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BASIC CAUSES OF AMPLITUDE MODULATION IN CLIMATIC/WEATHER PARAMETERS *

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ABSTRACT

The continuous interaction between the Earth's spinning motion and energy from the Sun gives rise to some (heat) energy oscillations in the Earth-atmosphere system (Njau, 1985a; 1985b; 1986a; 1986b). Recent results of large scale analysis of East African climatic records have proved that these oscillations significantly link the Sun to climatic/weather variations by systematically modulating key climatic/weather parameters like rainfall and air temperature (Njau, 1987a; 1987b; 1987c; 1987e; 1987f). In this paper, we re-develop the latter proof using a very different approach based upon theoretical analysis. The analysis has confirmed a general law suggested earlier (Njau, 1987d), that, with an exception of the diurnal cycle, any permanent cycle in the net solar energy incident upon a given part of the Earth-Atmosphere system gives rise to a quasi-permanent cycle whose period is approximately twice that of the former. Quasi-biennial as well as double sunspot cycles are shown to be a possible result of this general law.

1. INTRODUCTION

It has been shown that the spinning motion of our Earth constantly interacts with incoming solar energy to form some (heat) energy oscillations in the Earth-atmosphere system [Njau, 1985a; 1985b; 1987a]. These energy oscillations play a significant role in linking the Sun to climatic variations and hence may be involved in climatic prediction work [Njau, 1986a; 1986b; 1987b; 1987c]. On the basis of the latter references, these energy oscillations constantly cause slow to rapid changes (of differing spatial and time dimensions) in the distribution of heat over the globe and thus inevitably impress corresponding changes upon the atmospheric-oceanic circulation systems mainly in accordance with the laws of thermodynamics. It is mostly in the constant task of trying to force a balance within the Earth-atmosphere system between incoming energy, outgoing energy, and the internally transported energy that the atmospheric-oceanic circulation systems in turn influence and/or cause climatic variations.

According to the references cited above as well as others [eg. Njau, 1987d; 1987e; 1987f], some proof has been developed to the effect that the energy oscillations mentioned above do maintain the link between climatic variations and the Sun by constantly effecting significant (amplitude) modulations together with other variations in climatic parameters all over the globe. This proof was established mostly through a non-theoretical approach. The aim of this paper is to re-develop the same proof through a different approach that banks mostly on theoretical analysis. This analysis gives rise to formulations whose interpretation sheds light on new details about the role played by the oscillations mentioned above on Sun-climate links.

2. FORMULATION

The fairly regular cycles in (lower) atmospheric temperature or heat content whose permanent existence has so far received considerable support from meteorologists and atmospheric scientists include annual cycles which originate from the tilt of the Earth's rotational axis, 11-year sunspot cycles which originate from variations in sunspot numbers as well as the relatively longer-period Milankovitch cycles which arise from cyclic variations in the Earth's orbit and rotational axis [Hargreaves, 1979; Morner and Karlen, 1984; Burroughs, 1986; etc.]. We arbitrarily denote any of such quasi-permanent cycles, with respect to an arbitrary section or portion which is hereby labelled B, in the (lower part of the)

Earth-atmosphere system by the function $y(t)$. We also denote the latitude of this portion or section by L. Furthermore, we assume that $y(t)$ has period d_o and that it is fairly regular over an arbitrarily chosen time-length $T(>d_o)$. Let the magnitude of the constant component of incoming solar energy over portion B during the entire period T be denoted by C. It can be shown [eg. see Njau, 1982] that over the whole of period T, both $y(t)$ and the function $b(t)$ (where $b(t) = C$ within the entire period T and $b(t) = 0$ or undefined otherwise) will interact with the Earth's spinning motion to form (heat) energy patterns represented by $f_d(t)$, within portion B, such that

$$\begin{aligned} f_d(t) = (y(t)+C)\Big\{ & [\Psi(0)+\Psi(d_o)+\Psi(2d_o)+\ldots\ldots\ldots+\Psi(M-1)d_o)]+ \\ & [\Psi(k)+\Psi(d_o+k)+\Psi(2d_o+k)\ldots\ldots\ldots+\Psi((M-1)d_o+k)]+ \\ & [\Psi(2k)+\Psi(d_o+2k)+\Psi(2d_o+2k)+\ldots+\Psi((M-1)d_o+2k)]+ \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ & +[\Psi(Xk)+\Psi(d_o+Xk)+\Psi(2d_o+Xk)+\ldots\ldots\ldots\ldots\ldots\ldots \\ & +\Psi(M-1)d_o+Xk)]+\Gamma(t)\Big\} \end{aligned} \quad \ldots\ldots\ldots\ldots \quad (1)$$

where k = mean period of the diurnal cycle in portion B,

$\Psi(r)$ = the cosinusoidally edged window function $E(r)$ (detailed in Njau [1985a]) for $r \leq t \leq r + s$,
= 0 (or undefined) elsewhere,

M = total number of full wavelengths in the portion of $y(t)$ contained between $t = 0$ and $t = T$,

$$\Gamma(t) = \sum_{p=X}^{X+Z-1} \left\{ \sum_{n=0}^{M-1} \theta(t) g(pk + nd_o) \right\},$$

$g(pk+nd_0) = \sigma_L E(pk+nd_o)$ for $pk + nd_0 \leq t \leq pk + nd_0 + s$
= 0 (or undefined) elsewhere,

s = mean length of sunrise-sunset period in portion B,

$$Z = \left|\phi_e - \phi_i\right| \frac{d_o}{7\pi k} + \beta,$$

ϕ_e = end phase of the variations in $f_d(t)$,
ϕ_i = initial phase of the variations in $f_d(t)$,
$X = (d_o + \alpha)/k$,
$|\beta|$ $(<k)$ = constant,
$|\alpha|$ $(<k)$ = constant,
$\theta(t) = 0$ for all $(XM-1)k + s \geq pk + nd_o$
$= 1$ for all $(XM-1)k + s < pk + nd_o$,

and σ_L is a constant at latitude L.

The influences of atmospheric and oceanic circulation systems upon $f_d(d)$ are taken into account by making appropriate adjustments in σ_L as clearly evident from previous publications [eg. Njau, 1987f]. According to the latter reference, these influences on the average introduce some adjustments in the effective value of the parameter σ_L for each latitude. This means that whereas the formulation developed herein applies to any latitude, each latitude would have its appropriate value for the parameter σ_L since atmospheric and oceanic circulation systems basically re-distribute incoming solar energy within the Earth-atmosphere system such that some areas gain "extra" energy at the expense of some other areas.

We can change Eq. (1) through Fourier transformation into the following form:

$$F_d(w) = \left\{R(w) + C\,\delta(w)\right\} * \left\{\sigma_L J(w)\ \exp\ [i(M-1)\pi d_o w]\ x\ \exp[i(X-1)\pi kw]\ \frac{\mathrm{Sin}(M\pi d_o w)}{\mathrm{Sin}(\pi d_o w)}\ \frac{\mathrm{Sin}(X\pi kw)}{\mathrm{Sin}(\pi kw)} + \sigma_L J(w)\ \exp(i2\pi Xw)\exp[i(Z-1)\pi kw]\ x\ \frac{\mathrm{Sin}(Z\pi kw)}{\mathrm{Sin}(\pi kw)}\right\}, \qquad (2)$$

where $R(w)$ = Fourier transform of the signal $y(t)$,
$J(w)$ = Fourier transform of $E(t)$,
w represents radian frequency,
and $*$ denotes convolution operation.

If $y(t)$ is assumed to be a simple sinusoid with radian frequency w_o and amplitude A, then $R(w)$ will, to at least a good approximation, consist of nothing more than two quasi-delta functions, one centred at $w = w_o$ and the other centred at $w = -w_o$. On this basis, $|F_d(w)|^2$ may be obtained from equation (2) above as:

$$|F_d(w)|^2 \approx \Big|\sigma_L CJ(w)\ \exp[i(M-1)\pi d_o w]\exp[i(X-1)\pi kw]\ x\ \frac{\mathrm{Sin}(M\pi d_o w)}{\mathrm{Sin}(\pi d_o w)}\ \frac{\mathrm{Sin}(X\pi kw)}{\mathrm{Sin}(\pi kw)} + \sigma_L CJ(w)\ x\ \exp(i2\pi Xw)\exp[i(Z-1)\pi kw]\ \frac{\mathrm{Sin}(Z\pi kw)}{\mathrm{Sin}(\pi kw)} + \sigma_L AJ(w - w_o)\ \exp[i(M-1)\pi d_o(w - w_o)]\exp[i(X-1)\ x\ \pi k(w - w_o)]\ \frac{\mathrm{Sin}[M\pi d_o(w - w_o)]}{\mathrm{Sin}[\pi d_o(w-w_o)]}\ x\ \frac{\mathrm{Sin}[X\pi k(w - w_o)]}{\mathrm{Sin}[\pi k(w - w_o)]} + \sigma_L AJ(w - w_o)\ x\ \exp[i2\pi X(w - w_o)]\exp[i(Z-1)\pi k(w - w_o)]\ x\ \frac{\mathrm{Sin}[Z\pi k(w - w_o)]}{\mathrm{Sin}[\pi k(w - w_o)]}\Big|^2 \qquad (3)$$

An analysis of equation (3) shows that $|F_d(w)|^2$ theoretically consists of two sets of (solar-derived) heat energy oscillations. The oscillations in the first set have the following frequencies:

$$w_n = nw_o \qquad (4a)$$

$$w_n = w_o + \frac{2n + 1}{Md_o}\pi \qquad (4b)$$

$$w_n = w_o + \frac{2n + 1}{Xk}\pi \qquad (4c)$$

and $$w_n = w_o + \frac{2n + 1}{Zk}\pi$$

where n = 1, 2,..... Note that the oscillations represented by equation (4a) have relatively greater amplitudes, and in particular the oscillation at frequency w_o has an amplitude (approximately) equal to A. On the other hand, the oscillations that belong to the second set have the following frequencies:

$$w_n = \frac{2n + 1}{Md_o} \pi \quad (5a)$$

$$w_n = \frac{2n + 1}{Xk} \pi \quad (5b)$$

$$\text{and} \quad w_n = \frac{2n + 1}{Zk} \pi \quad (5c)$$

All these oscillations are superimposed upon any corresponding conditions or trends that exist at t = 0. Now let amplitudes of the n^{th} oscillations represented by equations (4a), (4b), (4c), (4d), (5a), (5b) and (5c) be denoted, respectively by A_n, A_{Mn}, A_{Xn}, A_{Zn}, a_{Mn}, a_{Xn} and a_{Zn}. An analysis of equations (4a) through to (5c) shows that y(t) will undergo clear "double sideband" amplitude-modulation as long as the following two conditions take place simultaneously:

$$w_o = \frac{2n + 1}{Md_o} \pi + \frac{2m + 1}{Md_o} \pi \quad \text{that is } M = 1 + m + n \quad (6a)$$

$$\text{and} \quad A_{Mn} = a_{Mm} \quad (6b)$$

where m is a positive integer. The modulating signal in this case has a radian frequency w_m given as

$$w_m = \frac{2m + 1}{Md_o} \pi \quad (7a)$$

Note that the amplitude of this modulating signal is equal to a_{Mn}. Now according to equation (7a), the modulating signal has a period T_m which is given as

$$T_m = \frac{2(Md_o)}{2m + 1} \quad (7b)$$

But since M = 1 + m + n (see equation (6a)), then provided that m + n = 2, 4, 6, 8,...., the value for M may be given as M = 2D + 1, where D = 1, 2, 3,.... On this basis, equation (7b) may be written as

$$T_m = \frac{2(2D + 1)\ d_o}{2n + 1} \quad (8a)$$

Of course the replacement of integer symbol m with integer symbol n on the right-hand side of equation (8a) does not introduce any difference or change in the formulation. Now, for those cases in which n = D i.e. 2D + 1 = 2n + 1, the period T_m is constant and quite independent of n or D so that

$$T_m = 2d_o \quad (8b)$$

Besides, for those other cases in which 2D + 1 = 3(2n + 1),5(2n + 1), 7(2n + 1), 9(2n + 1), 11(2n + 1),....., the period T_m is also independent of D and n, and takes the following values, respectively:

$$T_m = 6d_o,\ 10d_o,\ 14d_o,\ 18d_o,\ 22d_o,\ldots\ldots \quad (8c)$$

The impression developed hitherto is that equations (6a), (6b), and (8b) are valid whenever y(t) undergoes double sideband amplitude-modulation. It should be stressed though that, depending mainly on its effective magnitude, any of the oscillations whose frequencies are given by equations (4b), (4c), (4d), (5a), (5b) and (5c) may as well impress single sideband amplitude-modulation over y(t) as already verified through analysis of meteorological records [Njau, 1985a; 1985b; 1986a; 1987d]. This then implies that equations (8b) and (8c) are valid for both single and double sideband amplitude-modulations though under slightly different conditions for both cases.

Equation (8b) confirms an earlier suggestion [Njau, 1987d], that for each permanent cycle in the net solar energy incident upon a given part of the Earth-atmosphere system, there exists a quasi-permanent cycle with a period approximately twice that of the former. Clearly the diurnal cycle is excluded from this general law simply because it coincides with the basic process (i.e. the day-night sequence) with which our spinning Earth samples out incoming solar energy. Of course it is unusual to consider a sampling process as a generator of samples from itself ! It is quite apparent that the general law mentioned above is what gives rise to quasi-biennial oscillations and double-sunspot oscillations, respectively, in connection with the annual cycle and the sunspot cycle. This point is further discussed in sections 2.1 and 2.2.

It is worthwhile noting that equations(7a) and (7b) have already been derived before in connection with single sideband modulation [Njau, 1985b; 1987a]. If single or double sideband modulation over $y(t)$ is such that $m = n = 1$ in equation (6a), then we would expect a resultant modulation index equal to $a_{M1}/A_1 \approx 0.21$. In fact, not only has this category of modulation been observed over World temperature covering the last 5000 years [Collin, 1986] but it has also been observed over at least 96% of the past rainfall and temperature data in Tanzania [Njau, 1987a; 1987d]. On the other hand, if $A_1 = C$ then it follows that $A_{Mn} = a_{Mn}$ so that it is quite possible for the oscillations given by equations (7a) and (7b) to amplitude-modulate $y(t)$ synchronously and hence produce a maximum modulation index of about 0.88. Interestingly the latter value is approximately equal to the maximum modulation index over rainfall (ie. ~ 0.86) which was previously calculated using a slightly different approach [Njau, 1987a].

We have already observed that the conditions generally represented by equations (6a) and (6b) give rise to amplitude modulation over $y(t)$ and that the corresponding modulating signals represent quasi-permanent cyclic variations in lower atmospheric heat energy and temperature. Since the latter has been shown to have definite relationship with other climatological parameters like rainfall and cloud cover (e.g. Riehl, 1979; Henderson-Sellers, 1986), it follows that some sort of amplitude modulation will be mapped onto these other parameters like rainfall as soon as the conditions given by equations (6a) and (6b) hold. This is so especially in view of a recent conclusion by Mosetti (1987) that the various climatic parameters are characterised by fluctuations with mostly similar periodicities but with different amplitudes and phases. Let us denote the term Md_o in equation (6a) by T_p. Then the latter equation may be written as

$$f_o = \frac{1}{T_p}(1 + m + n) \quad \text{where } f_o = \frac{w_o}{2\pi},$$

ie. $$T_p = T_o(1 + m + n) \quad \text{where } T_o = \frac{1}{f_o},$$

ie. $$T_p \geq 4T_o \qquad (9)$$

if $n \neq m$ since $m \geq 1$ and $n \geq 1$. But since $T = T_p + \Omega$, where $0 \leq \Omega < T_o$ then in general $T \geq 3T_o$ (10)

regardless of whether $n \neq m$ or not. This is not surprising because according to equation (7a) the frequencies of the signals that modulate $y(t)$ are at least equal to $3/2T$. Hence a complete modulating pattern for each of these signals would be covered if $T \geq 3T_o$ as required by equation (10). The broad physical interpretation of equation (10) is that the amplitude modulation implied by conditions (6a) and (6b) generally takes place in the Earth-atmosphere system only if the time-length involved is at least about three times the period of the shortest quasi-permanent cycle in heat energy or temperature. Now if we take the diurnal cycle as the shortest permanent cycle, then equation (10) automatically rules out the accurrence of significantly complete and consistent amplitude-modulations solely within a time-length shorter than 3 days due to the Sun-Climate link mentioned in Section 1 of this paper. As detailed below, the physical changes produced in the Earth-atmosphere system as a result of double sideband amplitude-modulation over $y(t)$ depend basically on which cyclic variation is represented by $y(t)$.

2.1. CASE 1: FUNCTION y(t) REPRESENTS THE ANNUAL CYCLE.

In this case $d_o = 1$ year and A_1 denotes the amplitude of the annual cycles. We can estimate values of A_1 at different latitudes from Fig. 1 which has been adapted from Crowe [1971]. Table 1 has been developed on the basis of Fig. 1 as well as earlier work [Njau, 1985a; 1985b; 1986a]. This table indicates that although pairs of oscillations which fulfil the condition given by equation (6b) are available at all latitudes, such pairs are most numerous and have greater amplitudes in Tropical areas lying between latitudes 10°N and 10°S. The message evident from Table 1 is that at least one of the two conditions (ie. the condition given by equation (6b)) basically necessary for the occurrence of amplitude modulation over annual cycles does in fact take place in all latitudes but at differing extents. Now the second and last condition (ie. the condition represented by equation (6a)) is fulfilled considerably well in all latitudes notably those in which both A_{M1} and a_{M1} have equal values or have values which are not very different from each other. It should be noted, though, that this condition may possibly hold to relatively lesser extent in latitudes wherein A_{M1} is much different from a_{M1}. An interesting observation to make is that if, as it may happen in virtually all latitudes (eg. see Table 1), $m + n = 2, 4, 6, 8, \ldots$ then equation (6a) simplifies into the form

$$M = 3, 5, 7, 9, \ldots\ldots \qquad (11)$$

On the basis of information given earlier, equation (11) clearly implies that a continuous signal at a period of about 2 years modulates the annual cycle and hence leads to the formation of "quasi-biennial oscillations" in meteorological data. This establishment (which automatically follows from equation (8b)) agrees with previous observations in at least two ways. Since the annual cycle is not always a perfect sinusoid as previously assumed, the observed periods of the quasi-biennial oscillations are expected to spread between some value less than 2 years and another value larger than 2 years as already observed [Lamb, 1972]. Secondly since the possibility of the condition $A_{Mn} = a_{Mn}$ being fulfilled becomes relatively less at high latitudes as evident from Table 1, it follows that amplitudes of quasi-biennial oscillations will decrease at high latitudes as already observed [Barry and Perry, 1973]. Equation (8c) predicts oscillations at periods 6 yrs, 10 yrs, 14 yrs, 18 yrs and 22 years. All these periodicities have already been detected in meteorological parameters [eg. Lamb, 1972; 1982; Pittock, 1983; Burroughs, 1986].

The account given above appears to explain the physical origins of the observed quasi-biennial oscillation in meteorological data which had not been explained before [Lamb, 1982]. In view of the account given above, quasi-biennial oscillations may be taken to involve an alteration in the pattern and rate of heat distribution affecting much of the World and its wind distribution. Since the "Southern oscillation" possesses these characteristics and has a period of about 2.3 years [eg. Lamb, 1972] it may be considered as being a specific case of the quasi-biennial oscillations discussed above. The same conclusion applies to the North Pacific oscillation (which affects the North Pacific high pressure and the Aleutian low pressure regions), the North Atlantic oscillation (which affects the Acores high pressure and the Icelandic low pressure regions) and the quasi-biennial wind changes in the stratosphere (which are closely linked and associated with corresponding temperature changes). Each of these oscillations has a period fairly close to 2years [Barry and Perry, 1973].

It would be proper to mention, at this point, that quasi-biennial oscillations are not only produced by the mechanism described earlier in this section.

Some oscillations that are not primarily involved in amplitude modulation may also be generated as a result of conventional interactions between some oscillations represented by equation (7a) and the corresponding annual cycles [Njau, 1987d; 1987e; 1987f].

According to equation (3), the oscillations represented in the middle column of Table 1 (hereinafter simply referred to as "M oscillations") have different phases compared to their counterparts in the right-hand column of the same Table (hereinafter simply referred to as "R oscillations). As explained earlier in this section, it is the M and R oscillations that interact together with the annual cycle under the already stated conditions to give rise to quasi-biennial oscillations. As far as (initial) phases are concerned, such an interaction is dominated by the M oscillations from latitude 0^{o} up to latitude $\sim 20^{o}$ but this domination shifts over to the R oscillations from latitude $\sim 20^{o}$ up to latitude $\sim 50^{o}$. This change-over of effective domination in (initial) phases which occurs at latitude $\sim 20^{o}$ apparently implies that quasi-biennial oscillations would undergo a significant change of phase at about latitude 20^{o}. Indeed such a phase change has been confirmed through observations [Miller and Thompson, 1970].

2.2. CASE 2: FUNCTION y(t) REPRESENTS THE SOLAR CYCLE.

In this case $d_o \approx 11$ years and that w_o represents the radian frequency of the 11-year sunspot cycle. Since the amplitude of the 11-year sunspot cycle in climate is less than about 20% of that of the corresponding annual cycle [Morner and Karlen, 1984], then it should be expected that at all latitudes, all the oscillations given by equation (4b) as well as at least some of the oscillations given by equation (5a) will amplitude-modulate the 11-year sunspot cycle provided that $A_{Mn} = a_{Mn}$ and that

$$w_o = \frac{2n + 1}{Md_o}\pi + \frac{2m + 1}{Md_o}\pi \qquad (12)$$

For any latitude at which $m + n = 2,4,6,\ldots.$ equation (12) may be simplified into the form

$$M = 3,5,7,\ldots\ldots \qquad (13)$$

Equations (13) and (8b) predict the continuous generation of double sunspot oscillations as the modulating signals for the 11-year sunspot cycle in climate. The exact period of these oscillations is expectedly close to but not necessarily exactly equal to 22-years mainly because the 11-year sunspot cycle is not always a perfect sinusoid as assumed in the formulation that finally led to equation (8b). Indeed, this view is supported by observations [Lamb, 1972] through which the double sunspot cycle in climate has been noted to have periods ranging from about 18 years to about 24 years. The theory just given provides some physical mechanism responsible for the existence of the double sunspot cycle in climatic data. According to equation (3), the oscillations at frequency w_n in equation (4b) are not always in phase with those given by equation (5a), and all these oscillations trigger off double sunspot cycles in climate according to the two conditions given earlier in this section. Now since, in addition, the physical significance behind the Hale or heliomagnetic cycle is simply the fact that the Sun's magnetic field changes direction with each new sunspot cycle, the Hale cycle should expectedly coincide with the double sunspot cycle in climate discussed above. Indeed, such coincidence has already been observed [eg. Barry and Perry, 1973]. Equation (8c) predicts the existence of oscillations at periods 66 yrs and 110 years both of which have been detected in meteorological records [eg. Lamb, 1972; 1982; Pittock, 1983; Burroughs, 1986].

Past observations have shown that the El Nino effect causes a complete reversal of the general circulation in the regions concerned once after about 11 years [Nieuwolt, 1977], thus making a complete cycle after about 22 years. So far it is apparently not known in exhaustive detail how the heat distribution patterns associated with the ~22-year period oscillations predicted by equation (13) do influence the general circulation system. A recent suggestion [Njau, 1987d; 1987f] points to the possibility that these heat distribution patterns are responsible, at least partially, for the ~22-year period variation in the general circulation system already observed over the regions directly influenced by the El Nino effect.

2.3 CASE 3: FUNCTION y(t) REPRESENTS ANY OF THE MILANKOVITCH CYCLES.

The Milankovitch theory of climate control identifies three cycles that together affect the incoming solar radiation at various areas on the Earth by up to ten percent (e.g. Schneider and Londer, 1984). These so called "Milankovitch cycles" have periods of about 100,000 years, 41,000 years and 21,000 years, and are caused by corresponding cyclic variations in the ellipticity of the Earth's orbit, the obliquity of the ecliptic and the longitude of the perihelion, respectively. All these three cyclic patterns have already been detected in proxy records of past climatic fluctuations (e.g. McCormac and Seliga, 1979; Lamb, 1977; Malone and Roeder, 1985).

If y(t) represents each of the three Milankovitch cycles in turn, then we would expect in accordance with equation (8b) the existence of cycles in climatic records at periods of about 200,000 years, 82,000 years and 42,000 years. Interestingly,all the latter periods have been detected in proxy records of climatic changes (Lamb, 1972; 1977; Pearson, 1978; McCormac and Seliga, 1979; Schneider and Londer, 1984; Malone and Roeder, 1985). Analysis of the same proxy records has yielded periodicities at ∿ 120,000 years, ∿ 210,000 years, ∿ 240,000 years, ∿ 400,000 years, ∿ 600,000 years and ∿ 1,000,000 years (Lamb, 1977; Pittock et al., 1978; Schneider and Londer, 1984; Malone and Roeder, 1985). Clearly the climatic indicators used in establishing these periods (i.e. historical records, tree rings, ice cores, pollen records, etc.) are not only difficult to calibrate but they essentially incorporate some dating uncertainty that in turn introduces some uncertainty onto the values of the periodicities indicated above. However, it is interesting to note that the six periods just mentioned are fairly close to the six periods which result after substituting d_o = 21,000 years, d_o = 41,000 years and d_o = 100,000 years in equation (8c) and then taking only the first two values of T_m in each case.

On the basis of the first part of this section, each Milankovitch cycle is amplitude-modulated by heat energy cycles which are effectively global, i.e. which act synchronously on both the North and South hemispheres. This explains why glacial-interglacial cycles occurred synchronously in the two hemispheres, a point that the Milankovitch mechanism has apparently failed to explain (Hansen and Takahashi, 1984).

It is generally agreed that the orbital insolation variations at least have a steering effect on the growth and decay of the Northern Hemisphere Pleistocene ice sheets, but it is also clear (e.g. Oerlemans, 1982) that internal (more or less free) oscillations play a dominant role and that such oscillations (which have not yet been clearly identified or established) do amplify the 100,000 years insolation signal. If y(t) represents the latter signal then the internal oscillations just mentioned are sufficiently established and accounted for in the initial portion of this section.

3. CONCLUSION.

We have verified theoretically that certain climatic/weather parameters including air temperature and rainfall undergo continuous amplitude-modulations principally as a result of interactions between energy from the Sun and the spinning motion of our Earth. Under certain specific conditions clearly derived and given in the text, these amplitude-modulations give rise to quasi-biennial oscillations and ~22-year period oscillations in the climatic/weather parameters mentioned above. One of the most important contributions of the paper is the confirmation of a previously suggested general law (Njau, 1987d) that, with an exception of the diurnal cycle, any permanent cycle in the net solar energy incident upon a given portion of the Earth-Atmosphere system gives rise to a quasi-permanent cycle whose period is approximately twice that of the former.

All the known periods of climatic variations ranging from 6 to 110 years [Morner and Karlen, 1984; Jakubcova and Pick, 1986] are given (in years) as follows: 6,7,8,9,10, 11,12,13,14,15,16,18,20, 22, 24,26, 30,34,45,58,90 and 110. It is interesting to note that each of the (twelve) underlined periods may be reproduced by substituting 1 year or 11 years for d_o in equation (8c) or (8b). Besides, each of the remaining (ten) periods may be reproduced by a physical interaction between a pair of the underlined periods or may possibly be a first harmonic of one of the underlined periods (see Table 2). Finally, interactions between the annual cycle and the quasi-biennial cycle or between one of these and each of the cycles listed in Table 2 would yield a string of oscillations whose periods range from about 1.3 years up to

about 6.0 years. Analyses of climatic records have detected quite a number of periodicities located in this range (e.g. Lamb, 1972; Berger, 1981; Morner and Karlen, 1984; Lamb, 1977; Pearson, 1978) which are given in years as 1.2, 1.6, 1.7, 2.0, 2.1, 2.2, 2.3, 2.5, 2.7, 3.0, 3.7, 4.0, 4.2, 4.7, 4.8, 5.0, 5.5 and 5.7.

NOTE ADDED

As already shown above, the analysis presented in the text predicts a string of periodicities in climatic variations stretching from about 1.3 years to about 6.0 years. As a further support to this prediction, elaborate analyses of rainfall records from Africa, Argentina, Australia, Brazil, China, India and New Zealand (see Kane and Trivedi, 1986) have detected the following periods which are given in years as follows: 2.07, 2.09, 2.11, 2.14, 2.19, 2.24, 2.26, 2.30, 2.32, 2.34, 2.37, 2.48, 2.60, 2.63, 2.66, 2.69, 2.72, 2.75, 2.77, 2.82, 2.85, 2.92, 2.95, 3.02, 3.05, 3.09, 3.20, 3.27, 3.30, 3.39, 3.47, 3.50, 3.51, 3.55, 3.63, 3.67, 3.72, 3.76, 3.85, 3.98, 4.07, 4.10, 4.27, 4.40, 4.42, 4.57, 4.62, 4.68, 4.70, 4.84, 4.95, 5.10, 5.40, 5.62, 5.69, 5.95 and 6.00.

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TABLE 1: The (heat) energy oscillations which give rise to amplitude-modulation over annual cycles.

LATITUDE	HEAT OSCILLATIONS EACH OF WHICH MAY BE PAIRED TO ANOTHER (FROM THE OTHER COLUMN) IN ORDER TO FULFIL CONDITION (6b) IN THE TEXT.	
	Oscillations at amplitude A_{Mn} and frequency w_n as given by equation (4b) in the text	Oscillations at amplitude a_{Mm} and frequency $w_m=(2m+1)\pi/Md_o$ as given by equation (5a) in the text.
0^o	ALL	Those for which $m \geq 3$
5^oS	ALL	Those for which $m \geq 2$
10^oS	ALL	ALL
20^oS	Those for which $n \geq 5$	ALL
30^oS	Those for which $n \geq 3$	ALL
40^oS	Those for which $n \geq 2$	ALL
50^oS	ALL	ALL
5^oN	ALL	Those for which $m \geq 3$
10^oN	ALL	Those for which $m \geq 2$
20^oN	Those for which $n \geq 5$	ALL
30^oN	Those for which $n \geq 3$	ALL
40^oN	Those for which $n \geq 2$	ALL
50^oN	ALL	ALL
60^oN to 90^oN and 60^oS to 90^oS	ALL	ALL (Note that the amplitudes of all these oscillations are negligibly small)

TABLE 2

A listing of all the known periods of climatic variations ranging from 6 to 110 years (Morner and Karlen, 1984; Jakubcova and Pick, 1986) and their possible causes based upon the analysis given in the paper.

OBSERVED PERIOD IN YEARS	POSSIBLE CAUSE
6	Predicted by equation 8(b) or 8(c)
10	"
11	"
14	"
18	"
22	"
26	"
30	"
34	"
58	"
90	"
110	"
7	First harmonic of cycle (shown above) at period 14 yrs
8	Interaction between cycles (shown above) at periods 6 & 11 yrs
9	" " " " " " 6 & 18 yrs
12	" " " " " " 11 & 14 yrs
13	" " " " " " 10 & 18 yrs
15	" " " " " " 11 & 22 yrs
16	" " " " " " 14 & 18 yrs
20	" " " " " " 11 & 110 yrs
24	" " " " " " 22 & 26 yrs
45	" " " " " " 10 & 18 yrs

FIGURE CAPTION

FIG. 1: A plot of the amplitude of annual change in incident solar energy versus latitude. Derived from Crowe (1971).

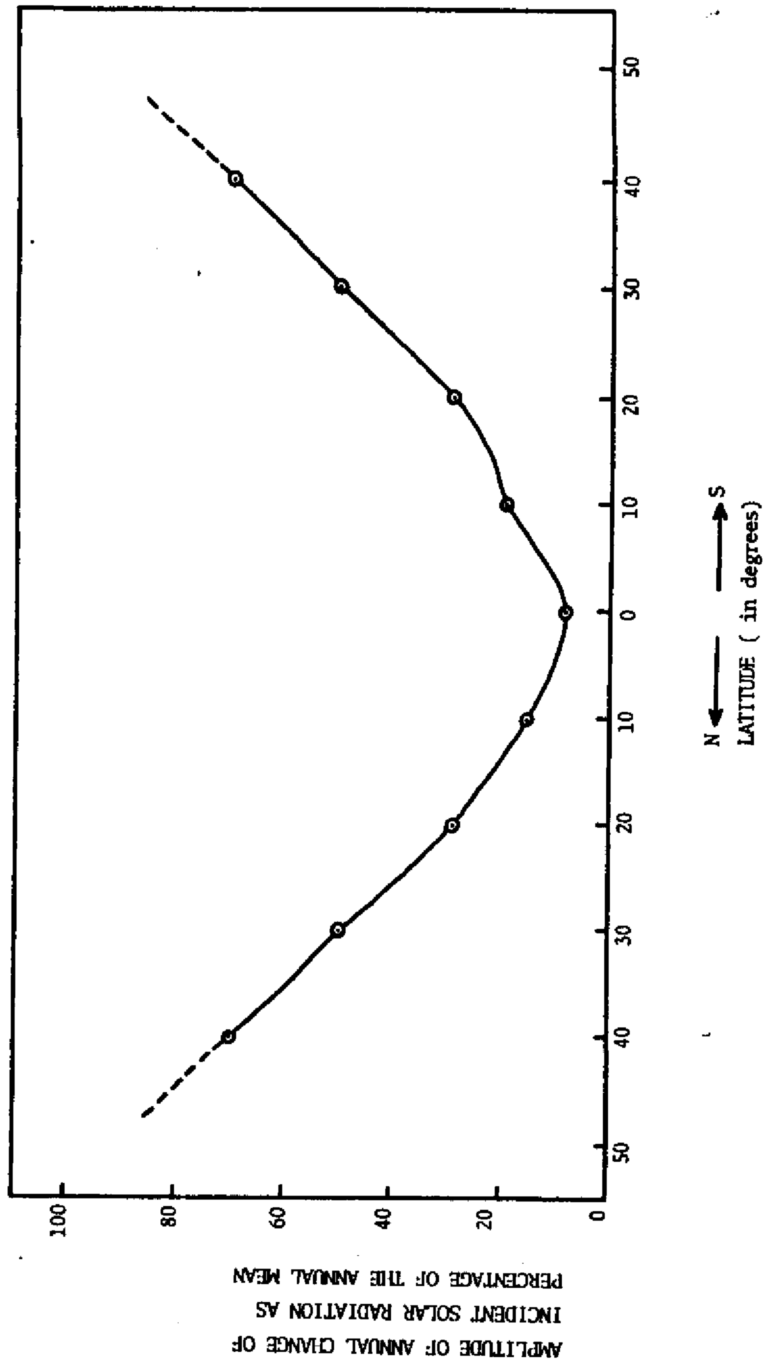

FIG. 1