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PION CONDENSATION IN SYMMETRIC NUCLEAR MATTER *

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ABSTRACT

Using a model which is based essentially on the chiral $SU(2) \times SU(2)$ symmetry of the pion-nucleon interaction, we examine the possibility of pion condensation in symmetric nucleon matter. We find that the pion condensation is not likely to occur in symmetric nuclear matter for any finite value of the nuclear density. Consequently, no critical opalescence phenomenon is expected to be seen in the pion-nucleus interaction.

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The Weinberg Lagrangian [1] for the pion-nucleon system is based on the non-linear realization of the chiral $SU(2) \times SU(2)$ symmetry and the relevant terms for the pion-nucleon scattering are given by

$$\mathcal{L}_W = \mathcal{L}_{\pi NN} + \mathcal{L}_{\pi\pi NN} , \tag{1}$$

$$\mathcal{L}_{\pi NN} = (f/\mu)\, \bar{\Psi} \gamma_5 \gamma_\mu \tau_i \Psi\, \partial^\mu \varphi_i , \tag{2}$$

$$\mathcal{L}_{\pi\pi NN} = (i/4f_\pi^2)\, (\bar{\Psi} i \gamma^\mu \tau_i \Psi)\, t_{ijk}\, \varphi_j\, \partial_\mu \varphi_k \tag{3}$$

The direct interaction represented by Eq.(3) may be replaced by the ρ-mediated interaction Lagrangian. However, in the calculation of the spin-averaged pion-nucleon scattering amplitudes along the forward direction, it does not make any difference in the second order field theory.

The πNΔ-interaction term can also be added to the Weinberg Lagrangian in the framework of the $SU(2) \times SU(2)$ symmetry [2),3)]. The most general form of the πNΔ-interaction Lagrangian, which contains only the first order derivative of the pion field and remains invariant under the point transformation of the interacting fields, can be written as [3)]

$$\mathcal{L}_{\pi N\Delta} = \frac{1}{\sqrt{2}} (g^*/\mu) \left[i \bar{\Psi}_\mu \Theta^{\mu\nu} T_i \Psi \partial_\nu \varphi_i + H.C. \right] \tag{4}$$

$$\Theta_{\mu\nu} = \left\{ g_{\mu\nu} + \left[\frac{1}{2}(1+4Z)A + Z \right] \gamma_\mu \gamma_\nu \right\} \tag{5}$$

where ψ_μ is the Ravita-Schwinger field and the T's are a set of matrices corresponding to the isospin-3/2. There are two parameters in Eq.(5), which are arbitrary except that $A \neq -\frac{1}{2}$. The parameter A also appears in the expression for the propagator of the spin-3/2 field. However, the physically interesting quantities do not depend on A. The parameter Z does occur in the expression for the scattering amplitudes and it measures the off-mass-shell effects of the Δ(1236) [3)-5)]. There has been a considerable work [2)-7)] - both theoretical and phenomenological - on the value of Z. In particular, the principles of the second quantization as well as the Lorentz invariance of the S-matrix demand that Z be equal to $\frac{1}{2}$ [3]. From the phenomenological analysis of πN scattering and the photoproduction of pions at low-energy, it is rather difficult to fix the value of Z uniquely, as there are other parameters in any model for the πN and γN interactions. However, it has been possible to place a bound on the value of Z [5),7)], that is,

$$|Z| \leq \frac{1}{2} \qquad . \tag{6}$$

In the present work, we take two values of Z, namely $Z = \frac{1}{2}$ and $Z = -\frac{1}{2}$ the former is, of course, the theoretically preferred value.

The remaining part of the discussion on this model is about the compatibility of our description of the low energy pion-nucleon interaction with the requirements of the current algebra and the PCAC [8)-10)], when one of the pions or both of them are off-mass-shell. In order to satisfy the constraints on the πN scattering amplitudes in the soft pion limit, we supplement the Weinberg Lagrangian by adding the well-known σ-term to it. More explicitly, our parametrization of the σ-contribution to the pion-nucleon scattering is given by [5),7),11)]

$$A_{\sigma}^{(+)}(\nu, \nu_B) = \frac{\sigma_{NN}(t=2\mu^2)}{f_\pi^2}\left[\frac{q^2+q'^2-\mu^2}{\mu^2} + \sigma'\,(4m\nu_B)\right], \tag{7}$$

where

$\nu = (s - u)/4m \quad , \qquad \nu_B = (t^2 - q^2 - q'^2)/4m \quad ,$

f_π = pion decay constant = 92.03 MeV ,

$\sigma_{NN}(t = 2\mu^2) = 63.5$ MeV, $\sigma' = 0.54\ \mu^{-2}$,

μ = pion mass , m = nucleon mass ,

and s,t,u are the Mandelstam variables with q and q' as the momenta of the incoming and the outgoing pion, respectively. The theoretical expectations from this model on the s- and p-wave scattering lengths and the s-wave effective ranges are in good agreement with the experiment [12),13)].

The first order self-energy of a negatively charged pion, propagating through an admixture of neutrons and protons and undergoing sequential collisions with separate nucleons, can be written as

$$\begin{aligned}-\Pi^{(1),\pi^-}(\omega,k) &= \int \frac{d^3p}{(2\pi)^3}\, n_n(p)\, D_{\pi^- n}(p,\omega,k) \\ &\quad + \int \frac{d^3p}{(2\pi)^3}\, n_p(p)\, D_{\pi^- p}(p,\omega,k) \\ &= \int \frac{d^3p}{(2\pi)^3}\left[n_p(p)+n_n(p)\right] D^{(+)}(p,\omega,k) \\ &\quad + \int \frac{d^3p}{(2\pi)^3}\left[n_p(p)-n_n(p)\right] D^{(-)}(p,\omega,k)\end{aligned} \tag{8}$$

$$q = q' \equiv (\omega, \vec{k}) \quad ,$$

where $n_n(p)$ and $n_p(p)$ are, respectively, the occupation functions for the neutron and the proton in the Fermi sea of non-interacting nucleons. The spin-averaged, forward πN scattering amplitudes are denoted by $(D^{\pm})$ and the superscripts $(\pm)$ refer to the even isospin and the odd isospin, respectively.

Similar expressions can be written for the first order self-energy of π^+ and π^0. However, in this work we are interested in the self-energy of a pion in symmetric nuclear matter. Therefore, the self-energy is the same for any charged state of the pion, as in the symmetric nuclear matter $n_n(p) = n_p(p) = n(p)$, where

$$\int \frac{d^3p}{(2\pi)^3}\, n(p) = \rho_n = \rho_p = \tfrac{1}{2}\rho$$

We have evaluated the nucleon contribution to the self-energy in terms of the Lindhard functions, whereas the γ- and Δ-contributions are obtained in the static approximation. To be more precise, in our work, we take

$$-\Pi^{(1)}_{\sigma,\Delta}(\omega,k) \simeq \rho\, D^{(+)}_{\sigma,\Delta}(\omega,k) \tag{9}$$

Further, at the πNN and $\pi N\Delta$ vertices we include a form factor $g(q^2)$ given by

$$g(q^2) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 - q^2} \quad (10)$$

In our numerical computations we have allowed Λ to vary from $\Lambda = 1$ GeV to $\Lambda = 1.5$ GeV [14].

We now introduce a function $U(\omega,k^2)$ defined by

$$\Pi^{(1)}_{N,\Delta}(\omega,k) = \Pi^{(1)}_{N,\Delta}(\omega,0) + k^2 U_{N\Delta}(\omega,k^2) \quad (11)$$

Taking account of the higher order corrections, the renormalized self-energy due to the nucleon and Δ-contribution can be written as

$$\Pi^{R}_{N+\Delta}(\omega,k) = \Pi_{N+\Delta}(\omega,0) + \frac{k^2 U_{N+\Delta}(\omega,k^2)}{1-g' U_{N+\Delta}(\omega,k^2)} \quad (12)$$

where g' is the spin-isospin correlation parameter and we have assumed the N-Δ universality. The most commonly used range of values of g', that is, $g' = 0.5 \sim 0.7$ [15], has been used as input in our computations.

Using the model presented here we have examined thoroughly the possibility of the transition, $N \to N\pi$, with $\omega = 0$. We find that the pion condensation is not likely to occur in symmetric nuclear matter. Consequently, it is not expected that any critical opalescence phenomenon [16] will be seen in the pion-nucleus scattering. Further, an interpretation of the EMC-effect [17),18] in terms of an increased pion number in the nucleus seems unlikely [19),20]. The details of our calculations will be published elsewhere.

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