Abstract

The steady-state temperature distribution and the stored energy in nuclear fuel elements are computed by analytical methods and used to rank, in the order of importance, the effects on stored energy from statistical uncertainties in modeling parameters, in boundary and in operating conditions. An integral technique is used to calculate the transient fuel temperature and to estimate the uncertainties in predicting the fuel thermal response and the peak clad temperature during a large-break loss of coolant accident.

The uncertainty analysis presented here is an important part of evaluating the applicability, the uncertainties and the scaling capabilities of computer codes for nuclear reactor safety analyses. The methods employed in this analysis merit general attention because of their simplicity.

It is shown that the blowdown peak is dominated by fuel stored energy alone or, equivalently, by linear heating rate. Gap conductance, peaking factors and fuel thermal conductivity are the three most important fuel modeling parameters affecting peak clad temperature uncertainty.

1. Introduction

Computer codes, such as TRAC-PF1 [1,2], play an important role in the assessment of nuclear reactor safety. The codes are used to predict many safety-related parameters during a variety of expected and postulated events, such as the peak clad temperature (PCT) during a large-break loss of coolant accident (LBLOCA). It is mandatory that reactor safety codes are quantitatively assessed concerning their applicability to a particular event, their ability to scale processes from test facility scales to full-scale power plants, and concerning their predictive uncertainties. The Nuclear Regulatory Commission has developed a new methodology for quantifying code uncertainties [3]. The analysis presented here is part of implementing this methodology, and of demonstrating its feasibility, for the TRAC-PF1/MOD1 computer code, as it is applied to LBLOCA analyses.

*Work performed under the auspices of the U.S. Nuclear Regulatory Commission.
The peak clad temperature (PCT) characterizes fuel integrity, i.e., the ability of the fuel cladding to retain fission products. Code uncertainties are quantified, therefore, on the basis of the code's ability to predict PCT accurately. During an LBLOCA, the clad temperature reaches, in general, two peaks; one during the earlier blowdown phase, the other one later, after the refill phase. Which one of the two peaks is higher depends on fuel design and coolant loop response characteristics. The clad temperature rises when the fuel element is suddenly surrounded by dry vapor. Then, more stored energy passes from the fuel pellets through the annular gas gap to the clad, then passes from the clad to the vapor, because the convective heat transfer is sharply reduced (dry-out or post-CHF*). The clad temperature reaches a peak when it decreases due to improved convective cooling after the arrival of liquid in the core or due to a decrease in pellet temperature. In either case, more heat is removed by convection from the clad than is supplied to the clad from the fuel pellet.

The PCT is the higher, the faster the clad temperature rises and the longer it rises. PCT uncertainty can therefore be decomposed into uncertainties of predicting the time rate of clad temperature rise and of predicting the time for liquid arrival in the core. The time rate of clad temperature change depends upon the energy initially stored in the fuel, primarily in the pellet, and upon the thermal response time of the fuel. The initially stored energy depends strongly on fuel parameters and on initial conditions. The thermal response of the fuel depends both on fuel parameters and on coolant conditions. The time for the reappearance of liquid in the core is dominated by break and pump flow characteristics for the first peak and by additional thermohydraulic loop phenomena for the second peak.

Thus, the energy stored initially in the fuel and the thermal response time of the fuel are key parameters affecting PCT. In fact, they will be shown to dominate PCT under the most plausible circumstances.

1.1 Purpose

The objective of the work presented here is to calculate the individual changes in initial fuel stored energy, associated with the known statistical uncertainties in input data specifications and with documented uncertainties of modeling fuel heat transfer in TRAC-PF1/MOD1. Specifically, the fuel stored energy variations are to be calculated for given 1o-level uncertainties of:

(i) initial fission power,
(ii) local power peaking factors,
(iii) fuel thermal conductivity,
(iv) cladding thermal conductivity,
(v) gap conductance,
(vi) fuel heat capacity,
(vii) cladding heat capacity,
(viii) radial fission power distribution in pellet, and
(ix) convective film coefficient.

It is also the objective of the work presented here to calculate the variations of the peak clad temperature for the early or blowdown peak as they

*CHF stands for critical heat flux.
are caused by the uncertainties in the above nine parameters. The analysis is
general in principle but evaluated for specific LBLOCA conditions in a
Westinghouse plant with 17x17 fuel arrays. The analysis is evaluated for
TRAC-PF1/M0D1 models and for reference conditions as computed by TRAC.

The effects from the above statistical uncertainties are to be ranked in
the order of their relative importance on peak clad temperature predictions.

1.2 Approach

The energy stored in fuel pellet and clad is computed by integrating the
steady-state temperature distributions in fuel pellet and clad. The temperature
distributions are obtained by closed-form integration of the steady-state
conduction equation, taking into account the temperature variation of thermal
conductivities and the radial variation of fission power in the pellet. This
is shown in Chapter 2 below.

An integral technique [4] is used to convert the partial differential
equations of transient heat conduction in fuel pellet and clad into two ordi-
nary differential equations for the volume-averaged temperatures in fuel pel-
let and clad [5]. The ordinary differential equations are numerically inte-
grated to obtain the transient clad temperature under prescribed conditions
for the coolant and for the fission power. The calculations are repeated,
while changing the previously listed parameters by their respective 1σ-level
uncertainties, to obtain the resultant changes in PCT. This is shown in
Chapter 3 below.

2. Steady-State Analysis for Fuel Stored Energy

2.1 Problem Description

In order to compute the energy E
stored in the fuel and in the clad, we
determine first the temperature distri-
butions \( T_f(r), 0 \leq r \leq R_1 \) and \( T_c(r), R_2 \leq r \leq R_w \) for the fuel and clad, respec-
tively. Consider the right circular
fuel pellet with the concentric clad,
forming an annular gap of width \( \delta \), as
shown in Figure 1.

We analyze steady radial conduction
in pellet and in clad, convection and
radiation between pellet and cladding,
and convection between clad and the
coolant at the known temperature \( T_w \).
The convective heat transfer coefficient
\( h_C \) is also known from hydraulics cal-
culations.

The radii are computed from known
specifications for cold conditions and
from correlations for thermal expansion
[6, p. 48d and p. 240]. Accordingly,
for the fuel pellet (UO$_2$), the relative radial expansion is

$$\frac{\Delta R_f}{R_{of}} = K_1 (T_f - T_0) + K_2 e^{-E_d/(k>T_f)}$$

(1)

where $\Delta R_f = R_f - R_{of}$, radial pellet expansion

$R_{of}$ = radius of cold pellet, at $T_0$

$K_1 = 1.0 \times 10^{-5} \text{ K}^{-1}$ (linear expansion coefficient [7])

$K_2 = 4.0 \times 10^{-2}$

$T_0 = 300 \text{ K}$

$<T_f>$ = area-averaged pellet temperature (see Ref. [7])

$E_d = 6.9 \times 10^{-20} \text{ J}$

$k = 1.38 \times 10^{-23} \text{ J/K}$

while for the cladding (zircaloy-2), the relative radial expansion is

$$\frac{\Delta R_c}{R_{oc}} = (6.72 \times 10^{-5})(<T_c> - T_0)$$

(2)

where $\Delta R_c$ = expansion of inner or outer clad radii

$R_{oc}$ = corresponding cold clad radii

$<T_c>$ = area-averaged clad temperature.

The area averages in Eqs. (1) and (2) are defined as

$$<T_f> = \frac{2}{R_{fo}} \int_0^{R_{fo}} T_f r dr$$

and

$$<T_c> = \frac{2}{R_w - R_{ci}} \int_{R_{ci}}^{R_w} T_c r dr$$

(3)

where

$R_{fo}$ = outer pellet radius

$R_{ci}$ = inner clad radius

$R_w$ = outer clad radius.

The equations in Reference [6] contain an additional constant each, from which the reference temperatures $T_0$ in Eqs. (1) and (2) above have been inferred.

The steady-state temperature distributions are derived from the nonlinear conduction equation with temperature-dependent thermal conductivities. The gap conductance for the annular gap between pellet and clad is computed with temperature-dependent thermal conductivity for the helium fill gas, and with temperature-dependent radiative surface properties for pellet and clad.

Calculations of the stored energy take into account the variation of specific heat with temperature.

2.2 Thermal Conductivities and Radiative Emissivities

The temperature dependence of thermal conductivity $k_f$ for UO$_2$ pellets is given in Reference [6, p. 23]. After the correction of typographical errors
in this reference, one finds for the range of temperatures encountered during normal operating conditions, that

\[
\frac{D \cdot (1 \text{ Wm}^{-1} \text{ K})}{(1-D)} \cdot \frac{c_v(T_f) \cdot (1 \text{ kg m}^{-1} \text{s}^{-1})}{(0.339 + 0.06867 \text{K}^{-1} \cdot T_f) \cdot (1 + 3 \Delta R_f/R_{of})} + 5.2997 \times 10^{-3} \text{ Wm}^{-1} \text{K}^{-2} \cdot T_f \cdot e^{-K_5/T_f} \left[ 1 + (2 + 0.169 \frac{K_5}{T_f}) \right] ,
\]

where \( D \) = fraction of theoretical density
\( T_f \) = fuel pellet temperature
\( c_v \) = fuel specific heat at constant volume for pellet
\( \Delta R_f/R_{of} \) = thermal strain, Eq. (1) evaluated with \( T_f \)
\( K_5 \) = 13,358 K.

The specific heat \( c_v \) is found in Reference [6, p. 23]. For UO\(_2\) pellets without plutonium, it is

\[
c_v = 296.7 \frac{J}{(kgK)} x^2 e^x/(e^x-1)^2 ,
\]

where \( x = 535.285 \text{ K}/T_f \)
\( T_f \) = fuel pellet temperature

Equation (4) is used in conjunction with Eqs. (1) and (5) to compute the fuel pellet thermal conductivity \((k_f)_{ref}\) for a reference temperature \((T_f)_{ref}\). For closed-form integration, the variation of thermal conductivity with temperature is then approximated by fitting results from Eq. (4), calculated in the temperature range 547K \( \leq T \leq 1,330K\) to an expression of this form:

\[
k_f(T_f) = (k_f)_{ref} \left( \frac{T_f}{(T_f)_{ref}} \right)^{b_f} ,
\]

where \((k_f)_{ref}\) = reference thermal conductivity, computed from Eq. (4)
\( b_f = 0.762 \ 303 \)

The maximum error of the curve fit in the above temperature range is 0.018 Wm\(^{-1}\)K\(^{-1}\) or 0.6%; the coefficient of determination is \( r^2 = 0.999 \ 86 \). The error of Eq. (6) is even less because \((k_f)_{ref}\) is exact, and the range of temperatures in the pellet is smaller than the range of the curve fit.
The temperature dependence of thermal conductivity $k_c$ for zircaloy-2 cladding is given in Reference [6, p. 217] as

$$k_c = [7.51 + (0.0209 \text{ K}^{-1})T - (1.45 \times 10^{-5} \text{ K}^{-2})T^2 + (7.67 \times 10^{-9} \text{ K}^{-3})T^3] \text{Wm}^{-1} \text{K}^{-1}. \quad (7)$$

Equation (7) is used to compute the thermal conductivity $(k_c)_{\text{ref}}$ of the clad at a reference temperature while the variation of thermal conductivity within the clad is taken to have the more convenient form

$$k_c = (k_c)_{\text{ref}} e^{b_c T}, \quad (8)$$

where $b_c = -7.71233 \times 10^{-4} \text{ K}^{-1}$.

Equation (8) is suitable for closed form integration. It introduces an error of less than $0.3\%$ and fits the results of Eq. (7) with the coefficient of determination, $r_d^2 = 0.9961$.

The thermal conductivity $k_g$ of the fill gas in the gas gap between fuel pellet and cladding is needed for calculating the gap conductance. The calculations are performed here for helium gas. Reference [6, p. 485] gives for pure helium

$$k_g(T) = A \left( \frac{T}{1\text{K}} \right)^B, \quad (9)$$

where

$$T = \frac{T_{fo} + T_{ci}}{2}, \text{ average gas temperature}$$

$$T_{fo} = \text{outer fuel pellet temperature}$$

$$T_{ci} = \text{inner clad temperature}$$

$$A = 2.639 \times 10^{-3} \text{ Wm}^{-1} \text{K}^{-1}$$

$$B = 0.7085.$$ 

Radiative emissivities are also needed to compute the gap conductance. The emissivity for UO$_2$ pellets is found in Ref. [6, p. 48] to be

$$\epsilon_f = c_1 + c_2 T_{fo}, \quad (10)$$

where

$$c_1 = 0.7856$$

$$c_2 = 1.5263 \times 10^{-5} \text{ K}^{-1},$$

while the emissivity of zircaloy-2 without oxide layer is, from Ref. [6, p. 230], a constant:

$$\epsilon_c = 0.9151. \quad (11)$$
This completes the definition of all temperature-dependent properties required for computing the steady-state temperature distribution in fuel elements.

2.3 Fission Power Variation

The fission power varies radially within the pellet due to neutron flux depression. The variation increases with burnup. With the objectives in mind of assessing TRAC code uncertainties, the fuel conditions presented here are those used in the reference TRAC calculation.

The fission power varies radially in accordance with [5, Eq. 114]

\[
q_f''(\xi) = \frac{n + 2}{n + 2(1 + m)} <q_f''>_f [1 + m\xi^n]
\] (12)

where \( q_f'' \) = local volumetric fission power density
\(<q_f''>_f \) = area-averaged fission power density
(see Eq. 3 for definition of \(< >_f \) operator)
\( \xi \) = \( r/R_1 \), normalized radius
\( R_1 \) = outer fuel pellet radius
\( r \) = radial coordinate
\( m, n \) = constants.

The form of Eq. (12) constitutes also the best type for fitting Westinghouse data as used in TRAC. The first coefficient on the right-hand side of Eq. (12) is needed to assure that averaging of Eq. (12) reduces to an identity. The coefficients \( m \) and \( n \) vary in the ranges of \( 0 \leq m \leq 3 \) and \( 2 \leq n \leq 4 \). Specified Westinghouse data require \( m = 0.403 \) and \( n = 3.917 \). This means that the fission power density is greater around the periphery of the pellet and that the temperature is more uniform than for constant fission power density.

2.4 Governing Equations for Steady Conduction

The combination of energy balance and Fourier's conduction law gives

\[
\nabla \cdot (k \nabla T) + q'''' = 0 ,
\] (13)

where all symbols have been defined previously. Since \( k \) depends on temperature, one introduces a new temperature \( \Theta \) via the Kirchhoff transformation [8]

\[
\Theta = \frac{1}{k_{ref}} \int_{T_{ref}}^{T} k(T) dT ,
\] (14)

where \( k_{ref} = k(T_{ref}) \) and \( T_{ref} \) is a known reference temperature. After substituting Eq. (14) into Eq. (13), one obtains this equation

\[
k_{ref} \nabla^2 \Theta + q''''(r) = 0 \] (15)
which is linear and easy to integrate. Substitution of Eq. (6) into Eq. (14) gives for the fuel pellet temperature

\[ T_f = (T_f)_{\text{ref}} \left[ 1 + \frac{b + 1}{(T_f)_{\text{ref}} \Theta_f} \right]^{b+1}, \]  

where \( \Theta_f \) is the transformed fuel pellet temperature. Similarly, substituting Eq. (8) into Eq. (14) and solving for the cladding temperature, \( T_c \), yields

\[ T_c - (T_c)_{\text{ref}} = \frac{1}{b_c} \ln(1 + b_c \Theta_c), \]

where \( \Theta_c \) is the transformed cladding temperature.

Equation (15) is written for radial conductance in the fuel pellet to give, in terms of the normalized radius \( \xi_f = r/R_1 \),

\[ (k_f)_{\text{ref}} \frac{1}{\xi_f} \frac{d}{d \xi_f} \left( \xi_f \frac{d \Theta_f}{d \xi_f} \right) + R_1 q_f^{\prime\prime\prime}(\xi_f) = 0, \]

where \( R_1 \) is the outer radius of the pellet. Equation (12) defines \( q_f^{\prime\prime\prime}(\xi_f) \). The appropriate boundary conditions for the fuel pellet are:

at \( \xi_f = 0 \) \quad \frac{d \Theta_f}{d \xi_f} = 0

at \( \xi_f = 1 \) \quad \Theta_f = 0.

From the last condition follows, by virtue of Eq. (14), that

\[ (T_f)_{\text{ref}} = T_f(R_1) = T_{fo} \quad \text{and} \]

\[ (k_f)_{\text{ref}} = k_f(T_{fo}) \]

Equation (15) reduces for the cladding to

\[ \frac{1}{r} \frac{d}{dr} \left( r \frac{d \Theta_c}{d r} \right) = 0, \]

since there is no heat absorption in the cladding. Equation (14) and the boundary conditions for Eq. (22) require that, at the outer clad radius, i.e., at \( r = R_w \):
Equation (23) implies, similarly to the last of Eqs. (19), that

\begin{align*}
(T_c)_{ref} &= T_c(R_w) = T_w \quad \text{and} \\
(k_c)_{ref} &= k_c(T_w),
\end{align*}

whence, Eq. (24) reduces to

\[
\frac{d\Theta_c}{dr} = -R_1^2 \frac{q_f'''}{f'''}f'(2R_w).
\]

Equations (18) and (19) and Eqs. (22), (23) and (27) define, via Eqs. (16) and (17), the temperature distributions in fuel pellet and cladding, provided the outer temperatures \(T_{fo}\) in Eq. (20) and \(T_w\) in Eq. (25) of pellet and clad are known. These two temperatures are determined from the conditions of heat flux continuity at the outer pellet surface. First, at \(r = R_1\):

\[
T_{fo} - T_{ci} = R_1^2 \frac{q_f'''}{f'''}f/\left[\pi(R_1 + R_2)h_{gp}\right],
\]

where

\begin{itemize}
  \item \(T_{ci}\) = inner clad surface temperature
  \item \(R_1\) = outer fuel pellet radius
  \item \(R_2\) = inner clad radius and
  \item \(h_{gp} = h_{gas} + h_{rad}\)
\end{itemize}

The gap conductance \(h_{gp}\) has a contribution from the conductance \(h_{gas}\) through the gas and a radiative contribution \(h_{rad}\). For the gas conductance

\[
h_{gas} = \frac{k_g(T)}{t},
\]

where \(k_g(T)\) was defined earlier in Eq. (9), and \(t\) is the effective gap width, taken here to be uniform around the circumference. According to Siefken et al. [9], the effective gap width is
\[ t = \delta + 3.2(\beta_f + \beta_c) + g_f + g_c \]  

where  
\[ \delta = R_2 - R_1 \]  
physical gap width  
\[ \beta = \text{surface roughness} \]  
\[ g = \text{temperature jump distance} \]  

and subscripts \( f \) and \( c \) designate fuel and clad, respectively. The gap width \( \delta \) is computed from radial thermal expansions according to Eqs. (1) and (2). Surface roughness \( \beta \) is known from measurements. The temperature jump distance \( g \) can be computed from temperature accommodation coefficients [9, p. 18], but it is normally derived from experiments to obtain measured fuel pellet temperatures. For the present analysis, \( g_f + g_c \) are taken to be the same as the specified TRAC values [1, p. 69] (cf. Sec. 2.6).

The radiative conductance is

\[ h_{rad} = \sigma \left( \frac{(T_f^2 + T_{ci})^2(T_f^0 + T_{ci})}{\frac{1}{\varepsilon_f} + \frac{R_1}{R_2} (\frac{1}{\varepsilon_c} - 1)} \right), \]  

where  
\[ \sigma = 5.6697 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \]  
\[ T_f^0 = \text{pellet outer surface temperature} \]  
\[ T_{ci} = \text{clad inner surface temperature} \]  
\[ R_1 = \text{pellet radius} \]  
\[ R_2 = \text{inner clad radius} \]  

and \( \varepsilon_f \) and \( \varepsilon_f \) are specified by Eqs. (10) and (11). Thus, Eqs. (28) through (32) determine the pellet outer temperature \( T_f^0 \).

Finally, the outer clad temperature \( T_w \) is given in terms of known coolant temperature \( T_m \) and convective film coefficient \( h_c \), averaged over the circumference of the clad. The energy balance for pellet and clad yields at \( r = R_w \):  

\[ 2R_w h_c (T_w - T_m) = R_f^2 q_{f'''} \]  

where all symbols have been previously defined. Equations (18), (27) and (33) suggest the simplification

\[ R_f^2 q_{f'''} = q_f'/\pi \]  

where \( q_f' \) = linear rate of fission power.
The linear heating rate $q'_r$ is independent of $R$, and remains, in contrast to $<q''_f>$, fixed during radial expansion of the pellet.

Equations (13) through (33) define the temperature distribution in the fuel element. Their solutions are presented below.

### 2.5 Steady-State Temperatures

#### 2.5.1 The Cladding Temperature $T_c(r)$

The cladding temperature $T_c(r)$ is found by computing first the outer wall temperature from Eq. (33). With Eq. (34), one finds

$$T_w = T_m + \frac{q'_f}{2\pi R w h_c}. \quad (35)$$

With $T_w$ known, one can now compute the thermal conductivity $k_c(T_w)$ for Eq. (26), integrate Eq. (22), subject to Eqs. (23) and (27), to obtain first the solution for $\theta_c(r)$ and then from Eq. (17)

$$T_c(r) - T_w = \frac{1}{b_c} \ln \left[ 1 + \frac{b_c q'_f}{2\pi k_c(T_w)} \ln \frac{R_w}{r} \right], \quad (36)$$

where $b_c$ is given below Eq. (8). Equation (36) must be evaluated for $r = R_2$ to give the inner clad temperature

$$T_{ci} = T_w + \frac{1}{b_c} \ln \left[ 1 + \frac{b_c q'_f}{2\pi k_c(T_w)} \ln \frac{R_w}{R_2} \right]. \quad (37)$$

#### 2.5.2 The Temperature Difference Across the Gap

The temperature difference across the gap is computed by combining Eqs. (28) through (32) with Eqs. (9) and (10) to get

$$Y(n) = c_1(n-1) \left( \frac{n+1}{2} \right)^B + \frac{c_2(n^k - 1)}{\phi_1 + (c_1 + c_3 n)}, \quad (38)$$

where

- $n = T_{fo}/T_{ci}$
- $c_1 = k_g(T_{ci})/t$
- $c_2 = \sigma T_{ci}$
- $c_3 = c_2 T_{ci}$
- $\phi_0 = q'_f/(2\pi R_1 T_{ci})$
- $\phi_1 = (R_1/R_2)(1/\varepsilon - 1)$

and the constants $B$, $c_1$ and $c_2$ are defined below Eqs. (9) and (10). All other symbols have been previously defined. Equation (38) is readily solved by Newton-Raphson iteration:
\[ \eta^{n+1} = \eta^n - \frac{Y(\eta^n) / Y'(\eta^n)}{\eta^n} , \tag{39} \]

where \( Y'(\eta^n) \) is the derivative \( dY/d\eta \), evaluated with the \( n \)-th iterate \( \eta^n \). The iteration is started with \( \eta^0 = 1,1 \). When two successive iterates differ by less than \( 10^{-6} \), then \( \eta \) for \( Y(\eta) = 0 \) is found and

\[ T_{fo} = \eta T_{ci} \tag{40} \]
gives the outer pellet temperature.

2.5.3 The Fuel Pellet Temperature is obtained by substituting Eq. (12) into Eq. (18) and by integrating Eq. (18), subject to Eqs. (19). The result is \( \Theta_f(\xi_f) \), which must be substituted into Eq. (16) to yield the fuel pellet temperature

\[ T_f(\xi_f) = T_{fo} \left[ 1 + \frac{b_f + 1}{2T_{fo}} \left[ R_1^2(q_{f''})_{cl} \right] \left[ 1 + \frac{4m}{(n+2)^2} \left( -\xi_f^2 (1 + \frac{4m}{(n+2)^2} \xi_f^n) \right)^{1/(b_f+1)} \right] \right] \tag{41} \]

Here, \( (q_{f''})_{cl} \) is the fission power density at the centerline and given from Eq. (12) as

\[ R_1^2(q_{f''})_{cl} = (n+2)q_f^2 / \{\pi[n+2(1+m)]\} \tag{42} \]

All other symbols have been defined in conjunction with Eqs. (6) and (12). The thermal conductivity \( k_f(T_{fo}) \) is evaluated from Eqs. (4) and (5).

2.6 Comparison of Analysis with TRAC Results

Even though the focus of this paper is on fuel stored energy, the results obtained so far provide an important assessment on fuel element simulations in TRAC. A conclusive assessment of TRAC modeling requires the output listing of important intermediate results which are not available from TRAC. Such results include radial expansions, gap width, thermophysical properties and the precise axial locations associated with listed temperatures and heat transfer coefficients. The comparison between TRAC results and this analysis is therefore limited to the comparisons of thermal conductivity and of temperature distribution.

The TRAC document \[1, p. 501\] claims that the fuel thermal conductivity is computed in TRAC in accordance with Reference \[6, p. 23\], but it is not. Evaluating the expressions in the TRAC document \[1, pp. 501 and 502\] for a fuel with 95% of theoretical density, one obtains the comparison with the MATPRO formulation \[6, p. 23\] as shown in Table 1 below. Also shown are the
results of an extremely simple correlation given by Malang [10], which is not only better than the TRAC formulation, but also produces results which are closer to the experimental data than the results from MATPRO [11, p. 3-33]. Table 1 shows that the values calculated by TRAC are too low and should result in higher fuel temperatures than those obtained with the MATPRO values. It will be shown below that this is not the case.

Table 1 Comparison of TRAC and MATPRO Results for Thermal Conductivity

<table>
<thead>
<tr>
<th>Temperature K</th>
<th>Thermal Conductivity W/(mK)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>MATPRO</td>
</tr>
<tr>
<td>300</td>
<td>8.28</td>
</tr>
<tr>
<td>500</td>
<td>6.09</td>
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<tr>
<td>750</td>
<td>4.47</td>
</tr>
<tr>
<td>1,000</td>
<td>3.58</td>
</tr>
<tr>
<td>1,250</td>
<td>3.03</td>
</tr>
<tr>
<td>1,500</td>
<td>2.68</td>
</tr>
</tbody>
</table>

Equations (35), (37), (40) and (41) were used to calculate the fuel temperature distribution in that rod and at that location, where TRAC computed the largest clad temperature at full power and under steady-state conditions in a Westinghouse PWR with 17x17 fuel arrays. It was the intent to have, for the reference calculation, the same conditions as in the TRAC-PF1/MOD1 calculation. These conditions are as follows:

linear heating rate, nominal 18.303 kWm⁻¹
local radial peaking factor 1.111
local axial peaking factor 1.050
hot rod peaking factor 1.215
linear heating rate, effective \( q'_f \) = 25.942 kWm⁻²K⁻¹
convective heat transfer coefficient \( h_c \) = 40.05 kWm⁻²K⁻¹
coolant temperature \( T_w \) = 599.44 K
pellet radius, cold \( R_{of} \) = 4.0960 mm
gap width, cold \( R_i \) = 0.0380 mm
outer clad radius, cold \( R_w \) = 4.7500 mm
pellet radius, @ full power \( R_1 \) = 4.11994 mm
inner clad radius, Eq. (2) \( R_2 \) = 4.14058 mm
outer clad radius, Eq. (2) \( R_w \) = 4.75756 mm
clad thickness (hot) \( h_g \) = 0.616981 mm

Since the convective heat transfer in TRAC is computed with the cold-value of outer clad radius, the heat transfer coefficient \( h_c \) had to be adjusted so that \( (h_c)_{TRAC}(R_w)_{COLD} = h_c \cdot R_w \). The effective gap width \( t \) in Eqs. (31) and (38) was computed to produce the same gap conductance as produced in the TRAC code: \( h_g = 10.960 \text{ kWm}^{-2}\text{K}^{-1} \). This was achieved by using first from TRAC [1, p. 69] for Eq. (31)
by computing the cladding radii $R_w$ and $R_2$ from Eq. (2), and then by computing
the gap width $\delta$ so as to produce the TRAC-computed gap conductance from Eq.
(38). The result is

$$\delta = 0.020639 \text{ mm}.$$ 

After substituting this gap width from $R_2$, the radius $R_1$ as shown above was
obtained. Had Eq. (1) been used to compute $R_1$, then the hot-condition gap
width would have been 13.801 $\mu\text{m}$ instead of 20.639 $\mu\text{m}$. *

All the expressions for thermophysical properties and Eqs. (35), (36),
(38) and (39), (40) and (41) were executed on an HP-41/CX programmable pocket
calculator. Table 2 below shows the comparison between the results from
closed-form integrations and from TRAC.

Table 2. Comparison Between Closed-Form Integration and TRAC-PFI Solutions

<table>
<thead>
<tr>
<th>Location</th>
<th>Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Closed-Form Int.</td>
</tr>
<tr>
<td>Fluid ($T_\omega$)</td>
<td>599.440</td>
</tr>
<tr>
<td>Outer Clad Surf. ($T_w$)</td>
<td>621.109</td>
</tr>
<tr>
<td>Inner Clad Surf. ($T_{ci}$)</td>
<td>654.932</td>
</tr>
<tr>
<td>Pellet Surface ($T_{fo}$)</td>
<td>746.368</td>
</tr>
<tr>
<td>In Fuel Pellet at $r/R_1=\sqrt{3}/2$</td>
<td>868.137</td>
</tr>
<tr>
<td>$\sqrt{2}/2$</td>
<td>999.148</td>
</tr>
<tr>
<td>1/2</td>
<td>1,140.99</td>
</tr>
<tr>
<td>0</td>
<td>1,296.16</td>
</tr>
</tbody>
</table>

The comparison reveals that the TRAC-computed centerline temperature is
too low, even though the surface temperatures have been made to agree fairly
well and even though the thermal conductivity $k_f$ of the fuel is too low in
TRAC as discussed above. Since the centerline temperature increases by ap-
proximately 16.4 K for a reduction in fuel thermal conductivity by 0.1 W/(mK),

*The gap width in TRAC was, in fact, increased to obtain fuel stored energy
as provided by Westinghouse.
and since, according to Table 1 above, the mean difference between MATPRO and TRAC values of thermal conductivity is -0.46 W/mK, it appears as if TRAC had a total computational error of -(4.6 * 16.4 + 5.2) K = -0.46 K. Notice that if TRAC accounts for fuel cracking or other phenomena which reduce the thermal conductivity but are not accounted for in the closed-form integration, then the deficit of the centerline temperature can only be larger.

2.7 Stored Energy

The thermal energy stored in fuel pellet and cladding is computed with respect to the energy at the coolant temperature $T_\infty$. It is the sum of energies stored in fuel pellet and in cladding, namely

$$E = E_f + E_c$$

where

$$E_f = 2\pi \int_0^{R_1} \rho_f \frac{T_f(r)}{T_\infty} [c_p(T)]_f \, dT \, dr$$

$$E_c = 2\pi \int_{R_2}^{R} \rho_c \frac{T_c(r)}{T_\infty} [c_p(T)]_c \, dT \, dr$$

and

$$\rho_f = 10,011.5 \text{ kg/m}^3, \text{ fuel density [10]*}$$

$$\rho_c = 6,487/5 \text{ kg/m}^3, \text{ clad density [10]}$$

$c_p$ = specific heat at const. pressure.

Subscripts $f$ and $c$ designate fuel and clad, respectively.

The specific heat for UO$_2$ pellets is given in Reference [6, p. 9]. For the oxygen to metal ratio of 2 for UO$_2$, it is

$$[c_p(T)]_f = c_v + K_6 T + \frac{K_7 E_D}{RT} e^{-\frac{E_D}{RT}},$$

where

$$K_6 = 0.0243 \text{ J/(kgK}^2)$$

$$K_7 = 8.745 \times 10^7 \text{ J/kg}$$

*$\rho_f$ and $\rho_c$ are computed with $<T_f>_f$ and $<T_c>_c$, respectively and kept constant.
and \( c_v \) is given in Eq. (5). Figure A-1.1 on p. 14 of Ref. [6] shows that \((c_p)_f\) increases linearly between \( T = 650K \) and \( T = 1300 K \). Thus, Eq. (46) is used to compute a reference value at \( T = T_{f0} \), and the \([c_p(T)]_f\) variation within the pellet, as needed in Eq. (44), is approximated by

\[
[c_p(T)]_f = [c_p(T_{f0})]_f [1 + K_8 (T_f - T_{f0})]
\]

where \( K_8 = 1.5299 K^{-1} \).

Equation (47) introduces an error of less than 0.6% in the calculation of the energy stored in the pellet, but it facilitates greatly the evaluation of Eq. (44).

The specific heat for zircaloy in its alpha phase \((300 \leq T_c \leq 1090 K)\) is given in Table B-1.1 of Reference [6, p. 206]. The data given in that table are represented best by

\[
[c_p(T)]_f \approx K_9 [T_c/(1K)]^{10},
\]

where \( K_9 = 80.20769 J/(kg K) \)

\( K_{10} = 0.220247 \).

Equation (48) fits the tabulated data with the coefficient of determination \( r^2 = 0.9983 \) and a maximum error of 0.6%, having a maximum impact on \( E \) in Eq. (43) of 0.01%.

Equations (41) and (36) were substituted in Eqs. (47) and (48). The respective results were then substituted into Eqs. (44) and (45). The integrals of Eqs. (44) and (45) were numerically evaluated on an HP-41CX programmable pocket calculator, using its built-in quadrature algorithm. This algorithm was executed to guarantee six significant digits. With the conditions listed in Section 2.6 above, the following results were obtained:

\[
E_f = 66.5903 kJ/m,
\]

\[
E_c = 1.4005 kJ/m\text{ and}
\]

\[
E = 67.9908 kJ/m.
\]

Linear, exponential and power-law regressions were tried throughout this work. The regression with the largest \( r^2 \) value was selected.
Figure 2 shows the dependence of fuel stored energy $E$ on normalized linear heating rate or fission power, the fission power being normalized by the value listed in Section 2.6 above.

It was indicated in Section 2.6 that the TRAC code appears to underpredict the fuel centerline temperature by 80K. With Eq. (43), one can determine the corresponding deficiency in fuel stored energy. TRAC appears to underpredict fuel stored energy by 6.05 kJ/m or by 8.9% of its reference value relative to the coolant temperature ($E = 0$ for $<T_f>_f = <T_c>_C = T_m$).

2.8 Uncertainties in Predicting Fuel Stored Energy

Expressions for thermophysical properties and transfer laws are derived from experimental data with known random errors. Secondly, reactor and fuel conditions are specified, but with statistical uncertainty. Thirdly, the mathematical models in TRAC-PF1/MOD1 were found to differ (cf. Table 1) from the documented models. The TRAC models have systematic errors because of such differences, because of parameter adjustments (cf. Sec. 2.6) and because of numerical approximations (computational errors). Parameter adjustments and computer errors are being assessed by comparison with data [3]. Their consequences are known, but only within statistical error bounds.

Thus, even though a computer code execution is deterministic, the statistical uncertainties in input data specifications and the often unpredictable consequences from systematic errors cause computer results to be afflicted with essentially random errors, the magnitudes of which must be estimated with quantifiable confidence.

In this section are summarized the uncertainties in fuel-related parameter specifications. They were taken from previously published uncertainty analyses, from the available code documentation and from Reference [6].

2.8.1 Uncertainties of Input Data Specifications. In the category of input data are (i) the initial power level or linear heating rate, (ii) the power distribution, (iii) the fuel dimensions at cold conditions and (iv) the condition of the fuel (burn-up conditions). Table 3 below summarizes the fuel-
related input parameters, their 1-σ uncertainties, associated probability distributions and the references. Unknown probability distributions are specified as uniform because equal probabilities reflect the maximum of ignorance.

<table>
<thead>
<tr>
<th>Table 3. Uncertainties in Input Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>-------------------------------------</td>
</tr>
<tr>
<td>Fission Power</td>
</tr>
<tr>
<td>Peaking Factors</td>
</tr>
<tr>
<td>Local Axial</td>
</tr>
<tr>
<td>Local Radial</td>
</tr>
<tr>
<td>Pellet Radius (cold)</td>
</tr>
<tr>
<td>Clad Radii (cold)</td>
</tr>
<tr>
<td>Neutron Flux Depression</td>
</tr>
<tr>
<td>m (Eq. 12)</td>
</tr>
<tr>
<td>n (Eq. 12)</td>
</tr>
</tbody>
</table>

The uncertainty in fission power reflects the random error of matching the calorimetric balance of the plant with instrument readings. Even though it is not a code uncertainty, it must be part of the peak clad temperature uncertainty. Peaking factor uncertainties account for random errors in specifying power distributions in the core. B. W. Sheron [12] implied normal distributions for power and peaking factors. These have been adopted here also.

Errors in pellet and cladding dimensions are random fabrication errors. Since quality control eliminates large deviations, a uniform distribution is suggested here. However, for the purpose of this analysis, these size uncertainties and the uncertainties of surface roughness [13, Table 2] are combined later into the gap conductance uncertainty, to which is assigned also a uniform probability distribution.

Uncertainties for fission power variation within the fuel pellet are estimated. No data could be found.

2.8.2 Modeling Uncertainties. Modeling uncertainties arise from random differences between heat transfer and thermophysical property measurements and the correlations used to represent the measurements in the computer code. Systematic differences, however, between the mathematical models in the code and the experimental data are code-specific and presented later in Section 2.8.3. In the category of fuel-related modeling uncertainties are the uncertainties from (i) fuel pellet thermal conductivity, (ii) cladding thermal conductivity, (iii) fuel pellet heat capacity, (iv) cladding heat capacity, (v) filler gas thermal conductivity, (vi) thermal expansion or gap width, (vii) radiative surface properties and (viii) convective heat transfer coefficients. Table 4 below summarizes 1-σ level uncertainties, approximate distributions and source references for these parameters.
Table 4. Uncertainties of Fuel-Related Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1-σ Uncertainty</th>
<th>Distinction</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel thermal cond., $k_f$</td>
<td>±0.2 W/(mK)*</td>
<td>Uniform</td>
<td>[6], p. 24</td>
</tr>
<tr>
<td>Clad thermal cond., $k_c$</td>
<td>±1.01 W/(mK)</td>
<td>Normal</td>
<td>[6], p. 218</td>
</tr>
<tr>
<td>Fuel heat capacity, $(\rho C)_f$</td>
<td>±30.035 kJ/(m³K)</td>
<td>Uniform</td>
<td>[6], p. 10†</td>
</tr>
<tr>
<td>Clad heat capacity, $(\rho C)_c$</td>
<td>±64.875 kJ/(m³K)</td>
<td>Uniform</td>
<td>[6], p. 211</td>
</tr>
<tr>
<td>Gas thermal cond., $k_g$</td>
<td>±0.0131 W/(mK)</td>
<td>Uniform</td>
<td>[6], p. 485, †</td>
</tr>
<tr>
<td>Effective gap width, $t$</td>
<td>±20.98 µm</td>
<td>Skewed</td>
<td>[14], p. 15</td>
</tr>
<tr>
<td>Pellet emissivity, $e_f$</td>
<td>±7%</td>
<td>Uniform</td>
<td>[6], p. 48.6</td>
</tr>
<tr>
<td>Clad emissivity, $e_c$</td>
<td>±0.10</td>
<td>Uniform</td>
<td>[6], p. 237</td>
</tr>
<tr>
<td>Convective heat transf.,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>single-phase, forced, turb.</td>
<td>-5% to +35%</td>
<td>Uniform</td>
<td>[16], p. 18; [17]</td>
</tr>
</tbody>
</table>

*Notice that Hobson's data listed in Table A-2.VII [6], are incorrectly plotted in Figure A-2.4 [6].
†Revision 8/81.

The probability distributions in Table 4 are specified as normal whenever they are unknown or known to be approximately normal.

The uncertainty of fuel thermal conductivity, $k_f$, is specified in Reference [6, p. 24] for all fractions of theoretical fuel density. For the fraction of 94.9%, Hobson's data as listed in Ref. [6] are $(0.32 \pm 0.09)\text{W/(mK)}$ higher than the MATPRO correlation, Eq. (4), in the temperature range between 547K and 1,330K. However, for this analysis we adopt the more universally applicable error range as specified in Reference [6].

Error brackets for heat capacities, $(\rho C_p)$, were computed from uncertainties specified for specific heats in Reference [6, pp. 10 (of 8/81 revision) and 211] and the densities given below Eq. (45). The uncertainty for density is trivial ($\pm0.5\%$ of theoretical density for fuel [13, Table 2]) and therefore ignored.

The uncertainties for important parameters affecting the gap conductance are combined into an overall gap conductance uncertainty, based on Eq. (3) and on the assumption of independence between the dominant gap width and gas conductivity uncertainties:

$$\Delta h_{\text{gp}} = \sqrt{\Delta_1^2 + \Delta_2^2}.$$  (49)
Here, the uncertainty from gas conductance uncertainty, $\Delta k_g = 0.0131 \text{ W/(mK)}$, is

$$\Delta_1 = \frac{\Delta k_g}{t} = 634.7 \text{ W/(m}^2\text{K)} \quad (50)$$

the effective gap width, $t$, being defined by Eq. (31) and computed with the values given in Section 2.6 to be $t = 25.039 \mu\text{m}$, and from gap width uncertainty,

$$\Delta_2 = \frac{k_g}{t + \Delta t} = 4,992.0 \text{ W/(m}^2\text{K)} \quad (51)$$

The gas thermal conductivity, $k_g$, is evaluated from Eq. (9), with the mean temperature computed from the values given in Table 2. The nominal gap conductance is $h_{gp} = 10.960 \text{ kW/(m}^2\text{K)}$ (cf. Sec. 2.6), as in the TRAC code. The gap width uncertainty, $\Delta t$, is taken from Reference [15] as shown there in Table 4. Uncertainties from radiative heat transfer (cf. Eq. (32)) are ignored because the entire contribution from radiative heat transfer to gap conductance is only 0.4% at normal full power conditions.

Substitution of Eqs. (50) and (51) into Eq. (49) yields the 1-$\sigma$ uncertainty of gap conductance which is

$$\Delta h_{gp} = 5,032.2 \text{ kW/(m}^2\text{K)} \quad (52)$$

and amounts to 45.9% of the nominal gap conductance.

Gap conductance uncertainty is dominated by the uncertainty of effective gap width. The effective gap width uncertainty, $\Delta t$, is slightly smaller than the effective gap width $t$ itself, but approximately as large as the physical gap width $s$. There is no general agreement in the literature concerning gap width uncertainties; most error ranges have been assumed without basis. Sheron [12, Table 2] lists an "assumed RMS error" of 25% for gap conductance, which appears low. In contrast, Lassmann and Hohlfeld [18] more recently compared nearly a thousand gap conductance measurements with results from the URAGAP computer code. The measurements were taken primarily from extremely well-controlled out-of-pile experiments, and URAGAP "has been fitted to the data of Ross and Stoute [19], Dean [20] and Campbell et al. [21]." Under such rather ideal but atypical conditions, Lassmann and Hohlfeld obtained the gap conductance uncertainty of 33.4% (1-$\sigma$ level). Cunningham et al. [22] concluded also from experiments that the gap width uncertainty is approximately 50% (p. C-2), based on an initial gap width of $229 \mu\text{m}$ (p. A-1) and a mean gap closure by 63% (p. C-2). This leaves, under normal full-power conditions (in a BWR fuel pin), a gap width of $108 \mu\text{m}$ with an uncertainty of $50 \mu\text{m}$. Cunningham et al. add a 100% uncertainty (p. C-6) in temperature jump distance uncertainty (cf. Eq. (31)) and a 10% uncertainty in the gap width coefficient of their STORE computer code. This tends to support the uncertainty analysis by Steck et al. [15] for PWR conditions, i.e. the reference from which Eq. (52) was derived.
For uncertainties of convective film coefficients, it was pointed out by Lellouche [16] that when heat transfer coefficients appropriate for conditions inside circular tubes are used for heat transfer in tube bundles, then the heat transfer coefficients are consistently underestimated. This being the case in the TRAC code, the ±20% range of 1-σ level uncertainties suggested by the comparison of the Dittus-Boelter correlation with data [17], has been shifted to [-5%, +35%] to account for the above enhancement of heat transfer regimes with phase change. This is addressed in Section 3 later.

2.8.3 Systematic Modeling Errors in TRAC-PF1/MQD1. Based on presently available code documentation [1] and on the written responses [23] from Los Alamos National Laboratory to specific questions posed by the author, TRAC-PF1/MQD1 has four systematic errors related to its fuel modeling:

1. The thermal conductivity is too low as shown in Table 1 (cf. [1, p. 10]). The mean bias is -0.46 W/(mK) relative to the correlation given in MATPRO [6, p. 23]. The consequences should have been an excess of initial fuel stored energy of approximately 6 kJ/m or 9%, and a 13% increase in pellet thermal response time. As pointed out in Section 2.6, however, TRAC results show a slight deficit of 1% in fuel stored energy when compared with results from closed-form integration. The cause for this discrepancy might be computational error; the exact cause cannot be identified without additional code documentation. The increase in thermal response time, on the other hand, appears to be corroborated indirectly by comparing the thermal response of the cladding as computed by TRAC and by the analysis presented in Section 3.

2. The gap width is increased from approximately 14 μm to 21 μm (under hot conditions). The adjustment is suggested in the TRAC LBLOCA report [24, p. 17], to achieve vendor-specified initial fuel-stored energy. The magnitude of the adjustment given here is inferred from hand calculations (cf. Eqs. (1) and (2)) because the necessary documentation is lacking in TRAC output listings. It is possible, by gap width adjustments, to obtain the correct initial fuel stored energy, but not without changing the pellet and cladding response times. It will be shown in Section 3, that any increase in gap width, i.e. decrease in gap conductance, increases blowdown peak clad temperature. This is true only in the vicinity of the fuel conditions calculated in TRAC for full-power conditions, but it shows that stored energy and with it pellet temperature is more important for peak clad temperature than gap conductance. Whether TRAC results are conservative, however, can only be determined from calculations carried out with correct fuel thermal conductivity, without significant computational error and without gap width adjustments.

3. Thermal expansion in radial direction are computed incorrectly in TRAC [1, p. 70], because (i) the formula for radial displacements is evaluated with the absolute temperature instead of with the excess temperature above the temperature of the strain-free state and (ii) total radial displacements are added to radii from previous time steps, rather than to radii of cold conditions. One should suspect documentation errors here, but it was not possible to resolved this issue [23] without better documentation.
4. The calculation of two-phase flow heat transfer coefficients in TRAC-PFI/MOD1 [1, p. 81, Eq. (140), p. 83, top equation; p. 63] is incorrect because (i) the equation on page 63 is wrong and (ii) a mixture heat transfer coefficient is assigned to the liquid phase, while the vapor phase heat transfer coefficient is not zero.

(i) Los Alamos National Laboratory claims [23], with unspecified references to M. Ishii and J. M. Delhaye, that the equation on page 63 of [1] for combining phasic heat transfer coefficients is derived by averaging. No reference for this averaging is offered. Since the equation contains neither a phasic perimeter fraction, nor a phasic wetted area fraction, nor a phasic wall residence time fraction, it is absolutely impossible to derive the equation on page 63, nor Eq. (98) on p. 58 (for solid structures) either by space or by time averaging.

(ii) Los Alamos National Laboratory concedes [23], that mixture and vapor heat transfer coefficients should not be added as in the TRAC code, but they claim that it is permissible to add them because the liquid convective heat transfer coefficient \( (h_c^l) \), is at least 100 times greater than that for vapor, \( (h_c^v) \). This claim is wrong in principle (one does not introduce unnecessary errors, because they are small); it is also wrong because 25 randomly selected TRAC results, obtained from a PWR LBLOCA calculation, show that during the blowdown phase \( (h_c^l)/(h_c^v) = 16 \pm 14, (1-\sigma) \).

This systematic error does not affect single-phase coolant conditions, whence it does not affect the initially stored energy but it will affect the transient calculations.

2.8.3 Resultant Uncertainties in Fuel Stored Energy. Table 5 summarizes the changes caused in fuel stored energy by varying the dominating fuel-related parameters, each one individually, by its 1-\( \sigma \) uncertainty. The second column shows the parameter values for normal conditions, as they were used to obtain the results in Table 2, and the stored energy given in Section 2.7. The third column shows the parameter changes which bring about an increase in fuel stored energy and which correspond to a 1-\( \sigma \) variation. Notice that the convective film coefficient was reduced by 20%, one-half of the total variation listed in Table 4. Column four shows the extreme values used to calculate the changes in stored energy, while column five lists the corresponding values for stored energy. The last column shows the changes in stored energy relative to the reference value of \( E = 67.991 \text{ kJ/m} \) (cf. Section 2.7).

The values in Table 5 serve four purposes. Firstly, they serve to rank the parameters in their order of significance in affecting stored energy. Notice that, the convective film coefficient is in eighth place because it is assumed that its minimum value is only 5% below its computed reference value (on account of heat transfer enhancement in rod bundles, cf. Section 2.8.2), and because such a 5% decrease would increase the stored energy only by 0.313 kJ/m. Clearly, uncertainties in gap conductance, peaking factors and fuel thermal conductivity have the most serious influence on stored energy.
Table 5. Elements of Stored Energy Uncertainty

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Reference Value</th>
<th>$\Delta$ Change$^\dagger$</th>
<th>Max./Min. Values</th>
<th>$E_\text{ref}$</th>
<th>$E - (E)_{\text{ref}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gap Conductance</td>
<td>10.960 kW m$^{-2}$ K$^{-1}$</td>
<td>-5.028 kW m$^{-2}$ K$^{-1}$</td>
<td>5.932 kW m$^{-2}$ K$^{-1}$</td>
<td>84.928</td>
<td>16.937</td>
</tr>
<tr>
<td>Peaking Factors</td>
<td>1.41736</td>
<td>x1.03218</td>
<td>1.46297</td>
<td>70.675</td>
<td>2.685</td>
</tr>
<tr>
<td>Fuel Thermal Cond.</td>
<td>4.392 W m$^{-1}$ K$^{-1}$</td>
<td>-0.2 W m$^{-1}$ K$^{-1}$</td>
<td>4.186 W m$^{-1}$ K$^{-1}$</td>
<td>70.475</td>
<td>2.484</td>
</tr>
<tr>
<td>Power</td>
<td>486.49 MW/m$^3$</td>
<td>+2%</td>
<td>496.22 MW/m$^3$</td>
<td>69.655</td>
<td>1.664</td>
</tr>
<tr>
<td>Fuel Heat Capacity</td>
<td>3.028 MJ m$^{-3}$ K$^{-1}$</td>
<td>+30.035 kJ m$^{-3}$ K$^{-1}$</td>
<td>3.059 MJ m$^{-3}$ K$^{-1}$</td>
<td>68.651</td>
<td>0.661</td>
</tr>
<tr>
<td>Cladding Thermal Cond.</td>
<td>16.735 W m$^{-1}$ K$^{-1}$</td>
<td>-1.01 W m$^{-1}$ K$^{-1}$</td>
<td>15.728 W m$^{-1}$ K$^{-1}$</td>
<td>68.455</td>
<td>0.464</td>
</tr>
<tr>
<td>Burn-Up m</td>
<td>0.4034</td>
<td>-0.05</td>
<td>0.3534</td>
<td>68.346</td>
<td>0.355</td>
</tr>
<tr>
<td>Burn-Up n</td>
<td>3.9167</td>
<td>-0.30</td>
<td>3.6167</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Convective Film Coeff.</td>
<td>40.05 kW m$^{-2}$ K$^{-1}$</td>
<td>-20%</td>
<td>32.04 kW m$^{-2}$ K$^{-1}$</td>
<td>69.241</td>
<td>1.251</td>
</tr>
<tr>
<td>Cladding Heat Capacity</td>
<td>2.145 MJ m$^{-3}$ K$^{-1}$</td>
<td>+64.875 kJ m$^{-3}$ K$^{-1}$</td>
<td>2.210 MJ m$^{-3}$ K$^{-1}$</td>
<td>68.033</td>
<td>0.042</td>
</tr>
</tbody>
</table>

$^\dagger$cf. Tables 3 and 4.
Secondly, the values listed in the third and last columns of Table 5 serve to estimate the sensitivity of fuel stored energy on modeling parameters. The ratio of listed stored energy change over a listed parameter change is an approximation to the partial derivative of stored energy with respect to that parameter and can be used to estimate the stored energy change for different parameter changes.

Thirdly, if one assumed that the conditions of linear error propagation are met, one could estimate the uncertainty in fuel stored energy from [25]

$$\sigma_E = \sqrt{\sum \sigma_i^2} = 17.7 \text{ kJ/m}$$

which is 26.1% of the reference value $E = 67.991 \text{ kJ/m}$. The $\sigma_i$ values in Eq. (53) are the values in the last column of Table 5, except that for convective heat transfer only 1/4 of the tabulated value is used (corresponding to the low range shown in Table 4). It must be pointed out, however, that Eq. (53) is only a rough estimate since the conditions of linear error propagation are not strictly met.

The fourth and last purpose served by Table 5 is to show that the effects of parameter variations on fuel stored energy can be simulated by simply varying the fission power or the power peaking factor. Since this is easy to carry out in the TRAC code for a large number of fuel elements, it is a powerful technique for developing response surfaces in support of statistical uncertainty analyses, because this technique saves computing resources. One needs to select from Table 5 the amount of needed change in stored energy for a given change of any of the listed parameters, and then to determine from Figure 2 the multiplier for peaking factor which is to be used for the steady-state calculations of initial conditions.

2.9 Conclusions Regarding Fuel Stored Energy Uncertainty

1. Initial fuel stored energy can be computed efficiently from closed-form integration of the steady-state conduction equation, taking into account the dependence of thermophysical properties on temperature and the radial variation of fission power due to neutron flux depression.

2. The uncertainty in fuel stored energy is dominated by uncertainties in gap conductance, peaking factors and fuel thermal conductivity.

3. The fuel stored energy per unit of length of fuel pin, in the fuel pin with the highest clad temperature at full-power conditions in a Westinghouse PWR core with 17 x 17 fuel arrays and 160,000 Md/t burnup is 68 kJ/m. The standard deviation of the stored energy is approximately 26%.

4. It is possible to approximate the effects on stored energy of all important uncertainties from fuel-related modeling parameters by corresponding variations in peaking factors.
3. **Thermal Response of Fuel**

It is recognized that peak clad temperatures are the higher the larger the initial fuel stored energy is and the more easily this stored energy can pass from the fuel pellet into the cladding. Also, one and the same change or uncertainty in important fuel modeling parameters can increase the stored energy while retarding the energy transfer from pellet to clad, thereby representing opposing mechanisms which determine peak clad temperature. Gap conductance and thermal conductivity of fuel pellets are two such important modeling parameters.

It is the purpose of this section to determine the effects which the parameters listed in Table 5 have on peak clad temperature. It is of particular interest to know which of these parameters have opposite effects on peak clad temperature and on fuel stored energy.

3.1 **Problem Formulation**

We seek the transient fuel and cladding temperature by integrating the transient conduction equation

\[
\rho c_p \frac{\partial T}{\partial t} = \bar{k} \frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) + q'''
\]  

in the two regions (subscripts \( f \) for fuel and \( c \) for clad), as shown in Figure 1. All symbols are defined as before, except

\[
\bar{\rho} c_p = \rho c(<T>), \text{ mean volumetric heat capacity}
\]

\[
\bar{k} = k(<T>), \text{ mean thermal conductivity},
\]

both evaluated, respectively, for fuel and clad with the area-averaged temperatures defined in Eqs. (3) by using Eqs. (4) and (5) for \( \bar{k}_f \), Eq. (7) for \( \bar{k}_c \), Eq. (46) for \( \bar{C}_f \), and Eq. (48) for \( \bar{C}_c \). The densities \( \rho_f \) and \( \rho_c \) are kept constant as specified below Eq. (45). We impose as boundary conditions the same four heat flux conditions as in Section 2.4 for the steady-state analysis: zero flux at the center line and flux continuity at all three surfaces. We also impose continuity of temperature at three surfaces of pellet and clad, taking properly into account the temperature jump distances \( g_f \) and \( g_c \) (cf. Eq. (31)). The surface roughness is insignificant and ignored.

3.2 **Solution Technique**

Equation (54) is first converted, by area averaging (cf. Eq. 3), into one ordinary differential equation each for pellet and for clad, as shown in Reference [26]. The result is

\[
\frac{d<T_f>}{dr} = \frac{<q'''>f>}{(\rho c_p)_f} - \frac{2g_f}{R_1 (\rho c_p)_f} \frac{T_{fo} - T_{ci}}{R_2 + g_c} \ln \frac{R_2 + g_c}{R_1 - g_f}
\]  

(55)
\[
\frac{d\langle T_c \rangle_c}{d\tau} = \frac{k_c}{sR_m (\rho_c c_p)_c} \frac{T_c - T_{ci}}{R_m^2 + s} + \frac{R_w}{s} N_{Bi} T_{m} - T_w, \tag{56}
\]

where

- \( c = R_2 - R_1 \), cladding thickness
- \( R_m = (R_1 + R_2)/2 \), mean radius of clad
- \( \bar{\alpha}_c = \frac{k_c}{(\rho c_p)_c} \), thermal diffusivity of clad
- \( N_{Bi} = \frac{s T_{m}}{k_c} \), Biot number of clad
- \( g \) = temperature jump distance

and all other symbols have been defined previously. The right-hand side of Eq. (55) represents clearly the balance between internal heat generation and heat transfer through the gap, while the right-hand side of Eq. (56) shows similarly the balance between heat supply from the pellet and heat loss to the coolant. Integration of Eqs. (55) and (56) yields the area-averaged temperatures defined in Eqs. (3) and needed to evaluate all properties. However, one needs to calculate the surface temperatures \( T_{fo} \), \( T_{ci} \) and \( T_w \), in order to integrate Eqs. (55) and (56).

These three temperatures are computed by approximating the transient temperature distributions in pellet and clad by two separate power polynomials in radius \( r \). The polynomials have time-dependent coefficients which are computed to satisfy the above-mentioned seven boundary conditions of heat flux and temperature continuity, plus two requirements imposed by Eqs. (3). The total of nine conditions defines the three temperatures \( T_{fo} \), \( T_{ci} \) and \( T_w \), all in terms of \( \langle T_f \rangle_f, \langle T_c \rangle_c \) and \( T_m \), and also six time-dependent polynomial coefficients \[26]. The three surface temperatures are:

\[
T_{fo} = F_1 \langle T_f \rangle_f + F_2 \langle T_c \rangle_c + F_3 T_m \tag{57}
\]

\[
T_{ci} = \langle T_f \rangle_f - \Omega_1 (\langle T_f \rangle_f - T_{fo}) \tag{58}
\]

\[
T_w = \left[ (\Omega_4 - \Omega_3) T_m - T_{fo} + (\Omega_3 + 1) T_{ci} \right]/\Omega_4, \tag{59}
\]

where

- \( F_1 = (\Omega_1 - 1) [\Omega_4 (\Omega_3 - \Omega_2) - \Omega_3 (\Omega_3 + 1)]/G \)
- \( F_2 = \Omega_3 \Omega_4 / G \)
- \( F_3 = -\Omega_3 \Omega_3 / G \)
\[ G = \Omega_1 \Omega_3 (\Omega_4 - \Omega_5) + (1-\Omega_1) (\Omega_2 \Omega_4 + \Omega_5) \]

\[ \Omega_1 = 4 \lambda_f \gamma_3 + 1, \quad \Omega_4 = \Omega_3 (N_B i + 2)/2 \]

\[ \Omega_2 = \gamma_6 - 2 \gamma_5, \quad \Omega_5 = \gamma_6 (\Omega_4 - \Omega_3) \]

\[ \Omega_3 = 2 \lambda_c \gamma_4 \]

\[ \lambda_f = \frac{\bar{k}_f}{\bar{k}_g}, \quad \lambda_c = \frac{\bar{k}_c}{\bar{k}_g} \]

\[ \gamma_3 = \ln \left[ \frac{R_2 + g_c}{(R_1 - g_f)} \right] \]

\[ \gamma_4 = \frac{\gamma_3 R_2}{s} \]

\[ \gamma_5 = \frac{(3R_2 + 2s)}{[3(2R_2 + s)]} \]

\[ \gamma_6 = \frac{(4R_2 + 3s)}{[6(2R_2 + s)]} . \]

All \( \gamma \)-values are fixed for fixed geometry; \( \lambda \), \( \Omega \), \( F \) and \( G \) values vary with temperature. One can integrate Eqs. (55) and (56) with Eqs. (57) through (58). The fuel center line temperature is given by

\[ T_c = 2 <T_f>(0) - T_f \quad (60) \]

The expressions for temperature distributions (parabolic in pellet and cladding, logarithmic in gap) can be found in Ref. [26]. They are not needed here. However, Eqs. (55) and (56) need initial values for \( \tau = 0 \). They are derived to render the derivatives in Eqs. (55) and (56) to be zero [26]. They are computed from:

\[ <T_f(0)> = \frac{(V_1 U_{22} - V_2 U_{12})}{(U_{11} U_{22} - U_{12} U_{21})} \quad (61) \]

\[ <T_c(0)> = \frac{(V_2 U_{11} - V_1 U_{21})}{(U_{11} U_{22} - U_{12} U_{21})}, \quad (62) \]

where

\[ V_1 = \frac{s/R_w}{N_B i} \cdot \frac{R^2 q_{f}^{\prime \prime \prime}(0)}{2k_f} + T \cdot \frac{\Omega_3 - F_3 [(1+\Omega_3)\Omega_1 - 1]}{\Omega_4} \]
\[ V_2 = \frac{R^2 \langle q^{'''}(0) \rangle_F}{2k_g} \gamma_3 - T \alpha F_3 (1 - \Omega_1) \]

\[ U_{11} = \frac{(1 + \Omega_3)[1 + \Omega_1 (F_1 - 1)] - F_1}{\Omega_4} \]

\[ U_{12} = F_2 [1 + \Omega_3 \Omega_1 - 1] \]

\[ U_{21} = (1 - \Omega_1) (F_1 - 1) \]

\[ U_{22} = F_2 (1 - \Omega_1), \]

and \( \langle q^{'''}(0) \rangle_F \) is the initial, area-averaged fission power; all other symbols are defined below Eq. (59).

The transient conduction model represented by Eqs. (55) through (62) was derived [5] on the basis of integral techniques described by Goodman [4]. The model applies to the thermally thin conduction regime. The criteria for its applicability are given by Wulff et al. [26]. Physically, the model implies thermal relaxation without delay; the results lead those of the exact solution by less than 12 milliseconds for the cladding and by less than 0.1 seconds for the pellet surface. This is deemed adequate for peak clad temperature calculations during LBLOCA simulations. See Reference [5] for more details concerning accuracy.

Equations (55) and (56) were integrated on an HP-41CX programmable pocket calculator, using a fourth-order Runge-Kutta integration algorithm. The HP-41CX executes Eqs. (4), (5), (7), (9), (46) and (48) for properties, Eqs. (61) and (62) for initial conditions and Eqs. (57) through (60) for fuel and cladding temperatures. The integration step size of 50 milliseconds guarantees a computing accuracy of \( \pm 0.05K \); five significant digits remained unaltered during eighty integration steps or 4 second of a null transient which was started from Eqs. (61) and (61).

Results were computed for two sets of boundary conditions. One set approximates the fission power and coolant conditions as calculated by the TRAC code while simulating an LBLOCA for reference Westinghouse PWR plant conditions. The other set consists of limiting step changes in coolant conditions to show the effects from DNB delay (departure from nucleate boiling) or from dryout delay, and also with changes of time for scram initiation. The results are presented in the following two subsections.

\[ \text{The Runge-Kutta algorithm was coded for the HP-41CX as part of this project. It needs 26 memory registers. The number of nonlinear ordinary differential equations, } \ddot{Y} = f(x,Y), \text{ that can be integrated is free and depends only on the complexity of } f \text{ and the number of available registers (max. 896).} \]
3.3 Effect of Fuel Parameter Uncertainty on Peak Clad Temperature

Eighteen transient fuel temperature calculations have been performed to assess the effects of parameter variations on thermal response and on peak clad temperature. The results are shown in Table 6.

Table 6 Thermal Response of Fuel and Peak Clad Temperature Change

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Step Change</th>
<th>TRAC Blowdown B.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial Clad</td>
<td>Change of Peak Clad</td>
</tr>
<tr>
<td></td>
<td>Rise K/s</td>
<td>K/s</td>
</tr>
<tr>
<td>Gap conductance</td>
<td>724.2</td>
<td>-7.2</td>
</tr>
<tr>
<td>Peaking factor</td>
<td>753.0</td>
<td>21.6</td>
</tr>
<tr>
<td>Fuel thermal conductivity</td>
<td>731.3</td>
<td>-0.1</td>
</tr>
<tr>
<td>Power</td>
<td>744.9</td>
<td>13.5</td>
</tr>
<tr>
<td>Fuel heat capacity</td>
<td>731.4</td>
<td>0</td>
</tr>
<tr>
<td>Cladding thermal cond.</td>
<td>732.0</td>
<td>0.6</td>
</tr>
<tr>
<td>Convective heat trans. coef.</td>
<td>722.0</td>
<td>-9.4</td>
</tr>
<tr>
<td>Cladding heat capacity</td>
<td>710.0</td>
<td>-21.4</td>
</tr>
<tr>
<td>Reference case</td>
<td>731.4</td>
<td></td>
</tr>
</tbody>
</table>

The first set of nine calculations was carried out to obtain the initial time rate of mean clad temperature rise from Eq. (56). Initial steady-state conditions were computed, according to Eqs. (61), (62) and (57) through (59), first with the parameters from TRAC-PFI/MOD1 as listed in Section 2.6, and then with all but the burn-up parameters (not modeled in Eqs. (55) and (56)) shown in Table 5 changed by the same \( \Delta \sigma \) uncertainty as listed in Table 5. The transient was induced by changing the following three boundary conditions from initial values:

\[
\begin{align*}
T_\infty &= 599.44K \\
\overline{h_c} &= 40.05 \text{ kW/(m}^2\text{K)} \\
\langle q_f'' \rangle_f &= 486.49 \text{ MW/m}^3 \\
\end{align*}
\]

for \( \tau < \tau' = 0 \) \hspace{1cm} (63a)

\[
\begin{align*}
T_\infty &= 666.80K \\
\overline{h_c} &= 0.8805 \text{ kW/(m}^2\text{K)} \\
\langle q_f'' \rangle_f &= 27.24 \text{ MW/m}^3 \\
\end{align*}
\]

for \( \tau \geq \tau' = 0 \) \hspace{1cm} (64a)

The boundary conditions were then varied as follows:

\[
\begin{align*}
T_\infty &= 599.44K \\
\overline{h_c} &= 40.05 \text{ kW/(m}^2\text{K)} \\
\langle q_f'' \rangle_f &= 486.49 \text{ MW/m}^3 \\
\end{align*}
\]

for \( \tau < \tau'' = 0 \) \hspace{1cm} (63b)

\[
\begin{align*}
T_\infty &= 666.80K \\
\overline{h_c} &= 0.8805 \text{ kW/(m}^2\text{K)} \\
\langle q_f'' \rangle_f &= 27.24 \text{ MW/m}^3 \\
\end{align*}
\]

for \( \tau \geq \tau'' = 0 \) \hspace{1cm} (64b)
which are the highest vapor temperature, the lowest heat transfer coefficient, both as calculated by TRAC-PF1/MOD1 for an LBLOCA blow-down phase, and 5.6% of initial power, representing the mean fission power during the first two seconds. The results are shown in the second and third columns of Table 6. The second column shows the initial mean clad temperature rise, \(d<T_c>/dt\) at \(\tau = 0\). The third column shows the effect of parameter variations on the thermal response of the cladding, relative to that of the reference calculation, which is shown in the last line. Indeed, gap conductance, fuel thermal conductivity, convective film coefficient and cladding heat capacity, when changed so as to increase fuel stored energy, decrease, as expected, the rate of thermal response by the cladding. This suggest that these four parameters have opposite effects on stored energy and peak clad temperature.

Figure 3 shows the transient outer cladding temperature, \(T_w(\tau)\) under reference conditions and with boundary conditions as specified by Eqs. (63) and (64). This figure suggests that the initial slope might not dictate the peak of the clad temperature \(T_w\). In fact, as shown in the first two columns of Table 6, the opposite effects suggested by the results in the second and third columns are unimportant with regard to peak clad temperature.

Far more importantly, Figure 3 shows that during the blowdown phase the clad temperature is limited by vapor cooling, without the arrival of droplets. The clad temperature peaks even when the pessimistic cooling conditions, Eqs. (64), prevail at once after scram and then remain. The blow-down peak clad temperature is unaffected by reactor pump degradation or by transition of critical break flow from subcooled to two-phase flow conditions, except in the highly unlikely event that liquid injection at core entrance should occur in less than approximately two seconds after dryout or departure from nuclear boiling (DNB). Aside from this exception, the blow-down peak clad temperature is dictated by fuel stored energy alone or, equivalently, by the linear heating rate (cf. Figure 2). Whence, blow-down peak clad temperature measurements are not affected by scale distortions outside the core in a test facility.

Turning now to the second set of transient fuel temperature calculations, we describe how the last two columns in Table 6 were obtained. Another reference and eight repeat calculations were carried out with the \(1-\sigma\) parameter variations listed in Table 5, only one parameter being varied at a time, to show the effects of these parameter variations on blow-down peak clad temperature. These nine parametric calculations were carried out, however, with boundary conditions representative of the ones imposed by the TRAC-PF1/MOD1 code during blow-down, on the fuel pin segment which had the highest clad temperature at full-power steady-state conditions. Figures 4, 5, and 6 for cooling rate, liquid temperature and vapor temperature, respectively were generated by TRAC and used to derive these approximate boundary conditions for coolant temperature, \(T_m\), heat transfer coefficient, \(\overline{h}_c\), and fission power, \(<q_f^'>\):

\[
T_m(\tau) = 599.44K - (21.41K/s)\tau \tag{65}
\]

\[
\overline{h}_c(\tau) = 40.05kW/(m^2K)\cdot\overline{f}_1(\tau), \tag{66}
\]
Figure 3. Transient Outer Clad Temperature for Step Change in Boundary Conditions at Time $\tau = 0$

Figure 4. TRAC-Computed History of Heat Transfer Rate from Fuel Rod During LBLOCA for Rod Location With Greatest Initial Clad Temperature

Figure 5. TRAC-Computed History of Liquid Temperature at Fuel Rod During LBLOCA for Rod Location With Greatest Initial Clad Temperature

Figure 6. TRAC-Computed History of Vapor Temperature at Fuel Rod During LBLOCA for Rod Location With Greatest Initial Clad Temperature
where 
\[
  f_1'(\tau) = 1 + (0.9812 \text{ s}^{-1}) \tau \quad \text{for} \quad 0 \leq \tau \leq 0.47 \text{s}
\]
\[
  = \max\{0.022314, 2.13745 - (1.43886 \text{ s}^{-1})\}
\]
for \(0.47 \text{s} \leq \tau \leq 4 \text{s}\)

and
\[
  \langle q_f'' \rangle = 486.49 \text{ MW/m}^3 \quad \text{for} \quad 0 \leq \tau \leq t'' = 0.2 \text{s}
\]
\[
  = 486.49 \text{ MW/m}^3 \left[1.236 - (1.180 \text{ s}^{-1})\tau\right] \quad \text{for} \quad t'' \leq \tau \leq t'' + 0.8 \text{s}
\]
\[
  = 27.24 \text{ MW/m}^3 \quad \text{for} \quad t'' + 0.8 \text{s} \leq \tau \leq 4.0 \text{s}
\]

(67)

All boundary conditions start out with steady-state initial conditions. The TRAC-computed cooling recovery at 2.81s after break initiation is intentionally omitted; the cooling rate is kept constant instead.

The outer clad temperature history, \(T_W(\tau)\), obtained with these reference conditions and with all parameters in Table 5 at their nominal values is shown as the solid curve, marked REF, in Figure 7. The comparison with TRAC results shows that the TRAC-computed peak clad temperature is approximately 28K lower and appears 0.4 seconds later than shown in Figure 7. The shape of the curve from TRAC, however, is the same as that of the solid curve in Figure 7, except for a small reduction in slope at the time (1.47s) of a computer re-start.

The fourth column in Table 6 shows the peak clad temperatures obtained by changing the listed parameters by the \(1-\sigma\) uncertainties listed in Table 5. The last column shows the corresponding changes in peak clad temperature relative to the reference value of 832.4K, given in the last line. In contrast to the conclusion arrived at from the thermal response rate differences in the third column of Table 6, all parameter changes, which produce an increase in stored energy, produce also an increase in peak clad temperature. This shows clearly that energy storage is more important for peak clad temperature than are heat transfer rates.

After taking the square root of the sum of squares (cf. Eq. (53) for stored energy) of all the differences in the last column of Table 6, one obtains a rough estimate for the \(1-\sigma\) uncertainty in peak clad temperature of 41.4K, as it results from the uncertainty of fuel-related parameters. This estimate is offered here with caution, because it is based on linear error propagation [25], of which the requirements can be met only approximately by the peak clad temperature modeling analysis. The dash-dot curve at the top in Figure 7 shows the expected peak clad temperature when all the \(1-\sigma\) changes of fuel-related parameters combine statistically. The curve is drawn for the \(1-\sigma\) uncertainty in gap conductance of 46%. The intermediate dash curve is drawn for the gap conductance uncertainty of 35% as proposed by Lassmann and Hohlefeld [18]. No attempt is made to assign confidence limits to these peak clad temperature uncertainties as these limits are the subject of an ongoing effort by the USNRC.
However, as before for stored energy, the peak clad temperature differences given in Table 6 can serve, along with their corresponding parameter changes listed in Table 5, to estimate the sensitivity of peak clad temperature on each of the fuel-related parameters. Finally, one can recognize the order of importance for the parameters concerning their effect on peak clad temperature; the order is the same as for fuel stored energy, as far as the three most important parameters are concerned. Notice that in this ordering the change of peak clad temperature due to a change in convective heat transfer coefficient should be divided by four because the coefficient is expected to change in the range from -5% to +35%, while the difference in peak clad temperature was obtained for -20%, as shown in Table 5.

Notice in Figure 7 that the clad temperature peaks without recovery of cooling due to droplet injection. This confirms the conclusions reached before for Figure 3, but now also for boundary conditions which approximate those computed by TRAC-PF1/M0D1.

3.4 Effect of Hydraulics Parameters on Peak Clad Temperature

In this section are presented the results from transient fuel temperature calculations carried out with limiting step changes in boundary conditions for the purpose of assessing the effects on peak clad temperature from (i) changes in heat transfer coefficients, (ii) delay of the time when DNB or dry-out occurs, and (iii) delay of the time when scram is initiated.
The Effect of Heat Transfer Coefficients can be seen by comparing the curves for outer clad temperature, $T_w(\tau)$, in Figures 8 and 9. These curves were computed with all parameters in Table 5 at their nominal values, with fission power as described by Eqs. (67), with the coolant temperature $T_w$ as specified by the first of Eqs. (63a) and (64a) and with

\[ \bar{h}_c = \begin{cases} 40.05 \text{ kW/(m}^2\text{K)} & \text{for } 0 \leq \tau \leq \tau' \\ 227.1 \text{ W/(m}^2\text{K)} & \text{for } \tau' \leq \tau \leq 4s \text{ in Fig. 8} \\ 56.8 \text{ W/(m}^2\text{K)} & \text{for } \tau' \leq \tau \leq 4s \text{ in Fig. 9} \end{cases} \]  

The lowest value of $\bar{h}_c$ computed during blowdown by TRAC-PFI/MOD1 is 880.5 W/(m$^2$K). This is for single-phase, turbulent vapor flow, with* the velocity of 4.62 m/s, at the pressure of 81 bar and the vapor temperature of 666.2K. Turbulent free convection by dry vapor would represent the pessimistic extreme in velocity calculation and produce 164 W/(m$^2$K). Thus, the heat transfer coefficient $h_c$ used for Figure 8 is slightly more than 1/4 of the minimum TRAC-computed value; in Figure 9 it is ~1/15, and even only ~1/3 of the free convection value. This is extremely low.

The clad temperature peaks in Figures 3 and 8 reaches its asymptotic value $T_w(\infty)$ from above, in Figure 9 it approaches its asymptotic temperature from below. The clad temperature is limited by vapor cooling; a maximum occurs when the asymptotic wall temperature

\[ T_w(\infty) = (T_v)_\text{max} + R_1^2 \langle q''''_{\text{decay}} \rangle_f / [2R_w(\bar{h}_c)_{\text{min}}] \]  

is below the initial mean pellet temperature $\langle T_f(o) \rangle_f$, where

\[
\begin{align*}
(T_v)_\text{max} &= \text{maximum vapor temperature} \\
R_1 &= \text{pellet radius} \\
R_w &= \text{outer clad radius} \\
\langle q''''_{\text{decay}} \rangle_f &= \text{decay power density in pellet} \\
(\bar{h}_c)_{\text{min}} &= \text{minimum heat transfer coefficient}
\end{align*}
\]

Early arrival of liquid can turn down the clad temperature only if the heat transfer coefficient is unrealistically low or if the liquid arrives unrealistically early.

*Dittus-Boetter, $N_{Re} = 78,000, N_{Pr} = 1.12.$
The Effect of DNB or Dry-Out Delay is shown also in Figures 8 and 9. The delay $t'$ is measured from break initiation as shown in the top sketches of Figure 8 and 9.

Any delay in DNB or dry-out delays the temperature rise (horizontal shift of curves) and the peak by approximately the same amount $t'$. Moreover, any such delay reduces also the peak (vertical shift of curves), because extended cooling of the clad under pre-DNB conditions reduces the stored energy in the fuel prior to the temperature excursion.

The Effect of Scram Delay relative to the appearance of DNB or dry-out is shown in Figure 10. The three curves are obtained with $t' = 0.2s$ in Eqs. (63a) and (64a). The middle curve is obtained with the fission and decay power history described by Eqs. (67) with $t'' = 0.2s$, the top and bottom curves, respectively, with $t'' = 0.3s$ and 0.1s.

An advance in scram initiation reduces the peak clad temperature by approximately 300 K/s, while a delay of scram initiation increases the peak clad temperature by approximately 120 K/s.
3.5 Concluding Remarks

The above effects from variations in heat transfer coefficient in DNB or dry-out delay and scram delay include all effects from hydraulic and system-related parameters on the blowdown peak clad temperature, because droplet cooling is unimportant under realistic blowdown conditions. The effects from systematic modeling errors (cf. Section 2.8.3) on peak clad temperature uncertainty is not quantified here, because the necessary modeling documentation for TRAC-PF1/MOD1 is not available at this time.

Confidence limits for peak clad temperature will be developed in an ongoing USNRC project. The work will be extended from blowdown to reflood peak clad temperature.

4. Conclusions

The analysis presented here leads to these major conclusions:

1. The peak clad temperature during blowdown is limited by effective single-phase vapor cooling to less than 880K.
2. The blowdown clad temperature peaks, under all credible circumstances, before the arrival of liquid droplets in the core. Therefore, the peak clad temperature during blowdown is not affected by reactor loop characteristics (break flow or pump degradation). It is governed instead by fuel stored energy or, equivalently, by linear heating rate. It depends only slightly on vapor velocity which affects the convective heat transfer coefficient after DNB or dry-out.

3. Fuel stored energy and peak clad temperature during blowdown depend primarily on three fuel-related modeling parameters, with this order of significance: gap conductance, power peaking factor and fuel thermal conductivity. Uncertainties in stored energy and blowdown peaking factors are affected primarily by the uncertainties in these three parameters and potentially by systematic modeling errors in predicting these three parameters.

4. As a result of conclusions 2 and 3, blowdown peak clad temperature measurements are not affected by scaling distortions outside the reactor core (in downcomer, pumps, steam generators, etc.). Since test facilities with nuclear fuel (LOFT etc.) have full-scale fuel pins from the viewpoint of transient radial conduction, it can be stated that blowdown experiments with nuclear fuel yield the blowdown peak clad temperature without scale distortion.

5. The expected uncertainty in fuel stored energy is found to be 26%, based on 1-σ uncertainties in all governing parameters. Based on the same parameter uncertainties, the blowdown peak clad temperature has an uncertainty of 42K (1-σ level).

6. Based on its published documentation, the TRAC-PF1/MOD1 computer code has systematic modeling errors in fuel thermal conductivity, gap conductance and convective film coefficients. The first two errors compensate during steady-state, but not during transient calculations. The third error applies only to two-phase flow and heat transfer conditions. Additional documentation is needed to quantify the effects on peak clad temperature from these errors.
References


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