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CONFINEMENT MODELS FOR GLUONS *

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ABSTRACT

Confinement model for gluons using a 'colour super current' is formulated. An attempt has been made to derive a suitable dielectric function corresponding to the current confinement model. A simple inhomogeneous dielectric confinement model for gluons is studied for comparison. The model Hamiltonians are second quantized and the glueball states are constructed. The spurious motion of the centre of confinement is accounted for. The results of the current confinement scheme are found to be in good agreement with the experimental candidates for glueballs.

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1. INTRODUCTION

It has been well accepted that quantum chromo dynamics is a prime candidate for the theory of strong interactions. This is a nonabelian gauge theory governing the colour dynamics of quarks and gluons. The gluons which are the quanta of the colour field carry colour charges and they interact among themselves. Evidence for such interactions is the very existence of glueballs (the colour singlet bound states of multigluons). Thus the study of glueballs and its experimental confirmation is very crucial to the validity of quantum chromodynamics Since these coloured gluons are many in number (SU3-octet) the coupled equations obeyed by them are too complex to solve simultaneously. However an important feature of any theory describing colour dynamics should be the confinement of colour. The only indication for the confinement is from the lattice simulations (K.G.Wilson. 1974) apart from its experimental confirmations. Hence to understand the physical reality of its microscopic structure, one has to go for phenomenological models (de Rujula 1975; Isgur and Carl 1977).

Here we consider all the eight gluons described by the Yang-Mills field tensor which are of equal strength and the coupled nonlinear term in the field tensor corresponds to a source. It can be seen that this source in general is a function of the field potentials and its derivatives. This source is treated as a colour super current in analogy with Ginzburg-Landau's theory of super conductivity. In this picture we consider the gluon field as quasi Maxwellian field. The details are given in section-2.As a special case simple confinement scheme for gluons similar to that of relativistic harmonic (vector+scalar) potential model (RHM) for quarks (Khadkikar and Gupta 1983) is discussed in detail in section-3. The confined quasi gluon modes are obtained in the general frame of Lorentz gauge with a subsidiary condition called the oscilator gauge condition (Khadkikar 1985).These confined modes are quantized and the energies are calculated. In section-4 we obtained a dielectric function corresponding to the source in the above sections; to look at this scheme in line with the dielectric confinement model as proposed by T.D.Lee.The dielectric function we obtained is in general non local and inhomogeneous. As the dynamical dependence is neglected the function reduces to that of a simple inhomogeneous dielectric medium. The confinement model for the quasi gluons in such a dielectric medium is discussed in section-5.

The phenomenology of glueballs are described in section-6 and the di-gluon and tri-gluon colour singlet states are constructed taking into account the spurious motion of the center. Finally the parameter is fixed by fitting iota(0^{-+}) 1440 MeV as a di- gluon glueball candidate and predicted all other low lying di-gluon and tri-gluon states. We discuss our results in section-7 with the present status of the experimental results and compare with naive bag model results for glueballs.

2.GLUONS AS QUASI MAXWELLIAN FIELD POTENTIALS

For a pure colour gluon field the Lagrangian density is given by the Yang-Mills field tensor.

$$\mathcal{L} = -1/4\, F^{l}_{\mu\nu}\, F^{l}_{\mu\nu} \qquad (2.1)$$

$$\text{where} \quad F^{l}_{\mu\nu} = f^{l}_{\mu\nu} + G^{l}_{\mu\nu} \qquad (2.2)$$

$$\text{with} \quad f^{l}_{\mu\nu} = \partial_\mu A^{l}_{\nu} - \partial_\nu A^{l}_{\mu} \qquad (2.3)$$

$$\text{and} \quad G^{l}_{\mu\nu} = g\, f^{lmn}\, A^{m}_{\mu}\, A^{n}_{\nu} \qquad (2.4)$$

the colour indices (l,m,n) carry 1,2, ...8 and (μ,ν) are the four vector indices.By variational principle the equation of motion for the field is obtained as

$$\partial_\mu F^{l}_{\mu\nu} + g f^{lmn}\, A^{m}_{\mu}\, F^{n}_{\mu\nu} = 0 \qquad (2.5)$$

Substituting for $F^{n}_{\mu\nu}$ from eqns.2.2- 2.4 we get

$$\partial_\mu f^{l}_{\mu\nu} = -g f^{lmn} [A^{m}_{\mu}\partial_\mu A^{n}_{\nu} + \partial_\mu A^{m}_{\mu} A^{n}_{\nu}$$

$$+ A^{m}_{\mu}\partial_\mu A^{n}_{\nu} - A^{m}_{\mu}\partial_\nu A^{n}_{\mu}$$

$$+ g f^{n l' m'} A^{m}_{\mu} A^{l'}_{\mu} A^{m'}_{\nu}] \qquad (2.6)$$

g is the coupling constant· f^{lmn} is the SU(3) structure constant. The r.h.s of the eqn can be considered as a super current of the colour gluons in analogy with Ginzburg Landau's theory of super conductivity. And in an external gluon field say, A^1_μ with some uniform order parameter for all gluon pairs with the same scale parameter as that corresponding to the confinement, this colour super current is assumed as

$$J^1_\nu = \theta_{\mu\nu} A^1_\mu \qquad (2.7)$$

similar to that of the London equation in super conductivity. With this picture we treat the gluon field A^1 as quasi Maxwellian field potentials satisfying the dynamical eqn.

$$\partial_\mu f^1_{\mu\nu} = -J^1_\nu \qquad (2.8)$$

3. CURRENT CONFINEMENT MODEL FOR GLUONS (CCM) :

In this section we consider the gluon field in a quasi-Maxwellian theory with a confinement current assumed similar to that in eqn 2.7.

$$J_\mu = \theta_{\mu\nu} A_\nu \qquad (3.1)$$

where $A_\nu = (\underset{\sim}{A}, \Phi)$.

For simplicity and in analogy with the relativistic harmonic confinement model (RHM) for quarks (Khadkikar and Gupta 1983) we choose

$$\theta_{\mu\nu} = -\delta_{\mu\nu}\,\theta_\mu \qquad (3.2)$$

and

$$\theta_\mu = 2\alpha\,\delta_{\mu 0} - \alpha^2 r^2 \qquad (3.3)$$

The Lagrangian density for this quasi Maxwellian gluon can now be written as

$$\mathcal{L} = -1/4\, f_{\mu\nu} f_{\mu\nu} + 1/2\, \theta_{\mu\nu} A_\mu A_\nu + 1/2(\partial_\mu A_\mu)^2 \qquad (3.4)$$

where $$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad (3.5)$$

is the quasi colour gluon field tensor. Here the colour labels are omitted since all the fields carry same colour index.

From the variational principle.

$$\delta \int d^3r\, dt\, \mathcal{L} = 0 \qquad (3.6)$$

we find the field eqn. for the vector potential $\underset{\sim}{A}$ as

$$-\nabla^2 \underset{\sim}{A} + \theta \underset{\sim}{A} + \ddot{\underset{\sim}{A}} = 0 \qquad (3.7)$$

with the Lorentz gauge condition

$$\partial_\mu A_\mu = \underset{\sim}{\nabla} \cdot \underset{\sim}{A} + \dot{\Phi} = 0 \qquad (3.8)$$

where the number of dots above the variable represent the order of time derivatives.

To carry out the quantization in the Lorentz gauge we observe that Φ is a dependent variable. The conjugate momenta of $\underset{\sim}{A}$ is

$$\underset{\sim}{P} = \dot{\underset{\sim}{A}} + \underset{\sim}{\nabla}\Phi \tag{3.9}$$

and that of Φ is

$$\pi = \underset{\sim}{\nabla}\cdot\underset{\sim}{A} + \dot{\Phi} = 0 \tag{3.10}$$

as given by eqn.3.8.Consequently all the time derivatives of eqn.3.10 also vanish i.e.

$$\dot{\pi} = \frac{\partial\mathcal{L}}{\partial\Phi} = -\underset{\sim}{\nabla}\cdot(\dot{\underset{\sim}{A}} + \underset{\sim}{\nabla}\Phi) + \theta'\Phi = 0 \tag{3.11}$$

Thus from eqn. 3.9 and 3.11, we have

$$\Phi = (\underset{\sim}{\nabla}\cdot\underset{\sim}{P})/\theta' \tag{3.12}$$

where θ and θ' are the vector and scalar components of θ_μ associated with $\underset{\sim}{A}$ and Φ of A_μ respectively.Eliminating from eqns.3.8 and 3.9.

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{A} + (\underset{\sim}{\nabla}\cdot\underset{\sim}{P})/\theta' = 0 \tag{3.13}$$

and

$$\underset{\sim}{P} = \dot{\underset{\sim}{A}} + \underset{\sim}{\nabla}(\underset{\sim}{\nabla}\cdot\underset{\sim}{P})/\theta' . \tag{3.14}$$

In order to derive the Hamiltonian, we regard $\underset{\sim}{A}$ and $\underset{\sim}{P}$ as

independent variables where as Φ is a function of $\underset{\sim}{P}$ from eqn.3.12.Thus the Hamiltonian

$$H = 1/2\int d^3r\,[\,\underset{\sim}{P}^2 + (\underset{\sim}{\nabla}\cdot\underset{\sim}{P})^2/\theta' + \theta\underset{\sim}{A}^2 - (\underset{\sim}{\nabla}\cdot\underset{\sim}{A})^2 - \underset{\sim}{A}\cdot\nabla^2\underset{\sim}{A}\,]. \tag{3.15}$$

The Hamiltons eqns. of motion for $\underset{\sim}{A}$ and $\underset{\sim}{P}$:

$$\dot{\underset{\sim}{A}} = \partial H/\partial\underset{\sim}{P} = \underset{\sim}{P} - \underset{\sim}{\nabla}(\underset{\sim}{\nabla}\cdot\underset{\sim}{P})/\theta' \tag{3.16}$$

$$\dot{\underset{\sim}{P}} = -\partial H/\partial\underset{\sim}{A} = \nabla^2\underset{\sim}{A} - \theta\underset{\sim}{A} - \underset{\sim}{\nabla}(\underset{\sim}{\nabla}\cdot\underset{\sim}{A}). \tag{3.17}$$

Taking the second time derivatives of $\underset{\sim}{A}$ and $\underset{\sim}{P}$,we get

$$\ddot{\underset{\sim}{A}} = \nabla^2\underset{\sim}{A} - \theta\underset{\sim}{A} + \underset{\sim}{\nabla}[\underset{\sim}{\nabla}\cdot\theta\underset{\sim}{A} - \theta'\underset{\sim}{\nabla}\cdot\underset{\sim}{A}]/\theta' \tag{3.18}$$

$$\ddot{\underset{\sim}{P}} = \nabla^2\underset{\sim}{P} - \theta\underset{\sim}{P} + [\theta\underset{\sim}{\nabla} - \underset{\sim}{\nabla}\theta']\,(\underset{\sim}{\nabla}\cdot\underset{\sim}{P})/\theta' \tag{3.19}$$

using eqn. 3.17. the Lorentz condition given by eqn 3.13 leads to

$$\underset{\sim}{\nabla}\cdot\theta\underset{\sim}{A} - \theta'\underset{\sim}{\nabla}\cdot\underset{\sim}{A} = 0 . \tag{3.20}$$

This eqn. with the choice of θ and θ' given in eqns 3.2 and 3. 3 leads to the oscilator gauge condition (Khadkikar S.B.

1985).

$$[\underset{\sim}{\nabla} + \alpha \underset{\sim}{r}] \cdot \underset{\sim}{A} = 0 . \qquad (3.21)$$

It has been seen that the requirement of the conservation of the induced four vector current also demands the oscillator condition. In terms of oscillator operators defined by

$$\underset{\sim}{a} = (2 \alpha)^{-1/2} (\underset{\sim}{\nabla} + \alpha \underset{\sim}{r}) \qquad (3.22)$$

and its Hermitian conjugate

$$\underset{\sim}{a}^{+} = (2 \alpha)^{-1/2} (- \underset{\sim}{\nabla} + \alpha \underset{\sim}{r}) \qquad (3.23)$$

Now the eqn. 3.21 reduces to

$$\underset{\sim}{a} \cdot \underset{\sim}{A} = 0 . \qquad (3.24)$$

Similarly the eqn. for $\underset{\sim}{A}$ taking the time variations of the field as exp (-iWt), becomes

$$(\underset{\sim}{a} \cdot \underset{\sim}{a}^{+} + \underset{\sim}{a}^{+} \cdot \underset{\sim}{a}) \underset{\sim}{A} = w^{2} \underset{\sim}{A} . \qquad (3.25)$$

The scalar oscillator wave function corresponding to this eqn. is given by

$$\Psi_{n'lm} = N_{n'l} [\underset{\sim}{a}^{+} \cdot \underset{\sim}{a}^{+}]^{n'} Y_{lm}(\underset{\sim}{a}^{+}) \Psi_0 \qquad (3.26)$$

where

$$\Psi_0 = [\alpha^{1/2} \pi^{-1/2}]^{3/2} \exp(-1/2 \alpha^{2} r^{2}) \qquad (3.27)$$

and

$$N_{n'l} = [4 \pi / 2n'!! (2n' + 2l + 1)!!]^{1/2} \qquad (3.28)$$

with oscillator eigenvalues

$$E_n = w_n^2 = (2n + 3)\alpha \quad n = 2n' + 1 \qquad (3.29)$$

where $Y_{lm}(\underset{\sim}{a}^{+})$ is the solid spherical harmonics. Hence the solution for $\underset{\sim}{A}$ can be written as

$$\underset{\sim}{A} = \underset{\sim}{e} \, \Psi_{nlm} \qquad (3.30)$$

e represents the direction of polarization of the field A. In the oscillator space, we choose

$$\underset{\sim}{e}^{1} \propto i \, \underset{\sim}{a} \times \underset{\sim}{a}^{+}$$

$$\underset{\sim}{e}^{2} \propto \underset{\sim}{a} \times (\underset{\sim}{a} \times \underset{\sim}{a}^{+})$$

$$\underset{\sim}{e}^{3} \propto \underset{\sim}{a}^{+} \qquad (3.31)$$

Then the oscillator gauge condition gives the third component of $\underset{\sim}{A}$

$$A_3 = 0 \quad \text{,classically .} \qquad (3.32)$$

Then the remaining transverse components of the fields are given by

$$A^1_{nJm} = N^1_{nJ}\, i(\underline{a} \times \underline{a}^+)\, \Psi_{nJm}\, e^{-iW_n t} \qquad (3.33)$$

$$A^2_{nJm} = N^2_{nJ}[\underline{a} - \underline{a}^+(1/\underline{a}\cdot\underline{a}^+)\underline{a}\cdot\underline{a}]\Psi_{n+1Jm}\, e^{-iW_n t} \qquad (3.34)$$

and the corresponding conjugate momenta

$$P^1_{nJm} = -M^1_{nJ}(\underline{a} \times \underline{a}^+)\Psi_{nJm}\, e^{iW_n t} \qquad (3.35)$$

$$P^2_{nJm} = M^2_{nJ}\, i[\underline{a} - \underline{a}^+(1/\underline{a}\cdot\underline{a}^+)\underline{a}\cdot\underline{a}]\Psi_{n+1Jm}\, e^{iW_n t} \qquad (3.36)$$

$$\text{and} \quad P^3 = \nabla_3 \,[\, (1/(\theta' - \nabla^2))\, \nabla \cdot \dot{A}^2 \,] \qquad (3.37)$$

where the normalisation constants are

$$N^1_{nJ} = [\, J(J+1)\, 2\, \alpha^{1/2}\, (2n+3)^{1/2} \,]^{-1/2}$$

$$N^2_{nJ} = [\, J(J+1)/(n+1)\cdot 2\, \alpha^{1/2}\, (2n+3)^{1/2} \,(1 - J(J+1)/\{2(2n+3)(n+1)\}\,)]^{-1/2}$$

$$M^1_{nJ} = [\, \alpha^{1/2}\, (2n+3)^{1/2} \,/\, 2J(J+1) \,]^{1/2}$$

$$M^2_{nJ} = [\, \alpha^{1/2}(2n+3)^{1/2}(n+3)/\{2J(J+1)\}\,(1 - J(J+1)/\{(2n+3)(n+1)\}\,)\,]^{1/2} \qquad (3.38)$$

where $\underset{\sim}{J} = \underset{\sim}{L} + \underset{\sim}{S}$ is the total angular momentum of the fields. $\underset{\sim}{L}$ is the angular momentum and $\underset{\sim}{S}$ is the spin operator.

For quantisation we expand the quasi gluon fields in the above eigen basis to get the gluon energy in terms of the frequency W_n

$$\underset{\sim}{A} = \sum_{nJM\lambda} [\, C_{nJM\lambda}\, A^{\lambda}_{nJM} + C^+_{nJM\lambda} A^{\lambda *}_{nJM}] \qquad (3.39)$$

where λ refers to the type of the mode (magnetic/electric). $C_{nJM\lambda}$ and $C^+_{nJM\lambda}$ are the annihilation and creation operators satisfying the commutation relations

$$[\, C_{nJM\lambda} \,,\, C^+_{n'J'M'\lambda'} \,] = \delta_{nn'}\, \delta_{JJ'}\, \delta_{MM'}\, \delta_{\lambda\lambda'}$$

and

$$[C_{nJM\lambda}\,, C_{n'J'M'\lambda'}] = [C^+_{nJM\lambda}\,, C^+_{n'J'M'\lambda'}] = 0. \qquad (3.40)$$

The condition on A_3 here is now replaced by

$$A_3\, |\text{physical}\rangle = 0 \qquad (3.41)$$

and that on $\underset{\sim}{P}^3$:

$$\left[\underset{\sim}{P}^3 - \nabla_3 \left(1/(\theta' - \nabla^2) \ \nabla \cdot \dot{A}^2 \right)\right] \mid \text{physical} \rangle = 0 \qquad (3.42)$$

However the Hamiltonian is independent of $\underset{\sim}{P}^3$. Finally the Hamiltonian from eqn 3.15 becomes

$$H = \sum_{nJM\lambda} W_n \left(C^+_{nJM\lambda} C_{nJM\lambda} + 3/2 \right) . \qquad (3.43)$$

From eqn 3.29 the frequency

$$W_n = (2n+3)^{1/2} \quad \alpha^{1/2} \qquad (3.44)$$

where $C^+_{nJM\lambda} C_{nJm\lambda}$ are the number operators in the quantum state $(nJM\lambda)$ of the modes $\lambda(=1, 2)$. The low lying digluon and trigluon glueball states are calculated in section 6.

4. EXPRESSION FOR A DIELECTRIC FUNCTION :

Classically the Maxwell s displacement current $\underset{\sim}{D}$ can be written as

$$\underset{\sim}{D} = \underset{\sim}{E} + \underset{\sim}{P} \qquad (4.1)$$

where $\underset{\sim}{E}$ is the electric field causing a polarization and $\underset{\sim}{P}$ is the polarization vector current. For a source free case

$$\nabla \cdot \underset{\sim}{D} = 0 \qquad (4.2)$$

and

$$\nabla \times \underset{\sim}{B} = \partial \underset{\sim}{D} / \partial t . \qquad (4.3)$$

Thus we get the induced polarization charge

$$\rho = - \nabla \cdot \underset{\sim}{P} \qquad (4.4)$$

and the corresponding induced polarization current density

$$\underset{\sim}{J} = \partial \underset{\sim}{P} / \partial t . \qquad (4.5)$$

Now considering the current $\underset{\sim}{J}$ defined in the previous sections 2 and 3 as a polarization current in an external gluon field then

$$J_\mu = (\partial \underset{\sim}{P} / \partial t \ ; - \nabla \cdot \underset{\sim}{P}) \qquad (4.6)$$

$$\underset{\sim}{P} = - \underset{\sim}{J} / iW \qquad (4.7)$$

from eqns. 3.1-3.4

$$\underset{\sim}{P} = - \theta A / iW \qquad (4.8)$$

using the expression for $\underset{\sim}{E}$

$$\underset{\sim}{E} = iW \underset{\sim}{A} - \nabla \Phi$$

and

$$\nabla \cdot \underset{\sim}{E} = iW \nabla \cdot \underset{\sim}{A} - \nabla^2 \Phi \qquad (4.9)$$

Using Lorentz condition, we eliminate $\underset{\sim}{A}$ from eqn. 4.9:

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{E} = -W^2\Phi - \nabla^2\Phi \quad . \tag{4.10}$$

Hence,

$$\Phi = -\left(1/(W^2 + \nabla^2)\right)\underset{\sim}{\nabla}\cdot\underset{\sim}{E} \quad . \tag{4.11}$$

then equation for $\underset{\sim}{P}$ reduces to

$$\underset{\sim}{P} = (\theta/W^2)\ [\underset{\sim}{E} + \underset{\sim}{\nabla}\Phi] \quad . \tag{4.12}$$

Substituting Φ from eqn. 4.11

$$\underset{\sim}{P} = (\theta/W^2)\ [\underset{\sim}{E} - \underset{\sim}{\nabla}\left(1/(W^2 + \nabla^2)\right)\underset{\sim}{\nabla}\cdot\underset{\sim}{E}] \tag{4.13}$$

$$= (\theta/W^2)\ [1 - \underset{\sim}{\nabla}\left(1/(W^2 + \nabla^2)\right)\underset{\sim}{\nabla}\cdot\ ]\ \underset{\sim}{E} \tag{4.14}$$

thus from eqn. 4.1

$$\underset{\sim}{D} = \epsilon(\theta, \underset{\sim}{\nabla})\ \underset{\sim}{E} \tag{4.15}$$

where the dielectric function is now a non local operator given by

$$\epsilon(\theta, \underset{\sim}{\nabla}) = 1 + \theta/W^2(1 - \underset{\sim}{\nabla}\left(1/(W^2 + \nabla^2)\right)\underset{\sim}{\nabla}\cdot\ ) \tag{4.16}$$

Thus to get an identical confinement scheme as the CCM one has to use the dielectric function

$$\epsilon(\underset{\sim}{r}, \underset{\sim}{\nabla}) = 1 - \alpha^2 r^2/W^2\cdot(1 - \underset{\sim}{\nabla}\left(1/(W^2 + \nabla^2)\right)\underset{\sim}{\nabla}\cdot) \tag{4.17}$$

5. DIELECTRIC CONFINEMENT MODEL FOR GLUONS (DCM) :

Here we consider the dielectric function obtained in section 4 eqn. 4. 17 as a confinement medium for gluons,as proposed by T.D.Lee (1979). Neglecting the non locality this fuction reduces to a simple inhomogeneous function.

$$\epsilon(r) = 1 - a^2 r^2 \quad . \tag{5.1}$$

where a is a nonzero constant parameter, r is the spatial coordinate.For any value of $r \neq 0$, $\epsilon(r) < 1$ gives the antiscreening property as required by QCD vaccum (T.D. Lee 1979) ;while when $r \to 0$, $\epsilon(r) \to 1$ corresponds to the asymptotic freedom. And as $r \to 1/a$, $\epsilon(r) \to 0$ corresponds to perfect dielectric nature of the medium: therefore the colour electric field is pushed inside the region, leading to confinement. In the case of bag models $\epsilon(r)$ is taken as a step function with $\epsilon(r) = 1$ inside the bag and $\epsilon(r) = 0$ outside the bag surface.Here we have a smooth radially varying function avoiding such sharp transition.

In this quasi-classical macroscopic picture of the medium, we have to solve the Maxwell type of eqns. to get the quasi gluon fields, characterised by the quasi gluon electric field $\underset{\sim}{E}$ and magnetic field $\underset{\sim}{B}$:

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \partial \mathbf{D}/\partial t$$

$$\nabla \cdot \mathbf{D} = 0 \qquad (5.2)$$

where $$\mathbf{D} = \epsilon(r)\,\mathbf{E} \qquad (5.3)$$

These field strengths can be expressed in terms of the quasi gluon potential ($\mathbf{A}$, Φ) as

$$\mathbf{E} = -\partial \mathbf{A}/\partial t - \nabla \Phi$$

$$\mathbf{B} = \nabla \times \mathbf{A} \,. \qquad (5.4)$$

Assuming the time variation of the field as e^{-iWt} ,we obtain the stationary wave eqn for $\mathbf{A}$:

$$\nabla^2 \mathbf{A} + W^2 \epsilon(r) \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - iW\,\epsilon(r)\nabla\Phi \quad . \qquad (5.5)$$

Because of the spatial dependence of the $\epsilon(r)$.eqn.5.5 for $\mathbf{A}$ cannot be made homogeneous as in the case of bag model cavity eigen mode for $\mathbf{A}$ (J.Kuti 1977).To obtain homogeneous eqn. for simple eigen modes we require a gauge condition such that

$$\nabla(\nabla \cdot \mathbf{A}) - iW\,\epsilon(r)\nabla\Phi = 0 \qquad (5.6)$$

Obviously, the Lorentz condition cannot provide this requirement for a general Φ. The above condition leads to Lorentz condition as $r \to 0$ (asymptotic regions).And as these quasi gluons move away from the asymptotic region to confinement region where $r \to 1/a$.

$$\nabla \cdot \mathbf{A} = 0 \,. \qquad (5.7)$$

Then in this region the two transverse eigen modes can be defined as

$$\mathbf{A}^{TE} = \mathbf{L}\,\Psi_{nlm} \qquad (5.8)$$

and

$$\mathbf{A}^{TM} = \nabla \times \mathbf{L}\,\Psi_{nlm} \qquad (5.9)$$

where TE and TM represent the transverse electric and magnetic modes. $\mathbf{L}$ is the angular momentum operator. Ψ_{nlm} is the solution satisfying the homogeneous scalar wave eqn.given by

$$\Psi_{nlm} = N_{nl}(\alpha_n r)^l \exp(-1/2\,\alpha_n^2 r^2)\, L^{l+1/2}_{(n-1)/2}(\alpha_n^2 r^2)\, Y_{lm}(\theta, \varphi) \,. \qquad (5.10)$$

The size parameter α_n is

$$\alpha_n = (a W_n)^{1/2} \qquad (5.11)$$

where

$$W_n = (2n+3) a \qquad (5.12)$$

$L_n^l(Z)$ is the associated Laguerre polynomial. the normalization factor is

$$N_{nl} = [2 \alpha_n^3 ((n-l)/2)! / 4\pi \Gamma((n+3+l)/2)]^{1/2} \qquad (5.13)$$

In terms of the vector spherical harmonics

$$A^{TM}_{nJM} = (2W_n \epsilon(r))^{-1/2} [(J/(2J+1))^{1/2} R_{nJ+1}(r)\, Y_{JJ+1M}(\hat{n}) + ((J+1)/(2J+1))^{1/2} R_{nJ-1}(r)\, Y_{JJ-1M}(\hat{n})] \exp(-iW_n t) \qquad (5.14)$$

with parity

$$P = (-1)^J \qquad (5.15)$$

and

$$A^{TE}_{nJM} = (2W_n \epsilon(r))^{-1/2} R_{nJ}(r)\, Y_{JJM}(n) \exp(-iW_n t) \qquad (5.16)$$

with parity

$$P = (-1)^{J+1} \qquad (5.17)$$

and the vector spherical harmonic satisfies

$$\int Y^*_{lJM}\, Y_{l'J'M'}\, d\Omega = \delta_{ll'}\, \delta_{JJ'}\, \delta_{MM'} \qquad (5.18)$$

where $\underset{\sim}{J} = \underset{\sim}{L} + \underset{\sim}{S}$ (5.19)

and $J^2\, Y_{lJM}(\hat{n}) = J(J+1)\, Y_{lJM}(\hat{n})$

$$J_z\, Y_{lJM}(\hat{n}) = M\, Y_{lJM}(\hat{n}) \qquad (5.20)$$

The Hamiltonian for this quasi gluon field is given by

$$H = 1/2 \int d^3r\, [\underset{\sim}{E} \cdot \underset{\sim}{D} + \underset{\sim}{B}^2] \qquad (5.21)$$

This Hamiltonian is second quantized as in section 3 using the eigen modes defined in eqns.5.14 and 5.16.to get the energy of the confined quasi gluons in terms of its frequency.

6. CONSTRUCTION OF THE GLUEBALL STATES :

The fact that no coloured objects are seen free in nature allows only colour singlet states to exist with finite energies.In addition to $q\bar{q}$ qqq states one expects the colour singlet states of colour gluons also to exist (Jaffe & Johnson 1976). Such states are refered to as gluonium or glueballs. Strong experimental evidence for such particles exist (D.L.Sharre etal 1980; S.J.LindenBaum etal 1985; K. Konigsmann 1986).Construction of digluon and trigluon colour singlet low lying glueball states in the case of CCM and DCM are discussed here.

The lowest gluon modes we obtained are $J^{PC} = 1^{--}$ for the transverse magnetic (TM/E) and $J^{PC} = 1^{+-}$ for the transverse electric(TE/M). J is the total angluar momentum of the gluon state ,P is the parity and C represents the colour charge conjugation. Since glueballs are bosonic hadrons the total wave function including colour should be symmetric. The colour part is governed by the combination of Gell-Mann's λ-matrices as

$$[\lambda_l , \lambda_m] = 2if_{lmn}\lambda_n$$
$$\{\lambda_l , \lambda_m\} = 4/3\,\delta_{lm} + 2d_{lmn}\lambda_n \qquad (6.1)$$

where (lmn) are the colour indices, f_{lmn} are completely antisymmetric in their indices while d_{lmn} are completely symmetric. For di-gluon states the colour coupling is of the form δ_{lm} giving C =+1, whereas in the case of tri-gluon states the colour symmetric coupling of the type d_{lmn} gives C = - 1 and that of the colour anti symmetric coupling of the type f_{lmn} gives C = +1 The colour singlet glueball states with orbital,spin and colour symmetries can have the following combinations.

Orbital	Spin	colour
S	S	S
S	AS	AS
AS	S	AS
AS	AS	S
(MS/MAS	$MS/MAS)_S$	S
(MAS/MS	$MS/MAS)_{AS}$	AS

where S, AS refer to symmetric and antisymmetric respectively; while MS, MAS refer to mixed symmetric and mixed antisymmetric respectively.

Accordingly, low lying states of the digluon systems are obtained as $E^2(0^{++}\ 2^{++})$, $M^2\ (0^{++}, 2^{++})$ and EM $(0^{-+}, 2^{-+})$. And the low lying states of the trigluon systems with colour coupling of the type d_{lmn} are $1^{+-(--)}\ 3^{+-(--)}$ and the colour coupling of the type f_{lmn} are $0^{++(-+)}$ for M^3 (E^3) combinations. The wave functions corresponding to these states are given by

$$\Psi_{J=3,1} = d_{lmn}\ \chi^{J=1,3}_{\overline{123}}\ \Phi_{\overline{123}}$$
$$\Psi_{J=0} = f_{lmn}\ \chi^{J=0}_{\widehat{123}}\ \Phi_{\widehat{123}} \qquad (6.2)$$

where χ_{123} and Φ_{123} are the spin and orbital wave functions respectively. The bar over the indices represents symmetric and the cap represents the antisymmetric combinations between the particle indices. A detailed account of the M^3 and M^2E glueball states are given by K. Senba and Tanimoto 1984. In similar fashion we obtain the low-lying ME^2 glueball states as $(0^{++}, 2^{++}, 1^{+-}, 3^{+-})$ and M^2E glueball states as $(0^{-+}, 2^{-+}, 1^{--}, 3^{--})$.The calculations of hyperfine splitting of these states are beyond the scope of this paper.

The spurious motion of the centre of the glueball containing A-gluons should also be taken into account while constructing the glueball states. This can be done in a simplified manner by keeping the centre always at the lowest possible eigen state.Finally the intrinsic energy of the gluon in A-gluon system can be obtained as follows.Let particles 1, 2,3,... .A be confined around a common centre C at a distance R;and $r_1, r_2, \ldots r_A$ are the distance of each particle measured from the centre; and $x_1, x_2, \ldots x_A$ are the position vectors of these particles.Then

$$\underset{\sim}{R} = \sum_{i=1}^{A} \underset{\sim}{x}_i / A$$

$$\underset{\sim}{R} + \underset{\sim}{r}_i = \underset{\sim}{x}_i \qquad (6.3)$$

Now the oscilator type of eqns.obtained in sections 2 and 3 in the relative coordinate with respect to the centre of confinement can be resolved in terms of $\underset{\sim}{R}$ and $\underset{\sim}{x}_i$ using 6.3. In the case of CCM

$$[-\sum_{i=1}^{A} \nabla_i^2 + \sum_{i=1}^{A} \alpha^2 x_i^2 - (-\nabla_R^2 + \alpha^2 R^2)]\Psi = W_n^2 \Psi \qquad (6.4)$$

where

$$(-\nabla_R^2 + \alpha^2 R^2)\Psi = E_N^C \Psi \qquad (6.5)$$

and

$$E_N^C = (2N+3)\alpha \qquad (6.6)$$

We construct the states with the centre of confinement in the lowest oscilator state $E_0^C = 3$ .Assuming an equal contribution of E_0^C/A due to each gluon at the centre and ϵ_n is the intrinsic energy of each gluon,then

$$(-\nabla_i^2 + \alpha^2 x_i^2)\Psi_i = (\epsilon_n^2 + E_0^C/A)\Psi_i \qquad (6.7)$$

where

$$\epsilon_n^2 + E_0^C/A = (2n+3)\alpha \; . \qquad (6.8)$$

Thus

$$\epsilon_n^{CCM} = (2n+3-3/A)^{1/2}\, \alpha^{1/2}. \qquad (6.9)$$

Similarly in the case of DCM,

$$\epsilon_n^{DCM} = (2n+3-3/A)a. \qquad (6.10)$$

We calculate the energies of low-lying di-gluon and tri-gluon states in DCM and CCM.The expressions for the gluon energy quanta and the intrinsic gluon energy expressions for the lower modes are given in table 1.The single parameter a in DCM and $\alpha^{1/2}$ in CCM are calculated by fitting the iota (1440 MeV) 0^{-+} state as a di-gluon glueball. Without considering the spurious motion of the centre and not subtracting the zeropoint energy, the glueball energies become just addition of respective W_n's. In this case the DCM parameter a and the CCM parameter $\alpha^{1/2}$ are obtained as 180 MeV and 363 MeV respectively. The energies are tabulated in table-2. While considering the spurious motion of the centre and removing the average zeropoint energy the parameters are obtained as a = 288 MeV and $\alpha^{1/2}$ = 466 MeV respectively by fitting the same iota (0^{-+}) as a di-gluon state, using the intrinsic energy expression and taking the possible linear combinations of the low-lying levels to ensure the centre of confinement remains at the ground state. The calculated energies for the di-gluon and tri-gluon low-lying states are given in table-3, comparing with naive Bag model results (J.Kuti 1977) and some of the experimental candidates.

7. <u>COMPARISON WITH EXPERIMENT AND DISCUSSION</u>:

The lightest glueballs are expected to have masses ranging from 1-3 GeV and to have spin-parities 0^{++}, 0^{-+} and 2^{++}. Observations of such particles are very crucial to QCD. This mass range is accessible in radiative J/ψ decays and these states are expected to dominate this decay. The first candidate iota (1440 MeV) 0^{-+} was found by Mark II in the decay mode $\iota \rightarrow K_s^0 \; K^{+-} \; \pi^{-+}$ (D.L.Sharre et al 1980). Being an oldest glueball candidate we fit our parameters and

predicted all other glueball states which are found to be in good agreement with other existing candidates (see table 3). Although the latest experimental results of J/ψ decay give very strong evidence for the state iota 0^{-+} (1459$\pm$5 in Mark III) to be gluonic, the present situation is such that it is not unambiguously possible to identify the state due to the strong mixing of the $q\bar{q}$ pseudoscalar mesons in this energy range (K. Konigsmann 1986). The other glueball candidate which we obtained a very good agreement is the θ 2^{++} (1700 MeV) discovered by the crystal ball group in the channel $J/\psi \rightarrow \gamma\eta\eta$ (C. Edwards et al 1982). A detailed analysis of this state conclusively agreed it to be a gluonic meson. But its decay patterns cause slight problems and hence a thorough study of these states has to be done before they are confirmed. Another glueball candidate is the three resonances $g_T(2120)$ $g_T'(2220)$ and $g_T''(2360)$ with J^{PC} = 2^{++} obtained in the reaction $\pi^- p \rightarrow \phi\phi n$ which breaks down the OZI suppression. The analysis of these resonances as a three-gluonic combinations explained all their features in a clear-cut and simple manner by S.J.Lindenbaum in 1985. Some recent differences regarding the degree of OZI forbiddeness of this reaction has been resolved by them and it was concluded that they are produced by glueball and strongly argued that alternate explanations are incorrect and do not fit the experimental facts. Our results for the 2^{++} tri-gluon state in this energy range is obtained from the EEM coupled modes whose energy is very close to the average energy of the g_T resonances (2233 MeV). Another candidate is the g_s $(1240)0^{++}$ obtained in the reaction $\pi^- p \rightarrow K_s^0 K_s^0$ n (A.Etkin et al 1982).The characteristics of this state satisfied that expected by a di-gluon state.This state shows poor agreement with our results, when the zeropoint energy is subtracted from the energy quanta.But it is in good agreement with the CCM lowest glueball energy (table 2) without the correction due to the spurious motion of the centre.While all other states are in good agreement with the experimental candidates only when the spurious motion of the centre is taken into account.Thus as in the case of RHM the success of CCM is also closely linked with the accounting for spurious motion of centre of mass. The DCM results are not satisfactory even though the EEM 2^{++} state is close to g_T.The discrepancy as seen from table-3 between the CCM and the naive bag model energies of the coupled modes are due to the fact that the lowest gluon energy state in our case is the electric mode with $l = 0, J = 1^-$ state while that of bag model is the magnetic mode with $l = 1$, $J = 1^+$. Most of the other potential models also provide the $l = 0$ solution but they are neglected for massless spin-1 fields. We feel that it will be incorrect to neglect such solutions in phenomenological models like DCM or CCM where the inhomogeneous medium may provide an effective mass to the field as the interaction grows when it moves away from the 'centre of confinement'.The $\theta_{\mu\nu}$ tensor in the sec. 2 and then in 3 can be considered as a dynamical gluon mass for low momentum modes (L.S.Celenza and C.M. Shakin 1986). A new gauge condition named the oscillator gauge obtained in section 3 is another feature of our model which helps us to get a consistent confinement mode similar to RHM. This gauge is found to be a linear combination of the Coulomb gauge and the axial gauge.

The non-local dielectric function obtained in section 4 corresponding to the CCM is momentum dependent and the asymptotic freedom at lower distances or at higher momentum transfer is built in it. There are indications from QCD about the momentum dependence for the dielectric function (Baker M. et al 1983) for confinement models.It remains to be seen how exactly the nonlinearity of the QCD can be viewed through the phenomenological models with proper gauge condition,and the link between the description of the gluon condensate and the dielectric medium.There are many more low-lying glueball states of which very few are true experimental candidates. It is very crucial to distinguish $q\bar{q}$, $q\bar{q}g$, gg and ggg states among the vast experimental data for the exotic states in the energy range 1-3 GeV. And it is very important to see how these states lead to the understanding of the strong interaction between the nucleons at a more fundamental level.

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Table-1

Energy expressions for the lowlying gluon states

Quanta n	$\omega_n = (2n+3)a$	DCM $\epsilon_n = (2n+3 - \frac{3}{A})a$, A=2	A=3	$\omega_n = (2n+3)^{1/2}\alpha^{1/2}$	CCM $\epsilon_n = (2n+3 - \frac{3}{A})^{1/2}\alpha^{1/2}$, A=2	A=3
0	$3a$	$\frac{3}{2}a$	$2a$	$\sqrt{3}\alpha^{1/2}$	$\sqrt{\frac{3}{2}}\alpha^{1/2}$	$\sqrt{2}\alpha^{1/2}$
1	$5a$	$\frac{7}{2}a$	$4a$	$\sqrt{5}\alpha^{1/2}$	$\sqrt{\frac{7}{2}}\alpha^{1/2}$	$2\alpha^{1/2}$
2	$7a$	$\frac{11}{2}a$	$6a$	$\sqrt{7}\alpha^{1/2}$	$\sqrt{\frac{11}{2}}\alpha^{1/2}$	$\sqrt{6}\alpha^{1/2}$
3	$9a$	$\frac{15}{2}a$	$8a$	$3\alpha^{1/2}$	$\sqrt{\frac{15}{2}}\alpha^{1/2}$	$\sqrt{8}\alpha^{1/2}$

Table-2

Low-lying glue-ball energies without considering the spurious motion and without substracting the zeropoint energy

Coupled modes	J^{PC}	D C M	C C M
EE	0^{++} 2^{++}	1080 MeV	1257 MeV
EM	0^{-+} 2^{-+}	1440 "	1440 "
MM	0^{++} 2^{++}	1800 "	1623 "
EEE	0^{-+} 1^{--} 3^{--}	1620 "	1886 "
EEM	0^{++} 2^{++} 1^{+-} 3^{+-}	1980 "	2069 "
EMM	0^{-+} 2^{-+} 1^{--} 3^{--}	2340 "	2251 "
MMM	0^{++} 1^{+-} 3^{+-}	2700 "	2434 "

Table-3

Calculated low-lying glue-ball energy states in DCM and CCM with removal of the spurious motion of the centre in comparison with bag model results (J. Kuti 1977) and with experimental candidates

Coupled Modes	J^{PC}	Calculated energies in MeV			Experimental candidates	
		DCM	CCM	BAG	J^{PC}	Energy (in MeV)
EE	$0^{++}2^{++}$	864	1137	1796	0^{++}	1240(g_s)
EM	$0^{-+}2^{-+}$	1440	1440	1446	0^{-+}	1440(ι)
MM	$0^{++}2^{++}$	2016	1703	1096	2^{++}	1700(θ)
EEE	$0^{-+}1^{--}3^{--}$	1728	1971	2694	-	-
EEM	$0^{++}2^{++}$ $1^{+-}3^{+-}$	2304	2246	2344	2^{++}	(2120, 2220, 2360)(g_T)
EMM	$0^{-+}2^{-+}$ $1^{--}3^{--}$	2880	2489	1990	-	-
MMM	$0^{++}1^{+-}3^{+-}$	3456	2720	1644	-	-