(0)}$ can be maintained only if it is assumed to be complex:

$$\tilde{\xi}^{(0)}_{ab} = (1_{abcd} - \frac{1}{4}\delta_{ab}\delta_{cd} + \frac{i}{4}\delta_{ab}\delta_{cd})\chi_{cd} \qquad (\lambda = -1). \tag{5.13}$$

Here χ_{cd} is a real noise with correlation

$$\langle \chi_{ab}(x,s)\chi_{cd}(x',s')\rangle = 2 1_{abcd}\delta^{(4)}(x-x')\delta(s-s'). \tag{5.14}$$

As a consequence $h_{ab}(x,s)$ is a complex process. It converges to an equilibrium distribution, if (5.7) is modified by stochastic gauge fixing. Again putting $\lambda = -1$, the appropriate modification of (5.7) is of the form

$$\dot{h}_{ab} = -(P_{grav}h)_{ab} + \partial_a\Lambda_b + \partial_b\Lambda_a + \tilde{\xi}^{(0)}_{ab}. \tag{5.15}$$

The simplest choice for the gauge function Λ is

$$\Lambda_a = h_{ab,b} - \frac{1}{2} h_{bb,a}. \tag{5.16}$$

It implies

$$\dot{h} = \Box h + \tilde{\xi}^{(0)}. \tag{5.17}$$

The Fokker-Planck equation corresponding to (5.18) is

$$\dot{P} = \int d^4x \{\frac{\delta}{\delta h_R}[\ \Box h_R + (1 - \frac{\delta\delta}{4})\frac{\delta}{\delta h_R}] + \frac{\delta}{\delta h_I}[-\Box h_I + \frac{\delta\delta}{4}\frac{\delta}{\delta h_I}]\} P[h_r, h_I; s] \tag{5.18}$$

where we have used an obvious abbreviation for the two projection operators stemming from (5.13) and introduced real and imaginary parts as in (3.8). The equilibrium limit implied by the stationary solution of (5.18) is

$$P_{eq} \propto \delta[\frac{\delta\delta}{4} h_R]\delta[(1 - \frac{\delta\delta}{4})h_I] \cdot \exp[\frac{1}{2}\int h_R \Box (1 - \frac{\delta\delta}{4})h_R + \frac{1}{2}\int h_I \Box \frac{\delta\delta}{4} h_I]. \tag{5.19}$$

This means that in the partition function the conformal mode $\delta\delta h/4$ is integrated over along the imaginary axis. We have thus obtained from stochastic quantization the path integration contour of Gibbons et al. (1978), which solves the indefiniteness problem in perturbation theory.

As far as higher order perturbations are concerned, we shall confine ourselves here to a few sketchy remarks (for more details see (Rumpf, 1986)). For perturbations around flat space-time it is convenient to choose the reference metric as

$$G^{(0)AA'} = G^{AA'}(g^{(0)}), \qquad g^{(0)}_{ab} = \delta_{ab} \tag{5.20}$$

and the stochastic vielbein as

$$E_M{}^A \equiv E^{mn}{}_{\alpha\beta} = g^{-1/2} e^m{}_\alpha e^n{}_\beta \tag{5.21}$$

with $e^m{}_\alpha$ an orthonormal tetrad obeying

$$\delta_{mn} e^m{}_\alpha e^n{}_\beta = g_{\alpha\beta}. \tag{5.22}$$

For instance we may choose

$$e^m{}_\alpha(x, s) = (g^{1/2})_{m\alpha} \tag{5.23}$$

where $g^{1/2}$ is defined perturbatively as

$$g^{1/2} = (1 + 2\kappa^{1/2} h)^{1/2} = 1 + \kappa^{1/2} h - \frac{\kappa}{2} h^2 + \cdots, \tag{5.24}$$

1 denoting the 4×4 unit matrix. Substituting (2.3) and the above perturbative expansion of $E_M{}^A$ into the Langevin equation (5.4) ($\lambda = -1$), the stochastic equation for $h_{ab}(x, s)$ assumes the following structure:

$$\dot{h} + Bh = I(h, \partial h) + J(h)\tilde{\xi}^{(0)} + \tilde{\xi}^{(0)}. \tag{5.25}$$

Here I is the sum of all contributions of nonvanishing powers of $\kappa^{1/2}$ to the Einstein equations, while $J(h)$ has its origin in the perturbative expansion of the stochastic vielbein and is also a series of positive powers of $\kappa^{1/2}$. Eq. (5.25) can be solved iteratively by

$$h(s) = \int_0^\infty d\sigma H(s-\sigma)[I(h(\sigma), \partial h(\sigma)) + J(h(\sigma))\tilde{\xi}^{(0)}(\sigma) + \tilde{\xi}^{(0)}(\sigma)] \tag{5.26}$$

where

$$H(s) = e^{-Bs}. \tag{5.27}$$

The right hand side of (5.26) can be represented graphically by a sum of tree diagrams. This is quite analogous to the Yang-Mills case except for the contributions of $J\tilde{\xi}^{(0)}$. The latter are characteristic for gravity and require the introduction of a new type of vertex which may be called "stochastic vertex" (the order $\kappa^{n/2}$ contribution of $J^{(n)}\tilde{\xi}^{(0)}$ corresponding to a stochastic $(n+1)$-vertex). From (5.26) one may also deduce the diagrammatical structure of the stochastic n-point functions, which will involve as graphical elements the stochastic vertices as well as the "stochastic propagator" $\langle h(s)h'(s')\rangle$ which plays an analogous role as in non-gravitational theories.

We shall not deal with the problem of regularizing the divergencies that arise in the stochastic perturbation theory. They are of a similar type as those encountered in standard perturbation theory, and the non-renormalizability of quantum gravity certainly persists also in the stochastic approach. Recently a continuum regularization method employing a regularized field metric has been proposed by Halpern and collaborators (Chan and Halpern, 1987). Using this method explicit calculations in stochastic perturbation theory have been performed at the one-loop level.

We conclude with some remarks on the Lorentzian case. It is straight-forward to transcribe all the equations of this and the preceding section to that case simply by replacing the Euclidean by the Lorentzian metric and S_E by $-iS$. As explained in section 3, convergence to equilibrium will then hold only in a more refined methematical framework. The most important result that has been obtained in this framework is that the graviton propagator is causal for $-2 < \lambda < -1/2$, i.e.

$$K = \frac{1}{k^2 + i0} G^0(\lambda = -1) + \text{gauge variant terms}. \tag{5.28}$$

The propagator can be represented as

$$K = \lim_{m^2 \to 0} [V - (m^2 - i0)G^0]^{-1}, \tag{5.29}$$

i.e. it can be obtained from a certain massive extension of linearized gravity. In the case $\lambda = -1$ this massive extension is the unique one which contains no tachyon. We note that the spin-2 theory of Fierz and Pauli (see e.g. Wentzel 1949) which has the same property is not a genuine massive extension because of the van Dam-Veltman (1970) mass discontinuity.

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