ABSTRACT

The expansion of a multilayer plane target driven by an ion beam which has a range shorter than the thickness of the slab is described by means of a simple analytic model. The effect of a two-layer structure is studied and criteria for the optimization of the kinetic energy of the unheated part of the slab, the payload, are set.

The purpose of this paper is to develop a simple analytical model for the expansion of a plane target with a two-layer absorber and to find physical criteria for the optimization of the payload hydrodynamic efficiency. It is expected that these criteria can be applied also to spherical targets.

Piriz (1987) has shown that a target with uniform absorber expands in such a way that the center of mass surface remains at rest during the expansion. This behavior is due to the fact that...
the ion beam energy was assumed to be uniformly deposited in the absorber, which is a good approximation for heavy ions, and it is also consistent with a linear profile for the expansion velocity in the absorber (DEVORE et al., 1984; LONG and TAHIR, 1986a and 1986b; PIRIZ, 1987; STANYUKOVICH, 1960). Thus, the region ahead of the center of mass surface was defined as the tamper. From these results it becomes evident that the payload hydrodynamic efficiency can be increased if the tamper is composed by a high-Z material because in this way less energy is absorbed in the tamper, and as the momentum must be conserved, more energy is transferred to payload kinetic energy.

2. THE EQUATIONS OF THE MODEL

In order to show how the hydrodynamic efficiency is optimized, a previous model (PIRIZ, 1987) is modified here to describe the expansion of a two-layer absorber, and successive amounts of a light absorber on the front side are replaced by thickness of a high-Z material in such a way that the end of the ion range of energy $E_0$ remains as a constant. As in previous models, for simplicity, a constant intensity $W_0$ (in W/cm$^2$) irradiating the target is assumed, and the radiation loss, shortening of the range with the temperature increase, and ablation driven by thermal radiation are neglected.

The heating of the absorber is considered to be uniform in every material, and consistently the profile of the expansion velocity will be a linear one in each material. Of course, the slopes of each straight line can be different in the two materials and they will change in the time. However, from previous models (LONG and TAHIR, 1986a and 1986b, PIRIZ, 1987) can be seen that the slopes must be the same in each material for relatively short times, when the velocity is even uniform and close to zero, and also for relatively long times when they become a decreasing function of the time (as $t^{-1}$). During an intermediate period, the slopes are different and, as a consequence of the momentum conservation, the center of mass surface changes its position during this time and afterwards it goes back to its original position. Nevertheless, according to the simulation results for targets with a two-layer absorber (TAHIR and LONG, 1986; METZLER and MEYER-TER-VENN, 1984; VELARDE et al., 1986), the center of mass surface moves slightly from its original position and then, it is a reasonable approximation to consider that it remains at rest in every time during the expansion of the absorber. Therefore, the same slope of the velocity profile will be taken in the two materials and the position $x_{CM}$ of the center of mass is:

\[
N_{CM} = \frac{x_{Np}}{x_{NL}} + \frac{x_{NL}}{x_{NL}}
\]

where $x_{Np}$, $x_{NL}$, and $x_{N}$ are the initial positions of the payload surface, the interphase surface between the heavy and light parts of the absorber, and the front surface of the absorber respectively (Fig.1), $\rho_h$ and $\rho_l$ are the initial densities of the heavy and light parts, $m_p$ is the payload mass in g/cm$^3$ and $N_T = m_p + m_h + m_l$ is the target total mass in g/cm$^3$, $m_h = \rho_h(x_{NL} - x_{N})$ and $m_l = \rho_l(x_{Np} - x_{NL})$.

Taking the origin of coordinates on the center of mass surface, that is $x_{CM} = 0$, Eq.(1) yields:

\[
x_{Np} = \frac{[(m_h + m_l)d_L + m_h(d_h + d_L)]}{2N_T}
\]
where \( d_L = x_{p10} - x_{p0} \) and \( d_R = x_{p0} - x_{v0} \), and they are related by the condition that a change in \( d_R \) must be compensated by a change in \( d_L \) in order to make the end of the range, for a given energy \( E_0 \), remain on the payload surface. Then, as the energy deposition is constant in each material, the energy \( E_B \) of the ions on the surface \( x_B \) will be (TAHIR and LONG, 1983 and 1986; VELARDE et al., 1984; METZLER and MEYER-TER-VEN, 1984; LONG and TAHIR, 1986a and 1986b):

\[
E_B = E_0 \left[ 1 - \frac{d_R}{R_H(E_0)} \right]
\]  

(3)

where \( R_H(E_0) \) is the thickness of heavy material necessary to stop ions with energy \( E_0 \). On the other hand, taking the ion range in the light material as a power function of the incident energy, the thickness of light material to stop ions with energy \( E_B \) is:

\[
R_L(E_B) = a E_B^b
\]

(4)

and due to the fact that \( d_L = R_L(E_B) \):

\[
d_L = R_L(E_0) \left[ 1 - \frac{d_R}{R_H(E_0)} \right]^b
\]

(5)

Now, the energy balance for the internal region of the absorber \( 0 \leq x \leq x_{p1} \) can be written in a similar way than PIRIZ (1987) (see also SAYASOV, 1984):

\[
\frac{d}{dt} \left[ E_K + \frac{X_{p1}}{\gamma - 1} \right] = -P_{pl} v_{p1} + dW_0
\]

(6)

where \( v_{p1} = dx_{p1}/dt \) and \( x_{p1} \) are the payload velocity and position, \( P_{pl} \) is the thermal pressure on the payload, \( \gamma \) is the enthalpy coefficient (\( \gamma = 5/3 \)), \( \bar{P} \) and \( E_K \) are the mean pressure and the kinetic energy (per unit of area) of the absorber internal region:

\[
\bar{P} = \frac{1}{x_{p1}} \int_{0}^{x_{p1}} P \, dx
\]

(7)

\[
E_K = \frac{1}{2} \int_{0}^{x_{p1}} P_{pl} v_{p1}^2 \, dx
\]

(8)

and \( \omega \) is the fraction of the beam energy deposited into the region \( 0 \leq x \leq x_{p1} \), and since uniform absorption was assumed:

\[
\alpha = \begin{cases} 
1 - \frac{d_L}{R_H(E_0)} & \text{if } d_L < x_{p10} \\
1 - \frac{d_L + d_R - x_{p10}}{R_H(E_0)} & \text{if } d_L \geq x_{p10}
\end{cases}
\]

(9)

with \( R_H(E_0) \).

On the other hand, the conservation of momentum on the payload surface gives:

\[
\frac{dv_{p1}}{dt} = P_{pl}
\]

(10)

Assuming uniform pressure in the absorber, uniform density in each material, and linear profile of the expansion velocity, according to the simulation data (DEVORE et al., 1984; LONG and TAHIR, 1986a and 1986b), Eqs. (7) and (8) yield:

\[
\bar{P} = P_{pl}
\]

(11)

\[
E_K = \left( m_{p1} v_{p1}^2 / 2 \right) F
\]

(12)

where

\[
F = \frac{m_{pl} x_{p10}}{3 m_{p1} d_L} \left[ 1 + H(x_{p10} - d_L) \left( \frac{p_L}{P_{pl}} - 1 \right) \left( 1 - \frac{d_L}{R_H(E_0)} \right) ^3 \right]
\]

and \( H(z) = \begin{cases} 
0 & \text{if } z \leq 0 \\
1 & \text{if } z > 0
\end{cases} \) is the Heaviside's step function.

Introducing Eq. (10) into Eq. (6) and using Eqs. (11) and (12),
a first integration is performed:

\[
\frac{1}{2} (1 + F) m_p v_{p1}^4 + \frac{3}{2} m_p x_{p1} \frac{dv_{p1}}{dt} = \alpha w_0 t
\]  

(13)

As was noted by PIRIZ (1987), this equation should be solved with the initial condition of \( v_{p1}(t = t_0) = u_p \), where \( u_p \) is the shock velocity and \( t_0 \) is the time in which the payload begins to be uniformly accelerated (COOPER, 1973), but it is a good approximation to use Eq.(13) also for times \( t \geq t_0 \) and to adopt the following initial conditions:

\[
v_{p1}(t = 0) = 0 ; \quad x_{p1}(t = 0) = x_{p10}
\]

(14)

Introducing the following dimensionless variables:

\[
\xi = x_{p1} / x_{p10} ; \quad T = t / t_0 ; \quad P = p_{p1} / p_0
\]

(15)

where,

\[
t_0 = \left( \frac{m_p x_{p10}^3}{2 \alpha w_0} \right)^{1/3}
\]

(16)

\[
p_0 = \left( \frac{m_p}{x_{p10}^3} \right)^{1/3} (2 \alpha w_0)^{1/3}
\]

(17)

the Eqs.(10), (13) and (14) can be rewritten:

\[
\frac{T - (1 + F) \xi^4}{3 \xi} = \frac{\alpha}{2 + F}
\]

(18)

\[
P = \xi
\]

(19)

\[
\xi(0) = 1 ; \quad \xi(0) = 0
\]

(20)

The dots indicates temporal derivatives.

The Eqs.(18) to (20) now contain the dependence of the target expansion on the composition of the absorber, besides the dependences on the payload mass and on the ion energy, and then it is possible to use it in order to find the optimum thickness of the heavy material which gives the maximum hydrodynamic efficiency of the payload.

3. RESULTS AND CONCLUSIONS

Defining the hydrodynamic efficiency as \( \eta = E_{p1} / E_b \), where \( E_{p1} = m_p v_{p1}^2 / 2 \) is the payload kinetic energy and \( E_b = \alpha w_0 t \) is the beam energy, and using the dimensionless variables defined by Eq.(15), \( \eta \) will be:

\[
\eta = \frac{\alpha \xi^4}{2 + F}
\]

(21)

For simplicity, here will be considered only the situation for long times because in that case Eqs.(18) to (20) admit an analytical solution and it does not affect the main conclusions. Then, for \( T \geq 1 \):

\[
\xi = 2^{3/2} (2 + F)^{1/3}
\]

(22)

\[
\eta = \frac{\alpha}{2 + F}
\]

(23)

and \( \eta \) results to be:

To study the effect of replacing successive amounts of a light absorber by thicknesses of a heavy material, it is necessary to specify both of them, and therefore to give their densities \( \rho_L \) and \( \rho_H \) and the ranges for a given kind of ions of energy \( E_0 \). Here, as an example, it is chosen Pb for the heavy material (\( \rho_H = 11.3 \text{ g/cm}^3 \)) and PbLi alloy for the light material (\( \rho_L = 1.26 \text{ g/cm}^3 \)), and Bi-ions with incident energy \( E_0 = 10 \text{ GeV} \) are considered (TAHIR and LONG, 1983; METZLER and MEYER-TER-VEHN, 1984). The values of
R_\text{L} (E_p = 10 \text{ GeV}) and the coefficients a and b of Eq.(4) are inferred from the data given by METZLER and MEYER-TER-VRHN, (1984), being R_\text{L} (10 \text{ GeV}) = 1.58 \times 10^{-3} \text{ cm}, b = 1.18 and a = 10^{-3} \text{ cm}/(\text{GeV})^{1.18}.

In Fig.2 \eta is represented as a function of the ratio d/x_{p10} for different payload masses. For the construction of these curves, successive thicknesses of PbLl on the front side were replaced by Pb in such a way that the ions were stopped always on the payload surface. As can be seen, in all the cases the maximum hydrodynamic efficiency is achieved for d_{\text{L}} = x_{p10}, when the interphase between the heavy and light material is on the center of mass surface. On the other hand, Fig.2 shows that for a given value of d_{\text{L}}/x_{p10}, that is in this case, for a fixed absorber mass, there exists an optimal value of x_{p1} which gives the maximum \eta, such as it was found for targets with uniform absorber (PIRIZ, 1987). This result is clearly seen in Fig.3 where \eta was represented as a function of \text{x}_{\text{p1}}/M_\text{T} for d_{\text{L}} = x_{p10}. In Fig.3 it is also shown, for comparison, the behavior of \eta for the case of uniform absorber, and as can be seen, the effect of the heavy tamper is to increase the value of \eta by a factor of about two.

These results are not very sensitive to the exact values of the parameters R_{\text{L}} (10 \text{ GeV}), a and b. In fact, it is not very difficult to see that changes in those parameters do not affect the position of the maximum value of \eta in Figs.2 and 3, and the absolute values of \eta are not very sensitive to those changes. Actually, \eta depends on b and the ratio R_{\text{L}}/R_{\text{H}}, and changes of \pm 50% in these parameters around the considered values modify \eta in about \pm 3% and \pm 10% respectively.

Figs.2 and 3 show that payload hydrodynamic efficiencies of about 30% can be achieved in the interaction of a heavy ion beam with a multilayer plane target, in agreement with the simulation results reported by DEVORE et al. (1984). In conclusion, simple criteria for the design of an optimized target achieving high hydrodynamic efficiency can be set: the interphase between the heavy and light parts of the absorber must be close to the center of mass surface; or in other words, the tamper is the region ahead of the center of mass surface and it is convenient to construct it with a high-Z material. Besides, the payload mass must be about 10% to 30% of the target total mass (Fig.3).

It is expected that these two conditions can be also applied to spherical shell targets and it is noted that they are satisfied in the numerical optimized targets reported in the literature (TAHIR and LONG, 1983 and 1986; VELAREDE et al., 1984; METZLER and MEYER-TER-VRHN, 1984).

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FIGURE CAPTIONS

Fig. 1- Schematic diagram of the beam-target geometry.

Fig. 2- Hydrodynamic efficiency \( \eta \) versus the ratio \( d/L \) for different payload masses.

Fig. 3- Hydrodynamic efficiency \( \eta \) versus the mass ratio \( m_1/M \) for:
  a) target with uniform absorber,
  b) target with a high-Z tamper.
Fig. 1

Ion beam

Heavy material
Light material
Payload

x₀ x₀ x₀ xₚ₀

Absorber

Fig. 2

\( \eta [%] \)

\( m_{pl} = 2 \times 10^{-2} \text{ g/cm}^2 \)

\( m_{pl} = 3 \times 10^{-3} \text{ g/cm}^2 \)

\( m_{pl} = 10^{-1} \text{ g/cm}^2 \)

\( d_L / x_{plo} \)
Fig. 3

\[ d_L = x_{pl0} \]

\( \eta(\%) \)

\[ m_{pl}/M_T \]

uniform absorber

a)

b)