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ANALYTICAL MODAL DIFFUSION THEORY BASED ON FLUX SEPARABILITY

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ANALYTICAL NODAL DIFFUSION THEORY BASED ON FLUX SEPARABILITY

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The theory provides for an iterative solution of the mathematical problem of generating the assembly-wise power distribution in a LWR through the solution of the 2-group, multidimensional, diffusion equation. The companion problems of assembly pre-homogeneization and of pin power reconstruction [1] are of no direct concern presently.

The theoretical development stems from the assumption of flux separability in X, Y and Z. The assumption derives from the notion that separability holds in a great part of the interior of a LWR assembly. More important, well accurate power maps are generated with a code based on the theoreticial development yielded by the basic assumption.

Let $\underline{\underline{R}}$ denote a 2x2 matrix, $\underline{V}$ denote a 2x1 column vector, and $\underline{\underline{g}}$ dénote a 2x2 diagonal matrix. The mathematical development is briefly outlined next for a 2D case.

The diffusion equation is :

$$\underline{\phi}_{xx} + \underline{\phi}_{yy} + \underline{\underline{M}}\underline{\phi} = \underline{0} \qquad (1)$$

where

$$\underline{\underline{M}} = \begin{pmatrix} 1/D1 & 0 \\ 0 & 1/D2 \end{pmatrix} \begin{pmatrix} \frac{\nu\Sigma f1}{K} - \Sigma 1 - \Sigma 12 & ; & \frac{\nu\Sigma f2}{K} \\ \Sigma 12 & ; & -\Sigma 2 \end{pmatrix} \qquad (2)$$

with usual notation.

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The flux separability assumption is embodied in :

$$\underset{\sim}{\phi} = \underset{\approx}{\S}\ \underset{\tau}{X}\ \underset{\tau}{Y}\ \underset{\sim}{f} \tag{3}$$

$$\left.\begin{aligned} \underset{\tau}{X} &= \underset{\tau}{X}(x) \ ; \ \underset{\tau}{X}'' = -\underset{\tau}{\alpha}^2\ \underset{\tau}{X} \quad ; \quad \underset{\tau}{X}(0) = I \\ \underset{\tau}{Y} &= \underset{\tau}{Y}(y) \ ; \ \underset{\tau}{Y}'' = -\underset{\tau}{\beta}^2\ \underset{\tau}{Y} \quad ; \quad \underset{\tau}{Y}(0) = I \end{aligned}\right\} \tag{4}$$

Substitution of Eqs. (3) and (4) in Eq. (1) leads to three fundamental conclusions

$$\text{(i)}\ \underset{\approx}{\S} = (\underset{\sim}{\S}_1\ ,\ \underset{\sim}{\S}_2) \tag{5}$$

where $\underset{\sim}{\S}_1$ and $\underset{\sim}{\S}_2$ are eigenvectors corresponding to the eigenvalues λ_1^2 and λ_2^2 of the eigenvalue problem

$$\underset{\approx}{M}\underset{\sim}{\S} = \lambda^2\ \underset{\sim}{\S} \tag{6}$$

$$\text{(ii)}\ \underset{\tau}{\alpha}^2 + \underset{\tau}{\beta}^2 = \underset{\tau}{\lambda}^2 = \underset{\approx}{\S}^{-1}\ \underset{\approx}{M}\ \underset{\approx}{\S} \tag{7}$$

namely, the geometrical "buckling" matrices $\underset{\tau}{\alpha}^2$ and $\underset{\tau}{\beta}^2$, for the X and Y members of the solution, add up to a matrial "buckling" matrix, a property of the node.

$$\left.\begin{aligned} \text{(iii)}\ \underset{\sim}{L}_x &= \underset{\tau}{D}\ \underset{\approx}{\S}\ \underset{\tau}{\alpha}^2\ \underset{\approx}{\S}^{-1}\ \underset{\sim}{\phi} \\ \underset{\sim}{L}_y &= \underset{\tau}{D}\ \underset{\approx}{\S}\ \underset{\tau}{\beta}^2\ \underset{\approx}{\S}^{-1}\ \underset{\sim}{\phi} \end{aligned}\right\} \tag{8}$$

where $\underset{\sim}{L}_u$ is the net leakage out of the node through the faces perpendicular to u, and $\underset{\sim}{\phi}$ is the integrated nodal flux.

Eq. (8), providing an analytic expression for the leakage, is in distinction to the customary polynomial approximations employed for transverse leakages [2,3]. This equation is used in a twofold manner. One - to update $\underset{\tau}{\alpha}$ and $\underset{\tau}{\beta}$ at the conclusion of an iteration, using the numerical values for $\underset{\sim}{L}$ and $\underset{\sim}{\phi}$ gotten in that iteration. Two - to formally decouple the solution scheme for a line of nodes from fluxes in the transverse direction.

This latter scheme, for a solution along a line of nodes, derives from tri-flux algebric relation (bi-flux at boundaries or reflection planes) obtained by applying continuity of the total current and total flux at nodal interfaces. Thus the overall structure of the iteration procedure is a basic gaussian elimination of fluxes along a line, imbedded in a scheme for marching, alternatety, in rowes and columns. After a sweep of all rows and columns K is reevaluated, hence M updated, hence the geometrical bucklings α and β updated (with adherance to Eq. (7)), and the next iteration commenced.

The above development is applied in the code NOXER (Nodal Diffusion by Flux Separability). As of this writing the 2D option is complete and work on 3D is in progress. The convergence of assembly powers is accelerated by the use of the convergence ratio for extrapolation. The power distribution figures, normalised to an average of 1, converge to a promile in a manner of 12 to 15 iterations. For an 1/8 core symmetry this consumes $\sim$ 0.25 seconds on a CRAY1 XMP, or $\sim$ 5 seconds on an IBM 4361. The accuracy, compared with a high-order fine-mesh finit element calculation [4] is exemplified in the two figures.

Complete line decoupling renders the program well suited for paralel processing and potentially even more efficient. And the replacement of the continuity of the integrated face flux with the continuity of a judiciously chosen weighted face flux integral will render the method even more accurate.

Fig. 1 : The IAEA 2D BENCHMARK PROBLEM

K_∞	= K_∞ of loaded assembly
P	= (Power by a cubic^2 finit element in each 1/4 assembly) X1000
$\|\Delta P\|$	= | (P) NOXER - P | (NOXER calc. 1 mesh per assembly)

$|\Delta P|$ average = 1.1 %

$|\Delta P|$ max = 3.8 %

				1.11 592 38		
			0.69 473 14	1.11 688 16	1.11 605 5	
		1.05 1193 4	1.05 967 16	1.05 908 11	1.11 850 11	
	1.05 1469 3	1.05 1345 10	1.05 1179 8	1.05 1070 8	1.11 974 2	1.11 692 11
1.05 1434 3	1.05 1479 12	1.05 1314 2	1.05 1069 10	1.05 1035 6	1.05 948 14	1.11 731 21
0.69 747 31	1.05 1308 9	1.05 1453 7	1.05 1210 9	0.69 611 27	1.05 933 2	1.05 931 2	1.11 749 17

Fig. 2 : A TYPICAL ZION-2 LOADING

K_∞	= K_∞ of loaded assembly
P	= (Power by a cubic^2 finit element in each 1/4 assembly) X 1000
$\|\Delta P\|$	= |(P) NOXER - P| (NOXER calc. 1 mesh per assembly)

$|\Delta P|$ average = 1.1 %

$|\Delta P|$ max = 4.9 %

				1.28	1.15	
				972	490	
				49	28	
			1.23	0.92	1.28	
			1169	792	843	
			21	8	31	
			1.17	0.92	1.17	1.08	1.28
			1135	932	1106	951	605
			3	10	5	11	5
		1.17	0.97	1.17	1.03	1.17	1.14
		1121	1043	1221	1173	1092	718
		2	12	1	7	10	3
	1.17	0.92	1.17	0.97	1.17	0.98	1.28
	1088	920	1189	1137	1295	1030	863
	3	18	1	18	1	29	7
1.17	0.97	1.17	0.97	1.17	1.03	1.17	1.14
1123	986	1111	1070	1310	1252	1153	793
6	15	3	16	2	14	3	12

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