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ROLE OF CURRENT ALGEBRA IN $B \to p\bar{p}\pi(\pi)$ *

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ABSTRACT

We estimate in a current algebra framework the hadronic matrix elements relevant to the weak $B \to p\bar{p}\pi(\pi)$ transitions measured recently. The results so obtained are in agreement with the observed values.

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Recently the ARGUS Collaboration have claimed direct evidence of $b \to u$ transitions, by observing the $B^+ \to p\bar{p}\pi^+$ and $B^0 \to p\bar{p}\pi^+\pi^-$ non-leptonic decays [1]. Clearly this experimental result is of great importance in order to complete our knowledge of the quark mixing matrix elements $V_{q_iq_j}$ [2], and thus to severely test the Standard Model of electroweak interactions. Indeed, from the measured branching ratios

$$BR(B^+ \to p\bar{p}\pi^+) = (3.7 \pm 1.3 \pm 1.4)\,10^{-4} \tag{1}$$

$$BR(B^0 \to p\bar{p}\pi^+\pi^-) = (6.0 \pm 2.0 \pm 2.2)\,10^{-4} \tag{2}$$

the following lower bound has been derived [1]:

$$\left|\frac{V_{bu}}{V_{bc}}\right| \geq 0.07. \tag{3}$$

In the familiar valence quark model framework the estimate of the hadronic matrix elements of H_W relevant to the decays above is complicated, as usual, by the complexities of nonperturbative physics, in particular by the role of quark hadronization into baryons, which is not simple to parametrize phenomenologically.

In the present note we would like to point out that, alternatively, the scale of those matrix elements can be assessed in a straightforward way by using current algebra and PCAC techniques. In addition to being rather general and suited for processes involving pions, these techniques have the advantage that with a minimum of assumptions the desired matrix elements can be expressed directly in terms of well-defined phenomenological constants, for which either experimental measurements or theoretical determinations are available. As we will see, the resulting estimates are in good agreement with eqs.(1) and (2), once the values of V_{bc} and V_{bu} are chosen consistently with the present experimental indications [3], including eq.(3).

In short, in the current algebra approach the PCAC-reduced amplitude for the transition $B^+\to p\bar{p}\pi^+$ can be written as

$$M(B^+\to p\bar{p}\pi^+) \simeq -\frac{i}{\sqrt{2}F_\pi}\langle p\bar{p}|[\mathcal{J}^{I^-}, H_W]|B^+\rangle +$$

$$+ M_{Born} + \lim_{q_\lambda\to 0}\left[\frac{i}{\sqrt{2}F_\pi}q^\lambda\Gamma_\lambda - M_{Born}\right], \qquad (4)$$

where $F_\pi = 93$ MeV, $\mathcal{J}^I$ is the isotopic spin vector charge, and

$$\Gamma_\lambda = i\int d^4x\, e^{iq\cdot x}\langle p\bar{p}|T A_\lambda^{I^-}(x)\, H_W(0)|B^+\rangle, \qquad (5)$$

with $A_\lambda(x)$ being the familiar axial vector current with the pion quantum numbers.

In eqs.(4) and (5) H_W is the $|\Delta b| = 1$ non-leptonic weak interaction hamiltonian density. Neglecting Penguin interactions, H_W has the current-current form [4,5]:

$$H_W = \frac{G_F}{\sqrt{2}}V_{bu}V_{ud}\left[c_1(\bar{u}b)_L(\bar{d}u)_L + c_2(\bar{d}b)_L(\bar{u}u)_L\right] + h.c., \qquad (6)$$

where $(\bar{q}q)_L = \bar{q}\gamma_\mu(1-\gamma_5)q$, and the coefficients $c_1 \sim 1.15$ and $c_2\sim -0.35$ account for short distance perturbative QCD corrections (in the limit of free quarks one would have $c_1=1$ and $c_2=0$).

Eq.(4) can easily be derived by standard Ward identity techniques [6], using the fundamental property of the left-handed hamiltonian (6):

$$[\mathcal{J}_5, H_W] = -[\mathcal{J}, H_W], \qquad (7)$$

where $\mathcal{J}_5$ represents the axial charge.

Using

$$\mathcal{J}^{I^-}|B^+\rangle = |B^0\rangle, \qquad (8)$$

$$\mathcal{J}^{I^+}|(p\bar{p})_{I=1}\rangle = -\sqrt{2}\,|p\bar{n}\rangle;\quad \mathcal{J}^{I^+}|(p\bar{p})_{I=0}\rangle = 0, \qquad (9)$$

eq.(4) becomes:

$$M(B^+\to p\bar{p}\pi^+) \simeq \frac{i}{\sqrt{2}F_\pi}\left[\sqrt{2}\langle p\bar{n}|H_W|B^+\rangle + \langle p\bar{p}|H_W|B^0\rangle\right] +$$

$$+ M_{Born} + \lim_{q_\lambda\to 0}\left[\frac{i}{\sqrt{2}F_\pi}q^\lambda\Gamma_\lambda - M_{Born}\right]. \qquad (10)$$

We now model the $\langle N\bar{N}|H_W|B\rangle$ matrix elements in eq. (10) through the pion-exchange, tadpole-like diagram represented in Fig.1.

In the framework of Fig.1 the first matrix element needed in eq.(10) has an enhancement factor with respect to the second one, viz.

$$\frac{\langle p\bar{n}|H_W|B^+\rangle}{\langle p\bar{p}|H_W|B^0\rangle} \sim \sqrt{2}\,\frac{\langle\pi^+|H_W|B^+\rangle}{\langle\pi^0|H_W|B^0\rangle} \sim \sqrt{2}\,\frac{c_1}{c_2} \gg 1, \qquad (11)$$

having taken eq.(6) into account. Moreover, the analogous isoscalar η-exchange diagram should be suppressed, compared to the π-exchange, by $g_{\eta NN} \ll g_{\pi NN}$. This justifies the neglect of the $(p\bar{p})_{I=0}$ channel tacitly made in eq.(10). Therefore the $B^+\to p\bar{p}\pi^+$ transition amplitude will be entirely determined in practice by $\langle p\bar{n}|H_W|B^+\rangle$, since both M_{Born} and $\lim_{q_\lambda\to 0}\left[\frac{i}{\sqrt{2}F_\pi}q^\lambda\Gamma_\lambda - M_{Born}\right]$ arise due to the $\bar{n}$ pole, and thus also depend on that matrix element only. From Fig.1, the latter can be parametrized as:

$$\langle p\bar{n}|H_W|B^+\rangle = i\beta\,\bar{u}(p)\gamma_5 v(\bar{p}), \qquad (12)$$

where

$$\mathcal{B} = \langle \pi^+ | H_W | B^+ \rangle \frac{1}{m_B^2 - m_\pi^2} \sqrt{2}\, g_{\pi NN} . \tag{13}$$

As is well known $\left[\frac{i}{\sqrt{2} F_\pi} q^\lambda \Gamma_\lambda - M_{Born} \right]$ has a definite limit as $q_\lambda \to 0$, and it is easy to see that this limit together with M_{Born} in eq. (10) finally gives

$$M(B^+ \to p\bar{p}\pi^+) \simeq \frac{\mathcal{B}}{\sqrt{2} F_\pi} \bar{u}(p) \left[\gamma_5 + \frac{g_A}{\sqrt{2}} \left(1 + \frac{2 m_N \not{q}}{2\bar{p}\cdot q + q^2} \right) \right] v(\bar{p}) , \tag{14}$$

where $g_A \sim 1.25$ is the β-decay axial coupling constant.

Before proceeding further we wish to remark that according to previous experience [7] PCAC extrapolation corrections, which arise when in the Ward identity (10) only the limit $q^2 = m^2_\pi \to 0$ is taken instead of $q_\lambda \to 0$, should not abscure the order of magnitude of the matrix elements set by the "leading" current algebra terms of eq.(14). The point is that in the present case of three-body decay the final state following from the soft pion limit is a two-body state, so that $q_\lambda \to 0$ does not necessarily imply $m_i = m_f$ as in the case of two-body decays. Consequently, the effects of such corrections should not be so serious as they might be in two-body decays and we, therefore, neglect them.

Evaluating the tadpole-like transition $\langle \pi^+ | H_W | B^+ \rangle$ in eq.(13) by the familiar vacuum intermediate state saturation of current-current products [8]*, eq.(14) turns into the final expression

$$M(B^+ \to p\bar{p}\pi^+) = \mathcal{A}\, \bar{u}(p) \left[\gamma_5 + \frac{g_A}{\sqrt{2}} \left(1 + \frac{2 m_N \not{q}}{2\bar{p}\cdot q + q^2} \right) \right] v(\bar{p}) , \tag{15}$$

* The factorization of matrix elements in the context of heavy meson two-body decays is discussed in detail e.g. in Ref.[9]. Actually our factorization assumption is weaker, as it is applied here to single particle matrix elements of H_W only and has proven quite successful in the current algebra treatment of $D \to K\pi$ decays [10].

with

$$\mathcal{A} \simeq G_F V_{bu} V_{ud}\, c_1 \left(\frac{2 F_B}{F_\pi} \right) F_\pi\, g_{\pi NN} . \tag{16}$$

In terms of eq.(15) the partial width is given by

$$\Gamma(B \to p\bar{p}\pi) = \frac{m_B^3 |\mathcal{A}|^2}{768 \pi^3} \cdot \left(1 + \frac{g_A^2}{2} \right) \cdot \left[1 - \frac{3}{2} \frac{4 m_N^2}{m_B^2} + O\left(\frac{4 m_N^2}{m_B^2} \right)^2 \right] . \tag{17}$$

In eq.(16) F_B is the (yet unmeasured) B-meson leptonic decay constant, analogous to F_π. As a reasonable range of values, which encompasses a number of theoretical estimates, we may take $F_B/F_\pi \sim 1 - 1.6$ [11].

To appreciate the order of magnitude predicted by eqs. (15)-(17) we take as an example the values $V_{bu} = 0.07\, V_{bc}$; $V_{bc} = 0.05$; $V_{ud} = 0.97$; $F_B = 1.2\, F_\pi$ and $g_{\pi NN} = 13$. We find, correspondingly,

$$\Gamma(B^+ \to p\bar{p}\pi^+) \big|_{th} \simeq 1.6 \; 10^{-16} \text{ GeV}, \tag{18}$$

or using $\Gamma_B \simeq 4.5 \; 10^{-13}$ GeV:

$$BR(B^+ \to p\bar{p}\pi^+) \big|_{th} \simeq 3.4 \; 10^{-4} , \tag{19}$$

which agrees rather nicely with the experimental determination of eq.(1)*.

Regarding the $B \to p\bar{p}\pi\pi$ transition, the analysis gets complicated by the presence of more than one pion in the final state. In this case we attempt a gross estimate by rescaling the $B \to p\bar{p}\pi$ amplitude by $\sqrt{2}\, F_\pi$ (as pertinent to pion emission), and by taking the ratio of four-body to three-body phase space. With the

* It might be interesting to observe that, with the same choice of input parameters, we would obtain from Fig.1 $BR(B^+ \to p\bar{n}) \sim 10^{-4}$.

aid of standard kinematical formulae we would obtain in this way:

$$\frac{\Gamma(B\to p\bar{p}\pi\pi)}{\Gamma(B\to p\bar{p}\pi)} \sim \frac{1}{12}\left(\frac{m_B}{4\pi\sqrt{2}F_\pi}\right)^2\left(1+O\left(\frac{4m_N^2}{m_B^2}\right)\right) \simeq 0.9 . \tag{20}$$

Combining eqs.(20) and (19) :

$$BR(B^\circ\to p\bar{p}\pi^+\pi^-)\Big|_{th} \simeq 3\ 10^{-4}, \tag{21}$$

which is in the range allowed by eq.(2). This encouraging result suggests that such a simplified procedure should be applicable to obtain predictions for the general multipion case as well.

We can thus conclude, summarizing our results, that the application of current algebra techniques allows simple, and sensible estimates of the $b\to u$ nonleptonic weak transitions measured recently. Within the uncertainties on some input parameters, the theoretical predictions so obtained nicely match the observed scales of the relevant hadronic matrix elements. In particular, eq.(16) clearly shows the interest of a measurement of the leptonic decay constant F_B.

A more detailed analysis would require to include in our simple treatment the resonance formation in the $B\to p\bar{p}\pi\ (\pi)$ final state. This effect is expected, and indeed is observed. From this point of view the results obtained here, as they stand, should be understood either as an overall matrix element scale assessment or, in more detail, as referring to the transitions into non-resonating "continuum" states. It should thus be interesting to try to distinguish experimentally the branching ratios into the resonant and the non resonant decay channels respectively.

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REFERENCES

[1] W.Schmidt-Parzefall, Talk given at the 1987 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, to be published in the Proceedings.

[2] M.Kobayashi and T.Maskawa, Progr.Theor.Phys. 49 (1973) 652.

[3] See e.g. Ref [1] and:
E.H.Thorndike and R.A.Poling, University of Rochester report UR-1022 (1987), to be published in Physics Reports;
P.Avery, University of Florida report UFTP-EXP-87-1 (1987), to appear in the Proceedings of the UCLA $B\bar{B}$ Factory Workshop, Los Angeles, 1987.

[4] M.K.Gaillard and B.W.Lee, Phys.Rev.Lett. 33 (1974) 108;
G.Altarelli and L.Maiani, Phys.Lett. 52B (1974) 351.

[5] For a review and further references see:
R.Rückl, Habilitationsschrift submitted to the University of Munich (1983).

[6] A review of the method can be found e.g. in:
R.E.Marshak, Riazuddin and C.P.Ryan: "Theory of Weak Interaction in Particle Physics", Wiley-Interscience, New York, 1969;
V.de Alfaro, S.Fubini, G.Furlan and C.Rossetti : "Currents in Hadron Physics", North-Holland, Amsterdam, 1973.

[7] C.A.Dominguez, N.Paver and Riazuddin, Phys.Lett. 182B (1976) 391.

[8] M.K.Gaillard and B.W.Lee, Phys.Rev. D10 (1974) 897.

[9] M.Bauer, B.Stech and M.Wirbel, Z.Phys. C34 (1987) 103;
A.J.Buras, J.M.Gerard and R.Rückl, Nucl.Phys. B268 (1986) 16.

[10] A.N.Kamal and M.D.Scadron, Phys.Rev. D32 (1985) 1164.

[11] For recent reviews and references see e.g.:
J.Körner, in Proceedings of the International Symposium on Production and Decay of Heavy Hadrons, Heidelberg, 1986;
A.Ali, DESY report 87-083 (1987), to appear in the Proceedings of the UCLA $B\bar{B}$ Factory Workshop, Los Angeles, 1987;
C.A.Dominguez and N.Paver, DESY report 87-060 (1987), to be published in Physics Letters B.

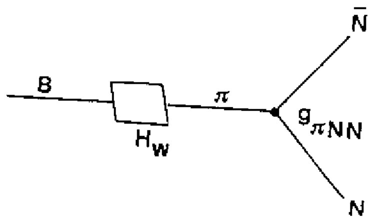

Fig.1

Diagram representing the mechanism used to estimate eq.(10).