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HADRON-QUARK PHASE TRANSITION IN DENSE STARS

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ABSTRACT

An equation of state is computed for a plasma of one flavour quarks interacting through some phenomenological potential, at zero temperature. Assuming that the confining potential is scalar and colour-independent, it is shown that the quarks undergo a first-order mass phase transition. In addition, due to the way screening is introduced, all the thermodynamic quantities computed are independent of the actual shape of the interquark potential. This equation of state is then generalarized to a several quark flavour plasma and applied to the study of the hadron-quark phase transition inside a neutron star.

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I. INTRODUCTION

Quark matter is expected to play an important rôle in cosmology and in astrophysics. In cosmology, if a quark-hadron transition actually occurred in the early universe, it could have a number of interesting consequences. First, it could be at the origin of density inhomogeneities[1),2)] (in our mind, this is probably the most important possible consequence because we do not know how to form galaxies, precisely the spectrum of density fluctuations must usually be added by hand in scenarii of galaxy formation). This transition might also provide an explanation to the missing mass problem, via the formation for instance of small black holes[3)] or strange nuggets [4),-8)]. Finally, if the transition occurred out of equilibrium, gravitational wave could have been emitted during the collisions of hadronic bubbles[4),9),10)] (this might be of interest in the future, because astrophysicists are trying to develop, in addition to conventional gamma-ray astronomy, ultraviolet astronomy, ..., new fields like neutrinos astronomy and gravitational wawes astronomy.)

In astrophysics, quark matter might be present in a number of objects: neutron stars with a quark core, pure quark stars -we will talk more about these objects below- and strange quark stars -see A.Olinto in the same issue of this review. In addition, quark matter might be the clue to understanding a number of anomalous events which occurred in cosmic rays such as the Centauro events or the - still controversial - Cygnus X-3 muon excess (see 11),4) and their followers).

However, *in all these possible applications of quark matter to cosmology and astrophysics, the main source of uncertainties is our lack of knowledge of the exact equation of state of the quark-gluon plasma.* This can be best illustrated by recalling briefly the history of quark core stars and quark stars.

The possibility that a quark core might exist in some type of stars, was first suggested by Ivanenko & Kurdgelaidze[12] (1969). Then Itoh[13] (1970) computed the maximum mass of quark stars, *neglecting totally interactions* between quarks. This mass, as in the *perfect* neutron gas of Oppenheimer & Volkoff[14] (1939), was very small: $10^{-3} M_{\odot}$. (For comparison, neutron star masses are usually observationally found to be of order 1.4 $M_{\odot}$.) The interactions between quarks were poorly known.

A few years later, when the M.I.T. bag model was invented[15], Brecher & Caporaso[16] (1976) showed that, by making use of it, higher maximum masses could be obtained for quark core stars. However, when first order corrections in the strong coupling constant were included in the quark M.I.T. equation of state, as done by Baym & Chin[17] (1976) and Chapline & Nauenberg[18] (1976), the density at which the quark phase should appear was much higher than the maximum central density reachable by stable neutron stars. So without the possibility of a quark-hadron phase transition inside neutron stars, the possibility that quark core stars or quark stars might exist was well diminished. (However as discussed by Kislinger & Morley[19] - both these papers made used of a high value of the coupling constant, obtained when fitting hadronic spectra. In addition, since the coupling constant was held fixed, the equation of state contained large logarithms involving the density).

In parallel to this phenomenological approach, Collins & Perry[20] (1975), demonstrated that asymptotic freedom which holds for large momentum transfers, also holds at high density, for exemple inside neutron stars. Keister & Kisslinger[21] (1976), using a perfect gas equation of state, and Chapline & Nauenberg[22] (1977), starting from an expression of the energy density computed to first order in the - density dependent- coupling constant, concluded again that the quark-hadron phase transition would occur at too large a density , so that the possibility of having a pure quark phase inside neutron stars could be ruled out. However, these calculations were too crude for the first one and not completely self-consistent for the second one as discussed by Freedman & Mc Lerrăn[23]. In 1978, it was shown independently by Kislinger & Morley[19] and Freedman & Mc Lerran[23] - and checked in a different approach by Baluni[24] -that when an expansion of the quark matter energy in the density dependent coupling constant was done, the quark-hadron phase transition was predicted to occur at densities lower than the maximum central density for a stable neutron star. It was also noticed by Freedman & McLerran, that this result still hold if one added a vacuum constant to the equation of state - thus defining an improved M.I.T. bag, without divergent logarithms when a proper choice of the substraction point was made. This was checked by Fechner & Joss[25] (1978), with and without vacuum constant, in the case of a first or a second order phase transition between hadrons and quarks, and for several nuclear-matter equations of state. They also showed that in addition to quark core stars, stable (pure) quark stars could even exist.

In addition to these studies, a number of other phenomenological approaches have been done. In a paper from 1977, Bowers & al.[26], extrapolated the Walecka model[27] of nuclear matter to quark matter, namely they assumed that quarks exchanged massive scalar and vector particles and treated them in the Hartree, or mean field approximation. They showed that for reasonable choices of their parameters, quark core stars might exist but that quark stars would not be stable. This model was re-interpreted by Alvarez & Hakim[28] in the context of the SLAC bag model: the scalar field was used to generate a first order phase transition from a state of massive particles to a state of particles of decreasing mass, the vector field was identified with gluons -massive from the beginning while in the SLAC model they acquire a mass through the Higgs mechanism. This allowed them to give an estimate of the parameters in their model, and their concluded that a quark-hadron phase transition within neutron stars was possible. Finally, starting simply from the QCD lagrangian with a Hartree-Fock approximation, Alvarez[29)] computed another equation of state. He concluded -see also the paper by Alvarez & Ibanez[30)]- that the quark-hadron phase transition would occur at a higher density than allowed in stable neutron stars and that pure quark stars could exist but would be instable. (In our mind, this equation of state always lies close to $p=\rho/3$ hence it is rather similar to that of Keister & Kisslinger and it should not come as a surprise if the quark-hadron transition occurs at too high a density).

So we have seen that the quark matter equation of state plays a crucial rôle when trying to draw conclusions on dense stellar objects. Because of the uncertainties in the parameters to be employed in both perturbation-based and M.I.T.-like equations of state, it is fruitful to investigate the consequences of other choices for the quark equation of state. In what follows, we will present a new way of deriving phenomenological quark matter equations of state (section 2), see how to apply this approach to the quark-hadron phase transition inside dense neutron stars (section 3) and discuss the avantages and drawbacks of this method, as well as possible improvements (section 4).

2. METHOD

a. The effective Lagrangian and the approximation

As a starting point, let us assume that the quarks interact via the following effective Lagrangian density

$$\mathcal{L} = \overline{\psi}(x)(i\not{\partial} - m)\psi(x) - i\int d^3z\overline{\psi}(x)\overline{\psi}(z)V(|x-z|)\psi(z)\psi(x) \qquad (2.1)$$

i.e. we treat the quarks as Dirac particles and their interactions are accounted for by a phenomenological potential V. (The idea of describing the interactions between quarks by phenomenological interquark potentials such as those of potential models used to fit experimental data on quarkonia, was first suggested by Wagoner & Steigman[31]. It was later taken up by Olive[32] annd Boal, Schachter & Woloshyn[33] to describe the quark-gluon plasma in the Thomas-Fermi approximation. Their type of equations of state was applied to studies of the primordial quark-hadron phase transition by Källman[34] and Schramm & Olive[35].)

From (2.1), one can derive the energy momentum tensor $T_{\mu\nu}$. On the other hand, one may assume that the medium is uniform and isotropic, in which case one also has

$$T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu - p g_{\mu\nu} \tag{2.2}$$

where p is the pressure and ϵ is the energy density. Form (2.1) and (2.2) it follows that

$$\epsilon = \frac{< T^{00} >}{V} = \int \frac{d^4p}{(2\pi)^4} \frac{p_0\gamma_0 + \vec{p}\vec{\gamma} + m}{2} G^<(p) \tag{2.3a}$$

$$p = \frac{< T^{ii} >}{3V} = \int \frac{d^4p}{(2\pi)^4} \frac{p_0\gamma_0 - \vec{p}\vec{\gamma}/3 - m}{2} G^<(p) \tag{2.3b}$$

So that all what is needed in order to calculate the equation of state, is the expression of $G^<(p)$ and this is what we now turn to.

The (Dirac) equation of motion which follows from (2.1) is

$$(i \not{\partial} - m) G(x, y) = \delta^4(x - y) - i \int d^3z G(x, z; y, z^+) V(| x - z |)_{|z_0 = x_0} \tag{2.4}$$

where the notation z^+ means that z^{0+} is infinitesimally greater than z_0.
In order to solve (2.4), one has to specify what is the form of the interquark potential V and how to express the two-particle Green function in terms of known or calculable quantities. First, we will assume the following structure for the interaction potential

$$V(r) = V_V(r) \gamma_\mu^{(1)} \gamma^{\mu(2)} - V_S(r) 1_D^{(1)} 1_D^{(2)} \tag{2.5}$$

In practice, one takes the vector term V_V to be the one-gluon potential expected to be dominant at short distances and the scalar term V_S to be the confining potential expected to be dominant at large distances [1]

In addition to specifying the Lorentz structure of the potential, its colour structure has to be determined. While it is normal to multiply the one-gluon exchange term by a factor of $\lambda^{(1)}.\lambda^{(2)}/4$, no decisive theoretical or experimental argument can be put forward to decide what to do for the confining part. So in addition to assuming that the confining potential is scalar, we will assume simply that it does not depend on colour[2]. (Note that such assumptions are also made in the MIT and SLAC models.) So

$$\begin{cases} V_S(r) &= V_C(r)1_c^{(1)}1_c^{(2)} \\ V_V(r) &= V_G(r)\vec{\lambda}^{(1)}\vec{\lambda}^{(2)}/4 \end{cases} \tag{2.6}$$

Finally, the two-particle Green function will be approximated by

$$G(x,y;z,t) \sim G(x,z)G(y,t) \tag{2.7}$$

(This in fact the Hartree approximation as will become clear later.)

Insertion of (2.7) into (2.4) then leads to an equation in terms of one-particle Green functions only

$$[i \not{\partial} 1_D 1_c - m 1_D 1_c + i\int d^3z G(z,z^+)V(\mid x-z \mid)_{|z_0=x_0}]G(x,y) = \delta^4(x-y)1_D 1_c \tag{2.8}$$

The Hartree potential is defined by the following matrix

$$U_H \equiv -i\int d^3z G(z,z^+)V(\mid x-z \mid)_{|z_0=x_0} \tag{2.9}$$

[1]Phenomenologically, a scalar component V_S is necessary because in the non-relativistic expansion of the Bethe-Salpeter equation, the spin-orbit term has opposite sign for scalar or vector potentials, the right sign to get the observed ordering of the ^3P_j levels of charmonium being that of scalar potentials[36]. *The confining potential is usually assumed to be this scalar component.* As a matter of fact, one does not know whether the confining potential does not have a more complicated structure. In reference 37 , we also studied the possibility that the confining potential has both a scalar and a vector component. In addition, potentials of the form (2.5) (i.e. without tensor, axial, etc, components) are the simplest ones leading to good theoretical predictions for quarkonia.

[2]We discussed briefly other possibilities for the colour dependence in ref. 37 and 38

Using (2.5) and (2.6) in (2.9), one obtains

$$U_H \sim -\int d^3z V_C(\mid x-z \mid).Tr[-iG(z,z^+)1_D 1_c].1_D 1_c + \int d^3z V_C(\mid x-z \mid).Tr[-iG(z,z^+)\gamma_\mu \lambda_a].\gamma^\mu \lambda^a \tag{2.10}$$

The last term in this equation is in fact null at equilibrium because then the Green function is proportional to the unity matrix in colour space. So that

$$U_H = U_S^H .1_D 1_c \tag{2.11}$$

where

$$U_S^H \equiv -\int d^3z [\alpha V_C(\mid x-z \mid)].Tr[-iG(z,z^+)1_D 1_c]$$

Equation (2.8) may be written in momentum space as below

$$\begin{array}{lll} [\not p - m_H]1_c G(p) & = & 1_D 1_c \\ \text{where} & & \\ m_H & = & m + U_S^H \end{array} \tag{2.12}$$

(m_H is designated thereafter as the effective mass).

The problem of a given quark having two-body interactions with others has been replaced by that of a free quark in the external field (2.9). The effect of this field is to change the mass m into the effective mass m_H.

b. Finite Hartree equations

(2.12) can be solved by using the methods of temperature-dependent quantum field theory[39] and we obtain the following expression for the quantity which we want to compute, the retarded propagator

$$\begin{array}{rcl} -iG(z,z^+) & = & \int \frac{d^4p}{(2\pi)^4} G^<(p) \\ & = & \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp[\beta(\sqrt{\vec p^2+m_H^2}-\mu)+1]} \frac{\gamma_0\sqrt{\vec p^2+m_H^2}-\vec\gamma\vec p+m_H}{2\sqrt{\vec p^2+m_H^2}}.1_c \\ & + & \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp[\beta(\sqrt{\vec p^2+m_H^2}+\mu)+1]} \frac{-\gamma_0\sqrt{\vec p^2+m_H^2}-\vec\gamma\vec p+m_H}{2\sqrt{\vec p^2+m_H^2}}.1_c \\ & + & \int \frac{d^4p}{(2\pi)^4} \frac{\gamma_0 p_0+\vec\gamma\vec p+m_H}{p_0^2-\vec p^2+m_H^2+i\epsilon}.1_c \end{array} \tag{2.13}$$

The first two integrals represent the contribution of matter while the last one accounts for vacuum fluctuations -and gives rise to a divergence. However, owing to the fact that our approach is essentially phenomenological, these fluctuations should not show up; the matter part only should be used throughout the calculations (see reference 37). Inserting the matter part of (2.13) into the expression of U_S^H in (2.11) then leads to the following self-consistent equation for U_S^H as a function of the chemical potential, also denoted by μ, *at zero temperature*:

$$U_S^H = -\int d^3z[\alpha V_S(|\, z - x\,|)].\{6\int_0^{p_F} \frac{d^3p}{(2\pi)^3}\frac{m_H}{\sqrt{\vec{p}^2 + m_H^2}}\} \tag{2.14}$$

where

$$p_F = \begin{cases} \sqrt{\mu^2 - m_H^2} & \text{if } \mu > m_H \\ 0 & \text{otherwise} \end{cases}$$

This equation is rather similar to that of the scalar plasma studied by Kalman[40] and Diaz-Alonso & Hakim[41] or of the scalar part of the nuclear matter model developed by Walecka and Chin[27]

$$U_S^H = -\frac{g_s^2}{m_s^2}.\{2\int_0^{p_F} \frac{d^3p}{(2\pi)^3}\frac{m_H}{\sqrt{\vec{p}^2 + m_H^2}}\} \tag{2.15}$$

The part of the ratio $\frac{g_s^2}{m_s^2}$ is played here by $\int d^3zV_C(|\, z - x\,|)$.

However contrarily to (2.15) expressions (2.14) may still contain infinities, since $\int d^3zV_C(|\, x - z\,|)$ is infrared divergent, for instance in the case of a linear confining potential. But, as advocated by Kogut and Susskind[42], infinitely rising potentials are expected to be screened by the creation of quark-antiquark pairs. As a first approximation, one may suppose that there exist some screening length c_S (to be determined from experiment), fixed whatever the value of μ is,then

$$\begin{aligned} U_S^H \;=\; & -\textstyle\int_0^{c_S} d^3rV_C(r) \\ & \text{or } -\textstyle\int d^3rV_C(r)\, \exp(-r/c_S) \\ & \text{or etc} \\ \times\; & \{6\textstyle\int_0^{p_F} \frac{d^3p}{(2\pi)^3}\frac{m_H}{\sqrt{\vec{p}^2+m_H^2}}\} \end{aligned} \tag{2.16}$$

according to the way screening occurs.

However, one expects c_S to depend on μ and it is possible to compute this dependence: U_S^H is an energy felt by a given quark, so that it should not exceed twice the Fermi energy $\epsilon_F = \mu$, else pair creation would occur (one must use the Fermi energy as a threshold, and not the quark mass, because in dense matter pairs can only be created above the Fermi sea). Hence one has

$$\begin{aligned} 2\mu &= \mid -\textstyle\int_0^{c_S} d^3r V_C(r) \\ &\quad \text{or} - \textstyle\int d^3r V_C(r)\,\exp(-r/c_S) \\ &\quad \text{or etc} \\ &\times \ \{6\textstyle\int_0^{p_F} \frac{d^3p}{(2\pi)^3}\frac{m_H}{\sqrt{\vec{p}^2+m_H^2}}\}_{|U_S^H=\pm 2\mu} \end{aligned} \tag{2.17}$$

In other words, the finite values of $\int d^3z V_C(\mid (x-z) \mid)$ is given by the following formula

$$\begin{aligned} & -\textstyle\int_0^{c_S} d^3r V_C(r) \\ & \text{or} - \textstyle\int d^3r[\alpha V_C(r)\,\exp(-r/c_S)] \\ & \text{or etc} \\ & = \frac{\pm 2\mu}{\{6\int_0^{p_F}\frac{d^3p}{(2\pi)^3}\frac{m_H}{\sqrt{\vec{p}^2+m_H^2}}\}_{|U_S^H=\pm 2\mu}} \end{aligned} \tag{2.18}$$

Eq. (2.18) hold for a given μ, whatever the value of U_S^H is, not just for $\pm 2\mu$, and so (2.14) can be replaced by

$$U_S^H = \frac{\pm 2\mu}{\{6\int_0^{p_F}\frac{d^3p}{(2\pi)^3}\frac{m_H}{\sqrt{\vec{p}^2+m_H^2}}\}_{|U_S^H=\pm 2\mu}} \times \{6\int_0^{p_F}\frac{d^3p}{(2\pi)^3}\frac{m_H}{\sqrt{\vec{p}^2+m_H^2}}\} \tag{2.19}$$

Note that in the equation (2.19), V_C has completely disappeared -because of the way screening was introduced- so the shape of the confining potential or the way it is dumped (i.e. is it exponentially dumped or does it stop sharply at a certain distance or etc) does not matter. Note also that only the minus sign is allowed in (2.9) is allowed, else the denominator would have $p_F = 0$ and vanish, so the finite value of $\int d^3z V_C(\mid (x-z) \mid)$ is positive.

c. Numerical results (for one flavour)

Equation (2.19) may be solved numerically. In order to get a first insight into the solution, let us assume that the screening length -and so the finite value of $\int d^3zV_C(\mid (x-z) \mid)$ as well- is constant. In figure 1a, we show the solution of the equation for the effective mass (2.16) for various positive values of $\int d^3zV_C(\mid (x-z) \mid)$ (multiplied by $3m^2/\pi^2$ for computational commodity, and labeled as Γ). In the general case (2.19), the curve for the effective mass is presented in figure 1b. It is rather similar to 1a, except for the fact that, due to screening, it terminates abruptly.

Once (2.19) is solved, the equation of state may be computed easily. By inserting the matter part of (2.13) into (2.3a-b), one gets at zero-temperature

$$\epsilon_Q = 3\int_0^{p_F} \frac{d^3p}{(2\pi)^3}\{\sqrt{\vec{p}^2+m_H^2} + \frac{\vec{p}^2+m_Hm}{\sqrt{\vec{p}^2+{}^2_H}}\} \tag{2.20a}$$

$$p_Q = 3\int_0^{p_F} \frac{d^3p}{(2\pi)^3}\{\sqrt{\vec{p}^2+m_H^2} - \frac{\vec{p}^2/3+m_Hm}{\sqrt{\vec{p}^2+m_H^2}}\} \tag{2.20b}$$

The density of quarks minus antiquarks is simply given by

$$n_Q = 6\int_0^{p_F} \frac{d^3p}{(2\pi)^3} \tag{2.20c}$$

In figure 2a, the pressure has been plotted as a function of the chemical potential: it shows that as the density increases a *first-order phase transition* takes place. This transition corresponds to the passage of a state of massive quarks to a state of quarks of decreasing mass (dashed-dotted line in figure 1b), as one would expect from perturbative QCD. In Fig. 2b, the energy per particle as a function of particle density is represented. At low density, it is smaller than m, so the quarks are in a collective bound state. This may be interpreted as the fact that the quark just start to go out of the hadrons. Thus the overall picture which is obtained is satisfying to describe quark matter at low density (where confinement is expected) and medium density (where the quark mass should start to decrease), so we can apply our method to neutron stars.

3. APPLICATION TO NEUTRON STARS

a. Generalization to several quark flavours

In the case of several flavour quark matter, one should replace (2.4) which holds for one flavour only, by

$$(i \not\partial - m_q)\, G^q(x,y) = \delta^4(x-y) - i\int d^3z \sum_{q'=u,d,s\dots} G^{q,q'}(x,z;y,z^+) V_C(|\,x-z\,|)_{|z_0=x_0} \tag{3.1}$$

Then proceeding as for the one flavour case, it is easy to show that

$$U_S^H = \frac{2\mu_L}{\sum_{q=u,d,s,\dots} 6\int_0^{p_F^q} \frac{d^3p}{2\pi^3}\frac{m_H^q}{\sqrt{\vec p^2+m_H^{q\,2}}}}_{|U_S^H=-2\mu_L} \cdot \sum_{q=u,d,s,\dots} 6\int_0^{p_F^q} \frac{d^3p}{(2\pi)^3}\frac{m_H^q}{\sqrt{\vec p^2+m_H^{q2}}} \tag{3.2}$$

where

$$p_F^q(U_S^H,\mu_q) = \begin{cases} \sqrt{\mu_q^2 - m_H^{q\,2}} & \text{if } \mu_q \geq m_H^q \\ 0 & \text{otherwise} \end{cases}$$

In the above expressions, μ_L designates the smallest among the Fermi energies of the various quark flavours; pairs of this flavour are created preferably to screen the interquark potential.

It can be seen that the Hartree field U_S^H is the same whatever the flavour, and the effective masses $m_H^q \equiv m_q + U_S^H$, which play a similar part to running masses, will all have a similar decrease.

Once (3.2) is solved, similarly to the one flavour case, the quark contribution to the equation of state will be obtained from

$$\epsilon_Q = \sum_{q=u,d,s,\dots} 3\int_0^{p_F} \frac{d^3p}{(2\pi)^3}\Big[\sqrt{\vec p^2 + m_H^{q\,2}} + \frac{\vec p_F^2 + m_H^{q\,2}}{\sqrt{\vec p^2+m_H^{q\,2}}}\Big] \tag{3.3a}$$

$$p_Q = \sum_{q=u,d,s,\dots} 3\int_0^{p_F} \frac{d^3p}{(2\pi)^3}\Big[\sqrt{\vec p^2 + m_H^{q\,2}} - \frac{\vec p_F^2/3 + m_H^{q\,2}}{\sqrt{\vec p^2+m_H^{q\,2}}}\Big] \tag{3.3b}$$

$$n_Q = \sum_{q=u,d,s,...} 6 \int_0^{p_F^q} \frac{d^3p}{(2\pi^2)^3} \tag{3.3c}$$

If electrons must be included to maintain charge neutrality, their contribution to the equation of state will be

$$\epsilon_e = \frac{\mu_e^4}{4\pi^2} \tag{3.4a}$$

$$p_e = \frac{\mu_e^4}{12\pi^2} \tag{3.4b}$$

$$n_e = \frac{\mu_e^3}{3\pi^2} \tag{3.4c}$$

b. Conversion of neutron matter to two flavour quark matter

Let us first study the case of neutron matter undergoing a phase transition to quark matter. Weak interactions do not have time to settle and charge neutrality simply reads

$$e(\frac{2}{3}n_u - \frac{1}{3}n_d) = 0 \tag{3.5}$$

Hence,

$$\mu_u \equiv \mu \tag{3.6a}$$

and

$$\mu_d = \begin{cases} [2^{2/3}\mu^2 + (1-2^{2/3})m_H^2]^{1/2} & \text{if } \mu \geq m \equiv m_u \equiv m_d \\ \mu & \text{otherwise} \end{cases} \tag{3.6b}$$

One sees that μ_d is greater or equal to μ_u, so $u\bar{u}$ pairs will be created rather than $d\bar{d}$ pairs, i.e. in (3.2), L=u. Note that quantities will now be plotted not as a function of μ_u or μ_d but as a function of the Gibbs energy per particle $G \equiv \sum_{q=u,d,s,...} n_q/n_Q = \mu_u + 2\mu_d$.

Equation (3.2)may be solved numerically. Its solution as a function of G is quite similar to that of the one flavour case in figure 1b, so we do not show it. Again the quark plasma will undergo a first order mass phase transition. In order to compute the Gibbs energy per particle at which it occurs, the pressure as a function of G is plotted in figure (3.a). The thermodynamically preferred state (i. e. G minimum for a given p) is p=0 from G=0 to G=1.575, and then p starts to increase abruptly. The phase transition takes place at this value of $G = 1.575$. In order to know at which density this corresponds, the baryon density has been depicted as a function of G in figure (3.b). One sees that when $G = 1.575$, the density is $n_t = 0.0085$.

In what precedes, all the quantities have been computed in unit of the quark mass, m, and indeed this is the only parameter of the two flavour model. It is in fact possible to find a lower bound for this parameter. First, n_t should be greater or equal to the nuclear matter density, so we must have

$$n_t \geq n_{nuc.matt.} = 1.28\,10^6\,Mev^3/m^3 \text{ which implies } m \geq 532.\,Mev \qquad (3.7)$$

The fact that we obtain 532.Mev as a lower bound for the constituent mass -usually thought to be of order $340.Mev$ is an indication that our model is reasonable but crude. In what follows, we are going to see that it is possible to get an upper bound for m as well.

c. Three flavour quark matter in chemical equilibrium

Once the transition is accomplished, the quarks will establish chemical equilibrium via the weak interactions

$$d \longleftrightarrow u + e + \bar{\nu}_e \qquad (3.8a)$$

$$s \longleftrightarrow u + e + \bar{\nu}_e \qquad (3.8b)$$

$$s + u \longleftrightarrow d + u \qquad (3.8c)$$

The weak interactions (3.8a-c) imply that

$$\begin{cases} \mu_d = \mu_s \equiv \mu \\ \mu_u + \mu_e = \mu \end{cases} \qquad (3.9)$$

and overall charge neutrality requires that

$$e(\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e) = 0 \tag{3.10}$$

Thus there is only one independent chemical potential, which we choose as being μ. One sees that $\mu_u = \mu - \mu_e$ is smaller or equal to μ_u, so again $u\bar{u}$ pairs will be created preferably.

Note that in the three flavour case there are two parameters: $m \equiv m_u$ and $r \equiv m_u/m_s$. Since m_u and m_s are constituent masses, we expect that $r \approx 500./340. = 1.47$.

Equations (3.2) for U_S^H and (3.10) for say, μ_e, can be solved simultaneously numerically for various values of μ, with the input (3.9). Once this is done, the thermodynamical quantities (pressure, energy density, baryon density, electronic density) can be computed by using (3.3a-c) and (3.4a-c). The behaviour of these various functions is rather similar to that of two flavour quark matter -so we only show $p(\epsilon)$ in figure (4) for $r = 1.47$. This does not mean that there will be another phase transition, once u and d quarks start to appear with a given density, they will be gradually depleted, the pressure needs not vary abruptly.

In figure (4), the curve terminates suddenly. This corresponds to a value of G equals to 1.7, which we will denote by G_{max}. Up to G_{max}, the equation of state is very non-perturbative. On the other side, one may compute the approximate value of the Gibbs energy per particle G_{pert} at which quark matter should start to be describable with a perturbative equation of state. Let us suppose that this happens when the coupling constant equals one [3]. We can get a rough approximation of G_{pert} by solving

$$\alpha_s(\mu^2) = \frac{6\pi}{(33 - 2\,N_f)\ln(\mu/\Lambda)} = 1 \tag{3.11}$$

[3] We could just as well take a half or any other value smaller than one, this would increase the upper bound we are looking for. So taking $\alpha_s = 1$ is more restrictive

This is the expression of the running coupling constant in the case of quarks of mass much smaller than μ.

If we take Λ to be 200. Mev for instance and $N_f = 3$, the solution of (3.11) is μ=401.Mev. So, if $r \approx 1.47$, the following constraint should be satisfied

$$G_{max} \leq G_{pert} = 3 \times 401./m \text{ which implies } m \leq 708.Mev \tag{3.12}$$

This upper bound is compatible with the lower bound (3.7). It corresponds to a transition density $n_t \sim 2.4 n_{nuc.matt.}$.

d. Some preliminary results for dense stellar objects

Now that we have an equation of state (expressed in function of m) and bounds for m, we may try to apply it to stellar objects. First one may wonder if strange stars could exist. As explained in Farhi & Jaffe[43) , quark matter containing strange quarks -in addition to u and d quarks- is more stable than *ordinary* nuclear matter if

$$G(u,d,s)_{p=0} \leq G_{nuc.matt.} = 930.\text{Mev} \tag{3.13a}$$

But one also needs to require that u-d quark matter be less favorable, energetically speaking, than *ordinary* nuclear matter, i.e.

$$G(u,d)_{p=0} \geq G_{nuc.matt.} = 934.\text{Mev} \tag{3.13b}$$

These equations imply

$$\text{if } r = 1.47 \text{ or } 1.47 - 10\%, m < 593. \text{ and } m > 593.$$

$$\text{if } r = 1.47 + 10\%, m < 596. \text{ and } m > 593. \tag{3.14}$$

So it does not seem very likely that strange matter would be stable and that strange stars would exist within this model, while it is possible if one uses the M.I.T. bag approach -see A.Olinto in the same issue of this review.

Next, one may want to compute mass-radius relations for quark core stars and pure quark stars and investigate their stability. This work is currently in progress. However, the stiffer the equation of state, the higher the maximum mass allowed for a star is. So a glance at figure (4) permits to conclude that our maximum mass should be higher than those obtained in the M.I.T. bag approach[44].

4. CONCLUSION

The methods of relativistic quantum many-body theory have been applied to the study of quark matter interacting through phenomenological potentials at zero temperature. It was shown that if the chosen confining potential has a scalar Lorentz structure and is colour-independent, the quark plasma undergoes a first order mass transition from a massive collective bound state at low density to a gas of particles of decreasing mass at high density. Moreover all the computed quantities are independent of the shape of the interquark potential, because of the way screening through pair creation has been implemented. This one-flavour model was then generalized to several flavours and applied to the hadron-quark phase transition in a cold plasma. It was shown that the u-d constituent quark mass had to be in the interval [532.Mev,704.Mev] -which is reasonable for such a simple model. Thus a satisfying description for quark matter seems to emerge from this approach.

However, some reservations must be made. First, instantaneous interquark potentials were used. At low-medium densities where quarks are massive, this should be a reasonable approximation. (Instantaneous potentials have been used widely to study nuclear matter in exactly the same range of densities we are interested in, namely at nuclear density -1.28 10^6 Mev^3- and slightly higher.) But at high densities where quarks become more and more relativistic, such an approach should not be used. As a matter of fact, we saw in section 2, that our equation of state is only computable for low-medium densities; at high densities, it should be matched with a perturbative equation of state. (Note that other phenomenological equations of state such as the M.I.T. one, cannot be used either -for other reasons- at high densities.)

Second, the confining potential may have a more complicated structure than just scalar and colour-independent. (In reference 37, we studied the possibility that the confining potential has both a scalar and a vector component. It might be actually interesting to develop the interquark confining potential with Gell-Mann matrices as can be done in the Clifford algebra with Dirac matrices- and study other more complex colour structures.)

Thirdly and finally, it is not completely obvious which kind of mass must be used for m. Looking at Eq. (2.1) for instance, one might favour a current mass - indeed this is what Olive[32)] did for example - because one would expect that, as a result of interactions with other quarks, this current mass is increased to the value of a constituent mass. However, it is not so: the Hartree field contribution to the quark mass is negative and $m_H/m \rightarrow 1$ as μ decreases as can be seen in figure 1.b. Also it may be shown that if one introduces non-zero temperatures in a gap equation such as (2.19), at a given density, m_H/m decreases as the temperature increases[41)], so that, in order that chirality be restored at high density or temperature, the choice of a constituent mass for m is most reasonable . In addition, since potential models of quarkonia and hadrons are fitted with constituent masses, and that our approach makes use of such potentials, it is also more self-consistent to use constituent masses (this is what was done as well for instance in reference 45 .)

It is also easy to imagine some possible improvement to this model. First Fock exchange terms are known to be negligible in the equation of state for nuclear matter and small for neutron matter. Since these terms are dependent of the shape of the potential, it would be interesting to check if they are also negligible for our quark matter model. This point is studied in reference 38.

Second, in the Hartree approximation, one replaces the interaction of a number of quarks on a given quark, by an exterior field acting on this quark. As a consequence of this, we get a collective bound state at low densities and not a soup of colour singlets (hadrons). In order to get , if not baryons, at least mesons, an approximation of a different type should be used.

Third, one can think of other ways of computing screening lengths in a self-consistent way. For instance, one can assume some value for the screening length, study the plasma oscillations of the quark gas, then from this compute the associated Debye shielding length and see if it agrees with the value initially assumed for the screening length -if not iterate. It would be interesting to compare this approach with the one followed here.

Fourth and last, this equation of state could be generalized to non-zero temperature. Non-virtual gluons could be considered as modes propagating in the quark plasma -in much the same way that a solid emits phonons- and their energy spectrum obtained by computing the plasma oscillations.

In summary, the equation of state presented here should be easy to improve. In addition, its use in astrophysics does not present any difficulty. In our mind, the main interest of this approach is that it allows one to utilize a new source of information: data from quarkonia spectroscopy, and perhaps in the future, results on the interquark potential obtained in lattice gauge theory simulations.

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FIGURE CAPTIONS

Fig. 1a Plot of the effective mass as a function of the chemical potential for different values of constant positive Γ. A phase transition occurs if $\Gamma \geq 2.5$.

Fig. 1b Plot of the effective mass as a function of the chemical potential. Due to screening, it terminates more abruptly than in figure 1a. A glance at Fig. 2a allows one to determine which states are not mechanically stable (dotted line) and which states are physically accessible (dashed line).

Fig. 2a Plot of the pressure as a function of the chemical potential. The transition is seen to be first order and is associated to a change of mass (dashed-dotted line in figure 1b). For curve designation, see figure 1b.

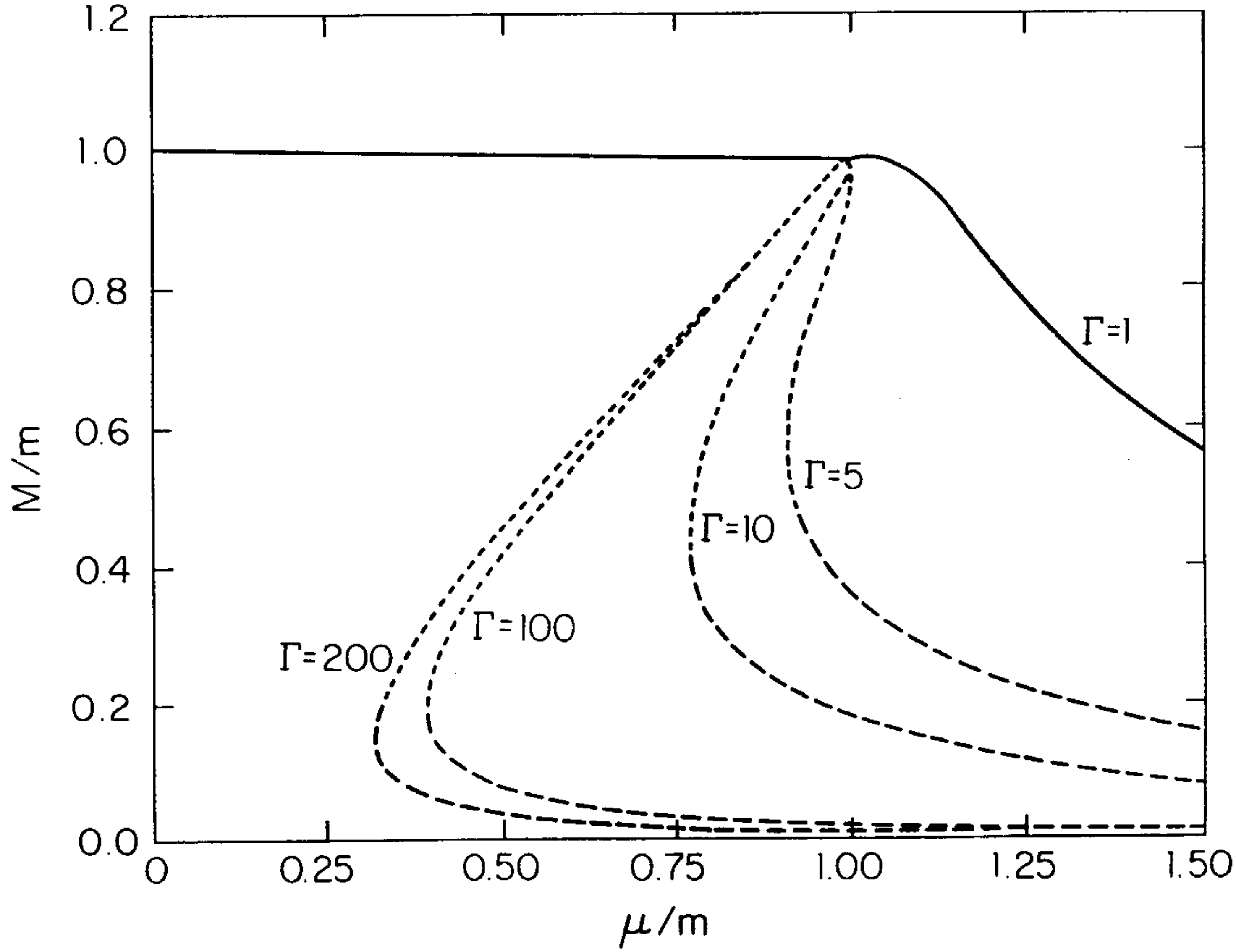
Fig. 2b Plot of the energy per particle as a function of particle density. The plasma is in a collective bound state at low densities.

Fig. 3a Plot of the pressure as a function of the Gibbs energy per particle, for several quark flavours. The transition occurs for G=1.575m.

Fig. 3b Plot of the density of quarks as a function of the Gibbs energy per particle, for several quark flavours. The transition takes place when $n_t = 0.0085m^3$.

Fig. 4 Plot of the pressure versus energy density for u,d,s quarks and electrons in chemical equilibrium, with m=595.mev and r=1,47 (dashed-dotted curve). For comparison, the M.I.T. bag equation (continuous curve) of state is shown (with[44] m=0, m_s=200.Mev, α_s=0.17) as well as the equation of state (dotted curve) for non-interacting massless u-d quarks and s quarks of mass 280 Mev (which are the effective masses corresponding to G=G_{max}).

Figure 1a



M/m
μ/m
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1.0
0.8
0.6
0.4
0.2
0.0
0
0.25
0.50
0.75
1.00
1.25
1.50
Γ=1
Γ=5
Γ=10
Γ=100
Γ=200

Figure 1b

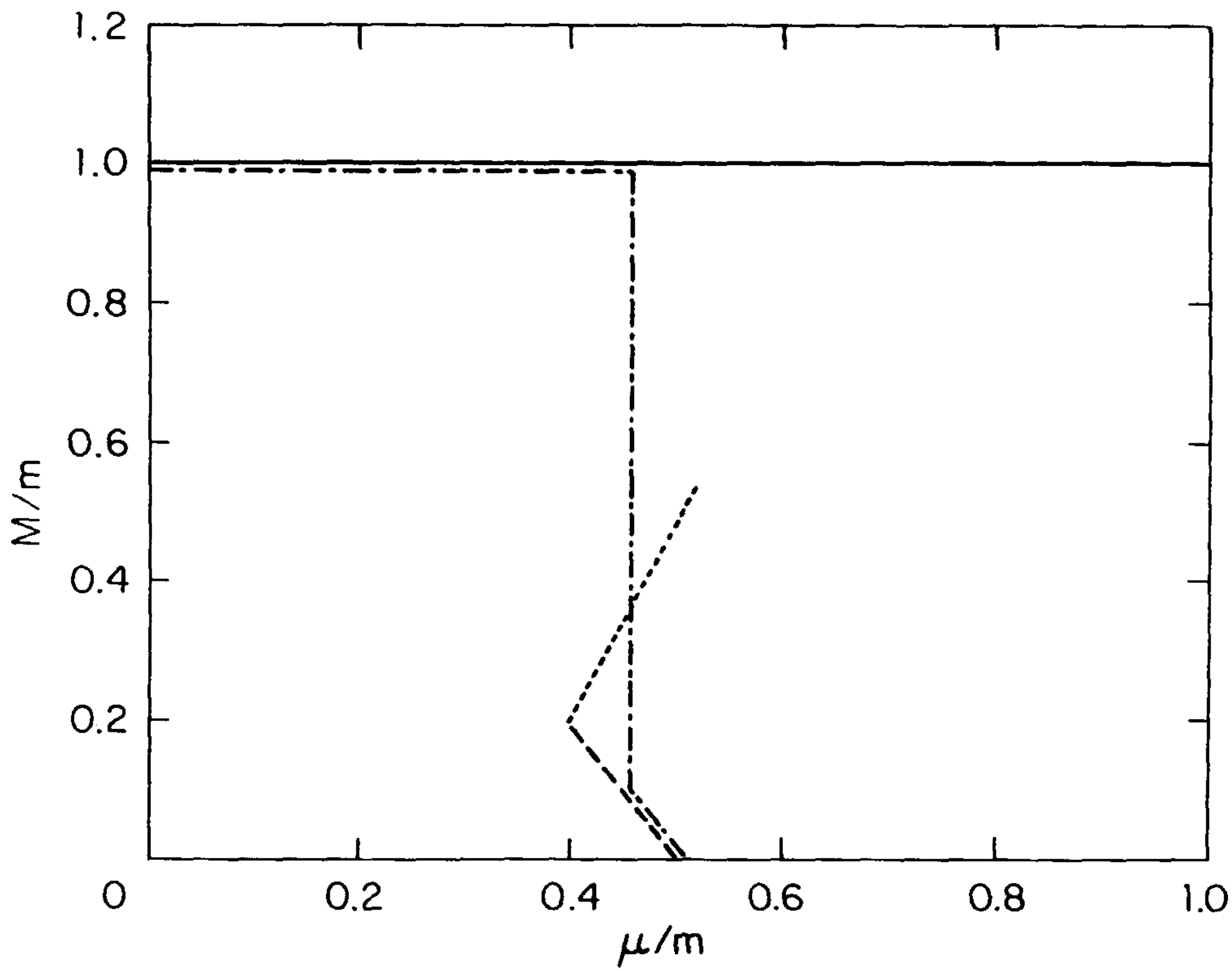


Figure 2a

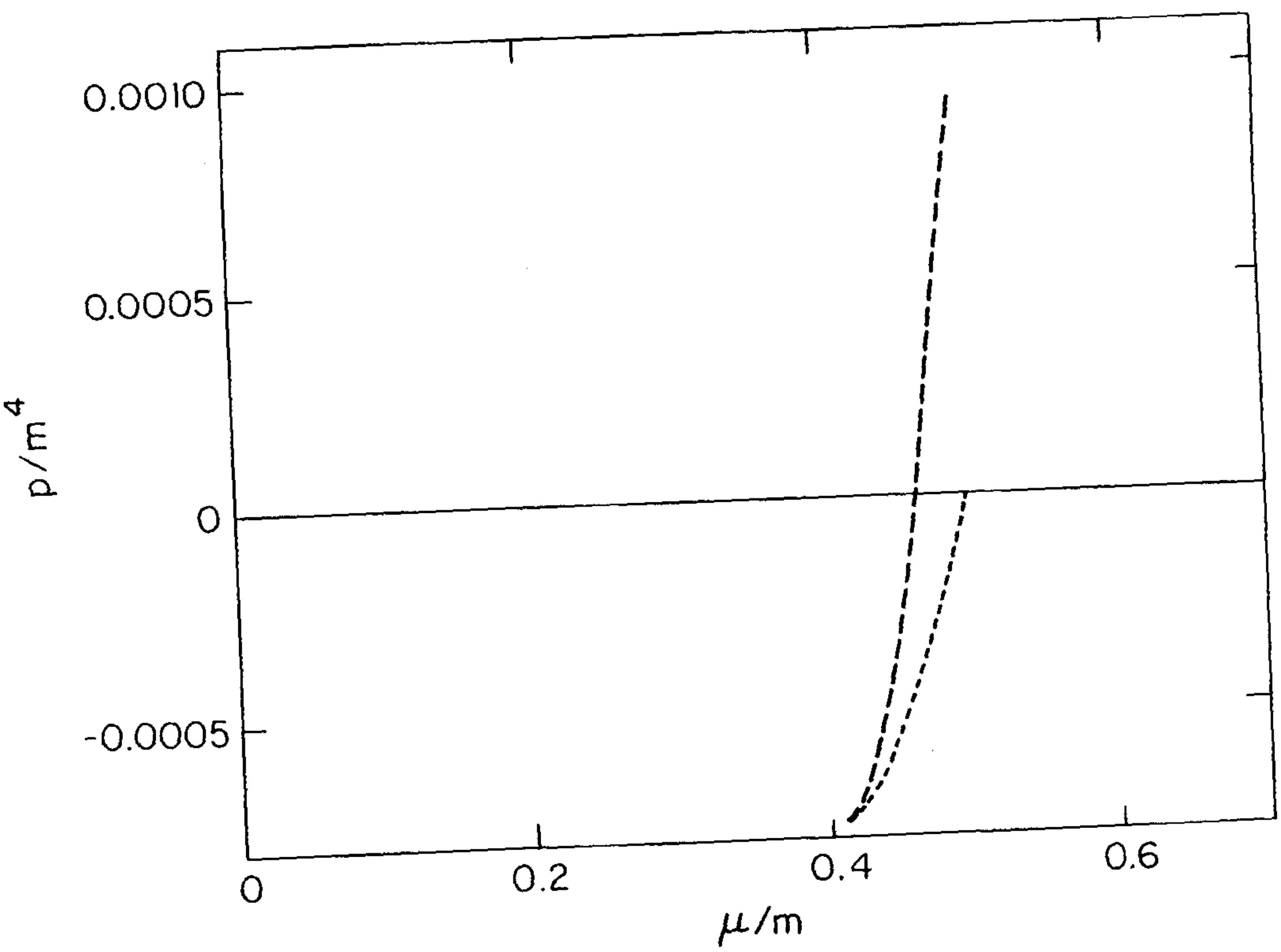


Figure 2b

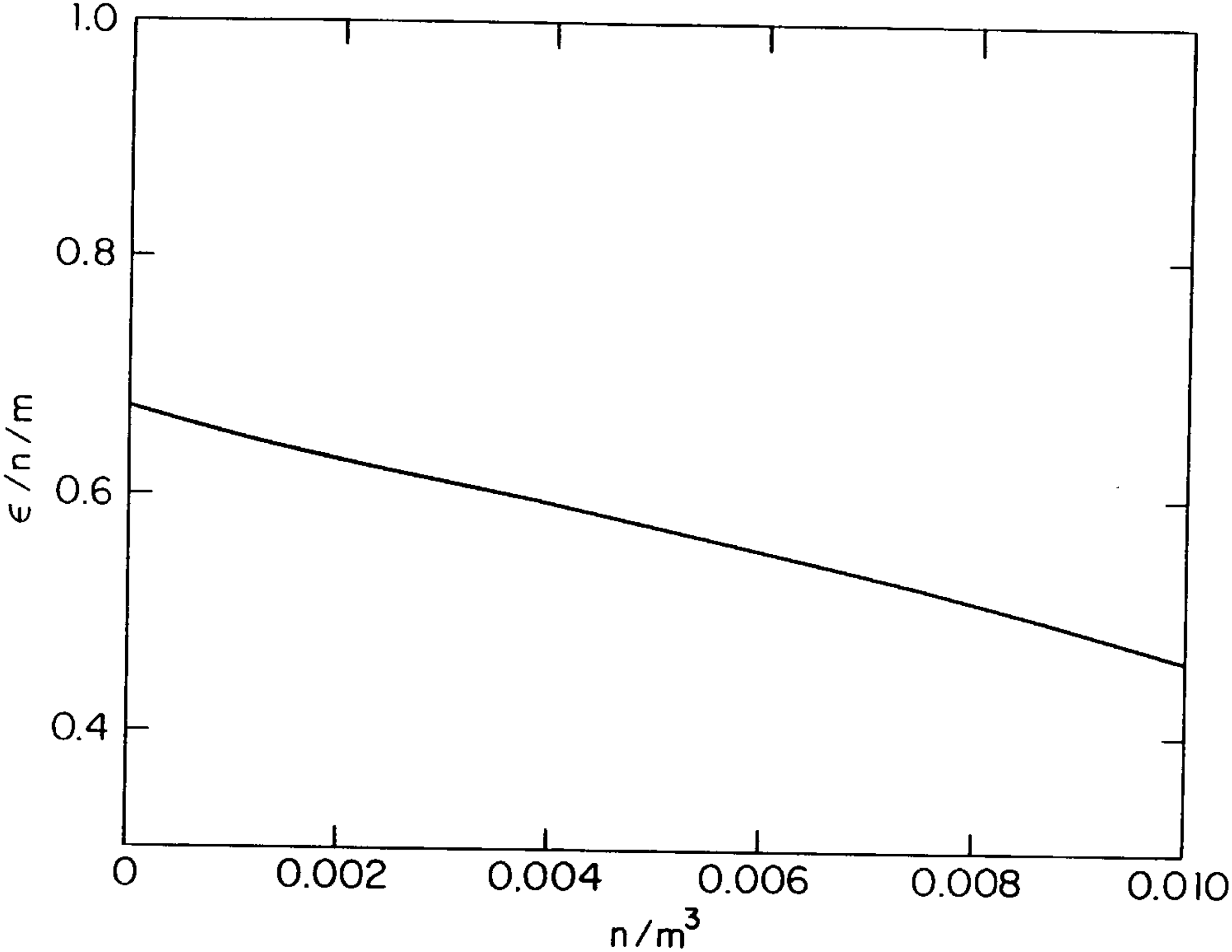


Figure 3a

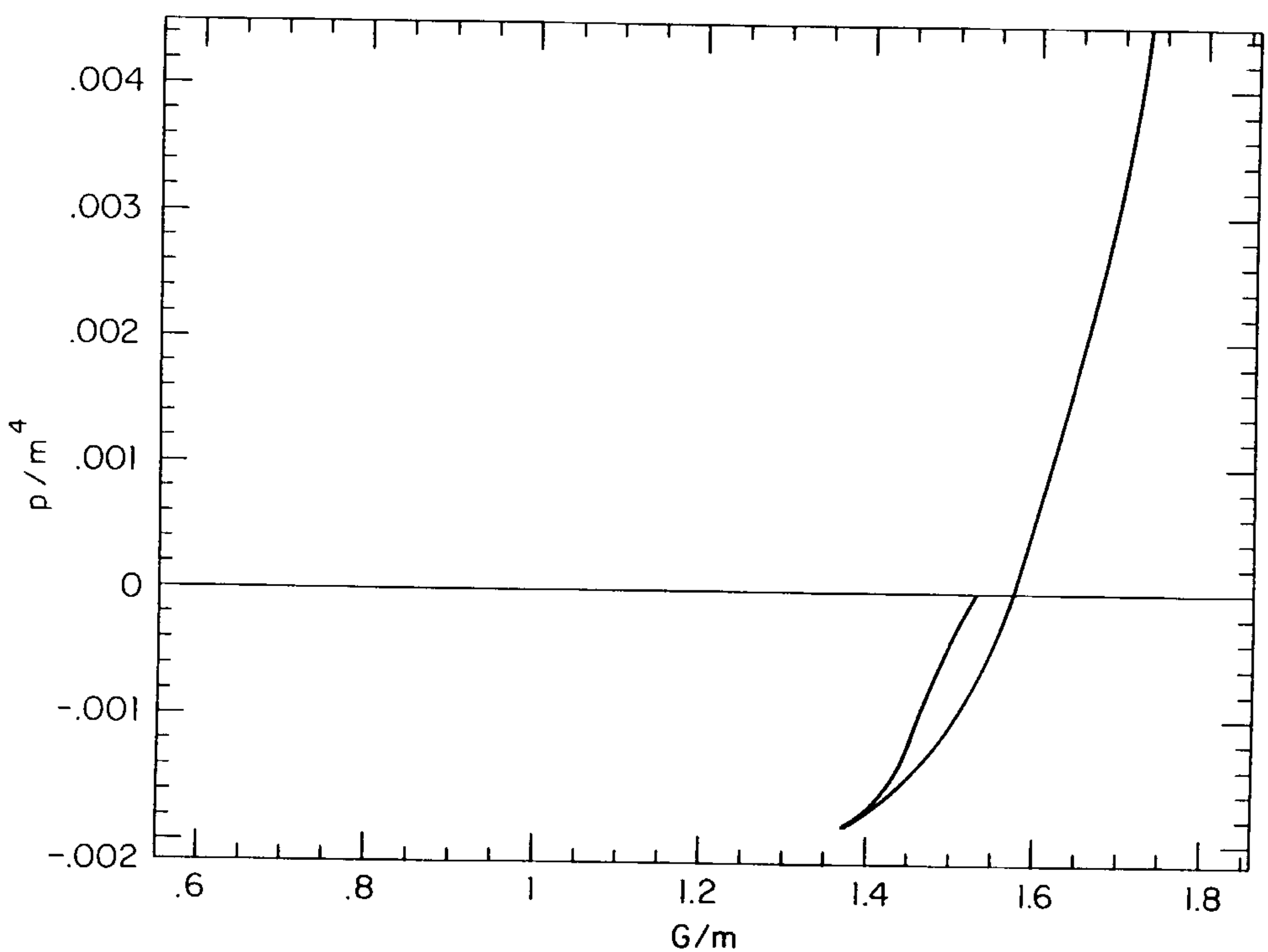


Figure 3b

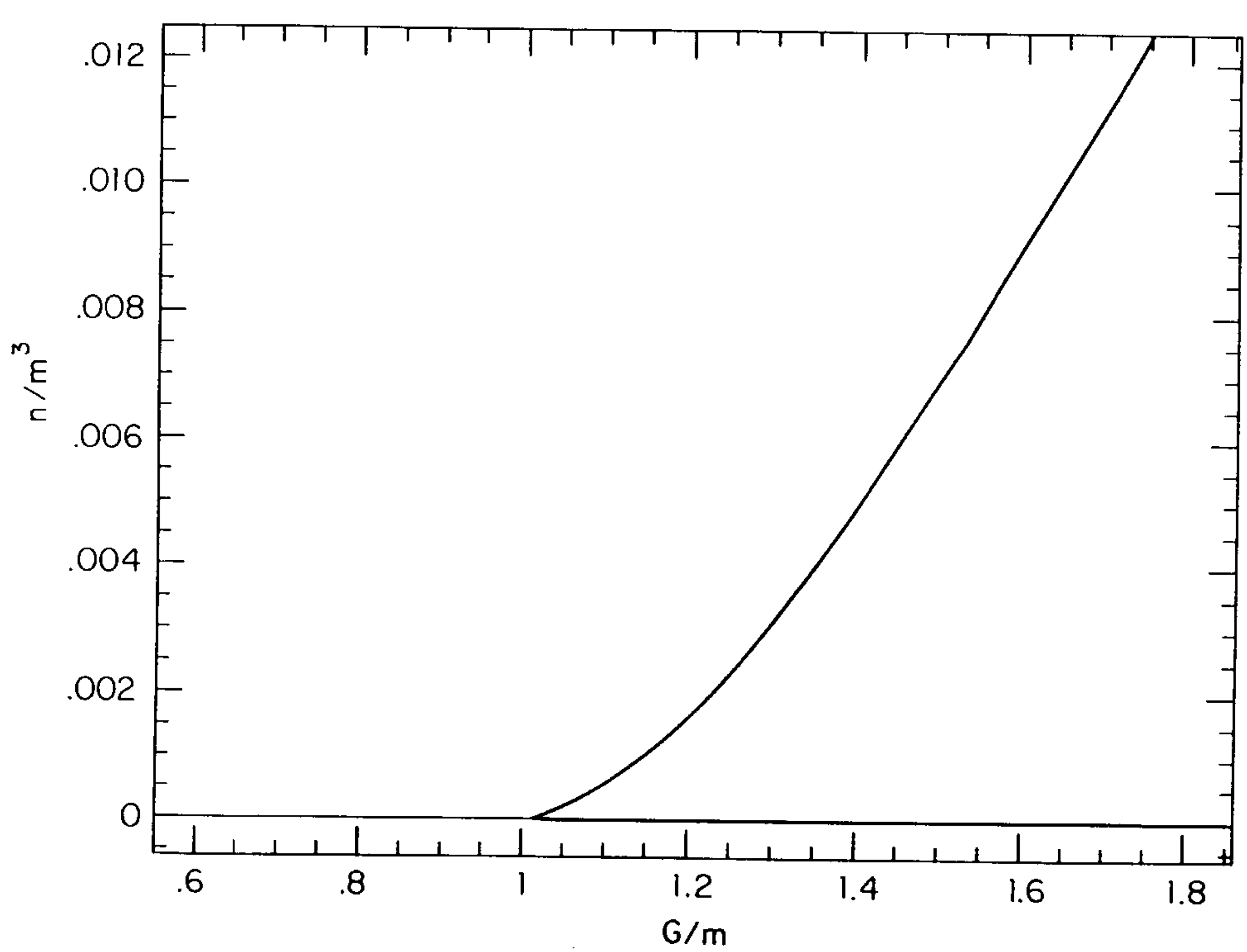
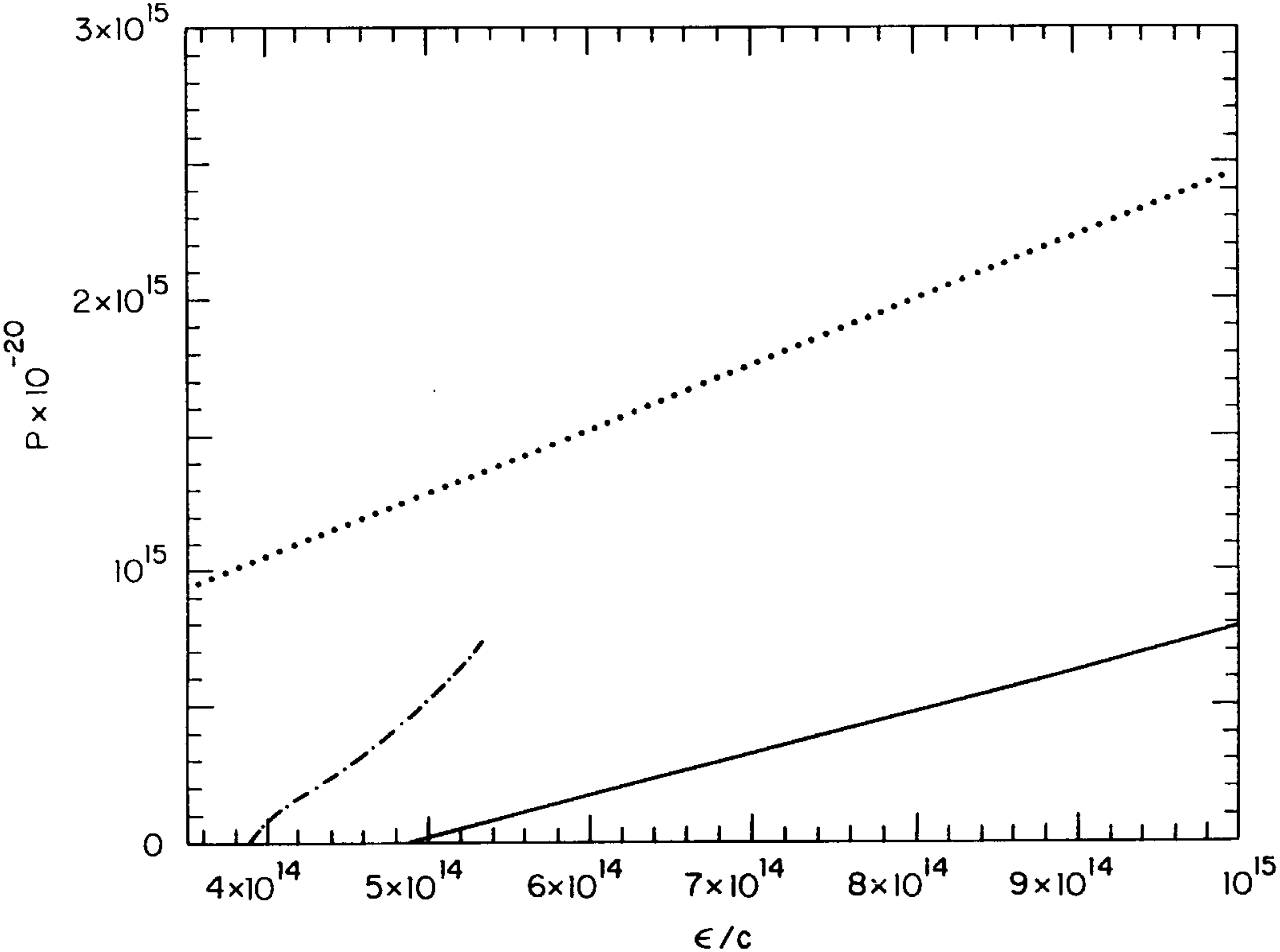


Figure 4



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2×10^15
10^15
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P × 10^-20
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5×10^14
6×10^14
7×10^14
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9×10^14
10^15
ε/c