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MONOPOLE STRENGTH AS A PROBE OF NUCLEAR SHAPE MIXING

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ABSTRACT

The monopole strength, MS, within a single set of nuclear shape excitations is compared with the MS between different shapes. After misconceptions are pointed out concerning the spin dependence of B(E2) values, MS properties are juxtaposed with gamma-ray and beta-decay properties of ^{70}Se, ^{96}Zr, ^{102}Pd, and the N=60 isotones to illustrate the utility of combined investigations and evidence is given for the observation of a two-phonon octupole multiplet. Finally, consideration is given to the dominance of the $^{3}S_{1}$ force in producing deformation in the N>50 1g nuclei.

1. Introduction

Monopole strength (MS) in nuclei has been well studied in the context of deexcitation within a particular nuclear shape and its excitations (e.g. the deexcitation of a beta vibration within a symmetric rotor description). However, there appears to be a poor understanding, to the point of misconception, concerning the role of MS with respect to its use as a probe of coexisting shapes. In part, this may arise from overlooking the fact that the MS is dependent on off-diagonal matrix elements rather than diagonal matrix elements, as is the case for most nuclear transitions. Thus, in a symmetric rotor, the breathing mode (or beta vibration) with a different radius from the ground state (gs) yields a large MS. Yet the MS is **strictly zero** for transitions between unmixed coexisting states that possess a very large difference between their radii. Thus one of the smallest known MS values was measured from the ^{238}U coexisting superdeformed band head (previously known as a fission isomer) to the ^{238}U gs.[1] Only when strong mixing occurs between two structures having different radii does large MS occurs. This is well illustrated by the MS between the strongly mixed spherical excited state of ^{100}Zr and its symmetric rotor gs. After a review of MS and illustration of its use in probing the mixing of shapes in nuclei, illustrations of the importance of combined transition strength investigations are given, including recent investigations by the JuBBuLLi collaboration whose principle investigators are:

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2. Monopole Strength

Although there was an initial intense exploration of MS in nuclei, there has been a dearth of recent literature that summarizes the various approaches. Kantele[2] has given the most complete recent survey of techniques used in the measurement of MS, summarized the extensive and elegant work performed by the Jyvaskyla group, and given a summary of the equations central to calculating the MS. Recently, we have explained in detail MS's and isotope shifts as a measure of nuclear shape mixing.[3] While details can be found in these reports, here we present a cursory review of some of the more useful formulas vis-a-vis the MS as a probe of nuclear shape mixing.

First, it should be recalled that unlike other multipole processes, the monopole interaction strictly vanishes except within the nucleus, where E0 transitions occur via the Coulomb coupling of the atomic electrons and the nuclear protons. All nuclear structure information is contained within the MS parameter ρ, which is defined as $\rho = \langle 0^+(f)|M(E0)|0^+(i)\rangle/eR^2$, where R is the nuclear radius ($1.2A^{1/3}$ fm). While protons are usually considered, neutrons are incorporated by the use of effective charges, which results in $M(E0) = e\Sigma_j r_j^2$ as the form of the monopole operator. Further, because the monopole (E0) transition probability, B(E0), is related to ρ by $B(E0) = e^2R^4\rho^2$, the quantity ρ^2 is commonly used. A good example of this is given by Kantele[2] who shows that in a typical case a 1 MeV E0 transition in $_{64}$Gd having a ρ^2 value of 0.01 has a partial half-life for the $0^+(2) \rightarrow 0^+(1)$ E0 K-conversion transition of 1 ns, which is much slower than the competing $0^+(2) \rightarrow 2^+(1)$ E2 transition with 1 Weiskopf unit (1 Wu) of speed, which would be about two orders of magnitude faster.

When the half-life of the level has not been measured, common practice has been to use the dimensionless ratio X_{ijk} =

$B(E0;0^+_i \to 0^+_j)/B(E2;0^+_i \to 2^+_k) = e^2R^4\rho^2/B(E2)$, where the experimental ratio X(E0/E2) is determined from the relative E0 and E2 conversion (electron or internal pair) intensities. However, because some types of states can have highly hindered E2 transitions, X_{ijk} values are not necessarily indicative of large MS. Although in the shell model low energy E0 transtions are strictly forbidden, they can occur as a result of pairing correlations,[4] where for two mixed single particle orbits j and k:

$$|0_i^+\rangle = a(j^2)_0 + b(k^2)_0 \text{ and } |0_f^+\rangle = -b(j^2)_0 + a(k^2)_0,$$

one has: $\langle 0_f^+|M(E0)|0_i^+\rangle = 2eab(\langle r^2\rangle_j - \langle r^2\rangle_i)$,

and for complete mixing (thus a = b): $\langle M(E0)\rangle_{sp} = M_{sp} = 1.0A^{1/3}\text{e-fm}^2$,

which leads to the harmonic oscillator result proposed by Bohr and Mottelson[4] of

$$\rho^2(sp) = (M_{sp}/eR^2)^2 = 0.49A^{-2/3}.$$

<u>2.1 Monopole Strength Units</u> Because the term ρ^2 generally is much less than one, it has become standard practice to use a milli scale. Consequently, we define the unit <u>"milli monopole unit", mmu</u>, where mmu = $10^3\rho^2$ for the MS.

<u>2.2 Single-Mode Model-Dependent Values</u> On order to form a basis of comparison for the MS that is observed in nuclei with coexisting shapes, it is first necessary to establish the MS expected within a single excitation mode. In this section we review the strength as predicted for deexcitation within a single nuclear mode (model).

<u>2.2.1 Vibrational Mode</u> For nuclei that are not symmetric rotors, perhaps the best MS estimate to use as a guide comes from the model for harmonic quadrupole motion where, to the lowest order, the E0 operator can be written as:[2-4] $T(E0) = (3/4\pi)ZeR^2\Sigma_\mu|\alpha_\mu|^2$ where the α_μs are the expansion coefficients for the

nuclear surface. This gives the selection rule $\Delta n = 0, \pm 2$ where n is the number of quadrupole phonons. In nuclei where there is either no coexisting state or, no or little mixing with coexisting states, the E0 strength for deexcitation of the 0^+ two-phonon state ($0^+(2)$) to the zero-phonon ($0^+(1)$) state is given by the spherical vibrator value of: $\rho^2(sv)=(2/5)(3/4\sim)^2 Z^2 \beta^4(rms)=2.28\times10^{-3} Z^2 \beta^4(rms)$ (It is important to note that in Kantele's review[2], his equation 17 has a typographical error in that the Z-exponent is incorrect), where $\beta(rms)$ is the root-mean-square beta value and can be deduced from the measured B(E2; $2^+(1)\text{-->}0^+(1)$) value (in a harmonic limit) through

$$\beta^2(rms) = B(E2;\ 0^+(1)\text{-->}2^+(1)/((3/4\pi)ZeR^2)^2$$

or (it is important to recognize that because we are dealing with a dynamical quantity, it is a root-mean-square value that is being dealt with)

$$\rho^2(rms) = (2/5)(3/4\sim)^2 \beta^4 (B(E2;\ 0^+(1)\text{-->}2^+(1))^2/e^4R^8$$

where experimentally, the $\beta(rms)$ takes on a value of circa 0.12, giving typical values of (Z in parentheses): 3.7 (28), 11.8 (50), and 32 (82) mmu.

<u>2.2.2 Symmetric Rotor Mode</u> The MS's in deformed nuclei have been dealt with by Rasmussen.[5] In this model the breathing mode of beta vibration gives rise to MS values of B(E2; $0^+(1)\text{-->}2^+(beta)$ = $4\rho^2/e^2R^4$. Thus, the deexcitation of a beta band head in a deformed nucleus such as $^{156}Gd_{92}$ has an MS value of ~160 mmu.

<u>2.2.3 Pairing Vibration Mode</u> Coherent pairing fluctuations (vibrations) have been well established to occur at $\sim 2\Delta$ in a wide range of nuclei and can be estimated to have MS values on the order of ~0.2 mmu for neutron pairing vibrations in N~100 nuclei and ~0.8 mmu for proton pairing vibrations in nuclei

with Z~68. However, because the E2 transition from this type of excitation is highly hindered, the pairing vibrations can exhibit unusually large X_{ijk} values.

<u>2.2.4 Interacting Boson Model E0 Transition Rates</u> Elsewhere,[3] we have given the formalism for the MS values within the SU(3), SU(5), and O(6) IBM limits.

<u>2.3 Monopole Strength between Modes as a Measure of Shape Mixing</u> It is extremely important to recognize that the MS between unmixed nuclear modes of excitation that are very different in shape (shape coexistence) is strictly zero. This is well illustrated by the MS value[1] of 1.7×10^{-6} mmu for the ^{238}U superdeformed state (fission isomer) while, some of the largest values (~100 mmu) arise as a result of large mixing between almost spherical and deformed states.[2,3,6,7]

For mixed states we can obtain an estimate of the MS value by taking two unperturbed configurations of vibrational (|vib>) and deformed (|def>) nature where

$$|0_i^+\rangle = a|vib\rangle + b|def\rangle$$

$$|0_f^+\rangle = -b|vib\rangle + a|def\rangle$$

then the MS results from: $\langle 0_f^+|M(E0)|0_i^+\rangle \sim abk\langle def|\beta^2|def\rangle = abk\beta^2$

and

$$\rho^2 = \{abk\beta^2/cR^2\}^2 = (3abZ/4\pi)^2\{1+(4\pi^2/3)(a_0/R)^2\}^2\beta^4$$

which for maximum mixing (a=b) gives:

$$\rho^2 = (3/8\pi)^2(\langle\beta^2\rangle_{def})^2 Z^2$$

Thus, as illustrated in Fig. 1A, the MS value has a very strong dependence on the effective deformation since it is proportional to the fourth power of the effective deformation. In summary, we stress that large MS in nuclei is not in itself a proof of shape coexistence but rather is an indication of strong mixing between modes of excitation with large differences in shape (radii). At the same time, it is important to recognize that, as in the case of the ^{238}U superdeformed isomer, small MS values do not preclude shape coexistence in a nucleus.

3. Shape Coexistence and Monopole Strength in Even-Even Nuclei

In Fig. 1B we show the MS for $0^+(2)$ to $0^+(1)$ transitions in even-even nuclei in the range $40 \leq A \leq 116$. As MS values in rare-earth deformed nuclei are outside the realm of our considerations, we exclude these data. Because we wish to investigate the occurrence of large MS values as a consequence of the mixing of near spherical vibrations with deformed states, a useful basis of comparison is the estimate that arises from the vibrational model (see Sec. 2.1.1). The variation of the vibrational model MS values over the mass range $40 \leq A \leq 116$ is shown in Fig. 1B for two typical values of β_{rms} (0.10 and 0.12).

3.1 Variation of Monopole Strength with Degree of Mixing As seen in Fig 1B, a large number of values in the mass 70 to 100 mass region well exceed the vibrational estimate. Recently, Heyde[3] has given a useful guide for the **trend** that should be expected for the MS values when shape coexistence occurs. Fig. 2A shows the growth of the MS from a low value where there is low interaction between the coexisting state occurring at high energy above the spherical state, to strong mixing as the two systems approach and become nearly degenerate, to finally, low values again when the coexisting state becomes the gs system. If the two systems exist at the end of such a deformed region, as shown in the figure, the reverse trend occurs.

3.2 Monopole Strength in 1g Nuclei The nuclei in the ^{28}Ni to ^{132}Sn region provide a unique opportunity to explore the role of the $^3S_1$ force and its strong interaction when there is large overlap between neutron and proton orbitals.

As reviewed by Federman and Pittel,[8] the strongest interaction occurs for spin orbit partners, while the next strongest is for neutrons and protons in the same orbit, and it increases as the orbital angular momentum increases. The ^{28}Ni to ^{132}Sn region affords the unique situation of having first both the neutron and proton in the $1g_{9/2}$ orbital in the N<50, nuclei while the maximum effect can be investigated in the N>50 nuclei where the protons are still in the $1g_{9/2}$ orbital but the neutrons are first denied occupancy because of a subshell closure and then gain occupancy at N~58 and the 1g system becomes the gs.

As shown in Fig. 2B, the general trend of the MS for the N=58 and 60 isotones is similar to that suggested in Fig. 2A, which was discussed in the previous section; here, we observe only half of the suggested trend. The factor of ~10 difference in the average MS can be understood within the framework of two factors. First, the two states are closer in energy in the N=60 isotones, hence will mix to a larger degree. However, the second and probably the main reason is that, while the N=58 nuclei have a dynamically deformed excited state, the N=60 nuclei possess a gs with static deformation. Since the MS is proportional to the fourth power of the deformation, even a change from a deformation of 0.22 to 0.36 will give a factor of 7 increase in the MS (c.f. $^{38}Sr_{60}$ is known[7] to have a deformation of circa 0.4).

A further manifestation of the nature of the coexisting structures arises from the general shape of the MS shown in Fig. 2B. Since, as we have proposed, it is the **n-p** interaction that gives rise to the coexisting state, we might expect that the general trend would follow the N_pN_n principle that has been discussed by Casten.[9] As seen in the figure, the peak of the values for the N=60 curve is shifted by two mass units with respect to those for the N=58 curve. However, if the two curves are considered as a function of N_pN_n, the maximum occurs at the same value. Also, as we discuss later, mixing between spherical and intruder states can also account for the cascade of three strong E0 transitions in both ^{96}Zr and ^{98}Zr (see Sec. 4.3.2).

4. Structural Analysis by Combined Transition Strength Studies

So far our consideration of the MS has been limited to its exclusive use in the investigation of shape coexistence and its mixing in nuclei. As is the case in most structural investigations, a more significant understanding arises from the combination of several techniques. This is of particular importance in the study of shape coexistence and its properties. In this section we illustrate this by combining the present findings of ongoing investigations into the nature of the 1g nuclei, i.e., those nuclei where the nucleons are filling the 1g orbit. A particular case in point is the joint investigation of ^{96}Zr by the JuBBuLLi collaboration (see first page for list of principal investigators). However, we first recall an important spin-dependent property of B(E2) values and show how the oversight of this property obfuscates the proper characterization of intruder configurations.

4.1 Consequences of Spin Dependent B(E2) Values It is important to recognize that the energy independent B(E2) value, the measurable electromagnetic quantity for transitions between levels, **is spin dependent** and that it is the transition matrix element that is of importance in the comparison of interaction strength and structure. This fact has been overlooked in a significantly large number of cases. Because of this oversight, "large discrepancies", "inverted band sequences", etc., have been cited. Here, we first recall the fundamental features of E2 transition strengths and then use this to show that a number of "highly collective" transitions from intruder band heads that have been reported in the literature are, in fact, slower than the 2^+(gs) to 0^+(gs) E2 transition reported for the same nucleus.

In general, when we wish to compare the structure of several states differing in spin by 2 units of angular momentum and, hence, connected by an electromagnetic quadrupole transition, it is the transition matrix element that contains the information we want. However, it is general practice to use the quantity B(E2) rather than the reduced matrix element. This is because the B(E2) value is the quantity that is obtained directly from experiment without a knowledge of the other level properties. In general, comparison is made for the spin-loss deexcitation of levels and, in general, to a common final state. Thus any spin dependence drops out. However, for the special case where we wish to compare the **spin-gain** E2 deexcitation of one level, even with the spin-loss E2 deexcitation of a common connecting level such as a $0^+(2)\text{--(E2)-->}2^+(1)\text{--(E2)-->}0^+(1)$ sequence, a significant difference arises for the relative B(E2) values as compared with the (true) transition matrix element (m_{ij}). This can be shown by recalling the basic quantities as given by Bohr and Mottelson,[4] where one starts with the Wigner-Eckart theorem and develops the general expression for the E2 transition probability of:

$$B(E2;\ I_1\text{-->}I_2) = \{1/(2I_1+1)\}|\langle I_2\|m(E2)\|I_1\rangle|^2.$$

Then, if we take a generalized case of level A deexciting to level B followed by deexcitation to level C, we arrive at:

$$B(E2;\ I_A\text{-->}I_B) = \{1/(2I_A+1)\}|\langle I_B\|m(E2)\|I_A\rangle|^2$$

$$B(E2;\ I_B\text{-->}I_C) = \{1/(2I_B+1)\}|\langle I_C\|m(E2)\|I_B|\rangle^2.$$

For simplicity we define $|\langle I_f\|\langle m(E2)\|I_i\rangle|^2$ as M_{ij}. Then, the comparative ratio becomes:

$$B(E2;A\text{-->}B)/B(E2;B\text{-->}C) = \{(2I_B+1)/(2I_A+1)\}\{M_{AB}/M_{BC}\}.$$

For the special case where $A=0^+(2)$, $B=2^+(1)$, and $C=0^+(1)$, we arrive at:

$$B(E2;\ 0^+(2) \dashrightarrow 2^+(1)/B(E2;\ 2^+(1) \dashrightarrow 0^+(1) = 5M_{AB}/M_{BC}$$

or

$$\frac{M[0^+(2) \dashrightarrow 2^+(1)]}{M[2^+(1) \dashrightarrow 0^+(1)]} = (1/5)\frac{B(E2;\ 0^+(2) \dashrightarrow 2^+1))}{B(E2:\ 2^+(1) \dashrightarrow 0^+(1))}$$

That is, when we compare the relative <u>transition strength</u> for the cascade $0^+(2)$--(E2)-->$2^+(1)$--(E2)-->$0^+(1)$, we must take 1/5 of the $0^+(2)$--(E2)-->2^+ measured B(E2) value and compare it with the full B(E2) value measured for the $2^+(1)$--(E2)-->$0^+(1)$ transition. Thus, for <u>equal transition strength</u> we would have to observe a 100 Wu speed for the $0^+(2)$--(E2)-->$2^+(1)$ B(E2) transition and 20 Wu for the $2^+(1)$--(E2)-->$0^+(1)$ B(E2) value.

Of course, a simple way to compare these values is to compare the B(E2) rate for the same spin increment (spin increase or decrease). Thus we can compare the B(E2) strength for the $2^+(1)$ to $0^+(1)$ transition with the <u>inverse</u> of the $0^+(2)$ to $2^+(1)$ B(E2) strength, i.e., the $0^+(2)$<<==(E2)==$2^+(1)$ B(E2) value rather than the $0^+(2)$--(E2)-->$2^+(1)$ value. That is, we compare 1/5 times the observed B(E2;$0^+(2)$-->$2^+(1)$) with the observed B(E2;$2^+(1)$-->$0^+(1)$) value.

<u>4.2 ^{70}Se and ^{102}Pd as Didactic Spin-Dependent B(E2) Cases</u> Recent studies of coexisting structures in the mass 100 region serve to illustrate the need to be aware of the spin dependence of B(E2) values.[2,10,11] These recent studies have concluded that "the B(E2; $0^+(2)$-->$2^+(1)$) value reveals strong collectivity". In the case of ^{70}Se, the B(E2) value was 53 Wu[10] while in ^{102}Pd the measured B(E2) value was 96 Wu.[11] Recently, we have accounted for the properties of the low-energy spectra of ^{102}Pd.[12] We have shown that the level properties of ^{102}Pd are consistent with its 1593- (0^+) and 1944-keV (2^+) levels being members of the $pd_{3/2}$-$nd_{5/2}$ intruder (i) state coexisting with an O(6)-like ^{102}Pd gs excitation spectrum. The branching ratios and the fact that the $0^+(i)$ states increase in energy from ^{102}Pd to ^{106}Pd were found to be consistent with this

description. However, as discussed in Sec. 4.1, to compare the B(E2) values we must take 1/5 of the $0^+(2)\text{-->}2^+(1)$ value or (1/5)(96)=19 Wu. This is less than the 33 Wu for the $2^+(1)\text{-->}0^+(1)$ B(E2) value. Elsewhere,[12,13] we have shown that mixing of the intruder and gs configurations can give rise to not only enhancement but cancellation of E2 transition strength. Thus configuration mixing can be taken as the cause of the relatively "slow" intraband $2^+(i)\text{-->}0^+(i)$ B(E2) value of 17 Wu (partial cancellation) in comparison to the 19 Wu for the $0^+(i)\text{<<==(E2)==}2^+(gs)$ transition (additivity).

Recently, in-beam conversion electron studies[10] were combined with previous lifetime measurements of levels in ^{70}Se. As in the ^{102}Pd study, a conclusion was reached that the largest discrepency in the concept that shape coexistence occurred in ^{70}Se was the "strong collectivity of 53 Wu for the $0^+(2)\text{--(E2)-->}2^+(1)$ B(E2) value". Again, if we instead compare the $0^+(2)\text{<<==(E2)==}2^+(1)$ value of (1/5)(53)~10 Wu with 21 Wu for the $2^+(1)\text{--(E2)-->}0^+(1)$ transition, we find this in the realm of understanding within the context of the mixing of coexisting systems.

<u>**4.3 Configuration Mixing in ^{96}Zr: An Example of Combined Studies**</u> Here, we give a partial summary of the JuBBuLLi collaboration's progress in the investigation of ^{96}Zr.[14-20]

<u>**4.3.1 Intruder-GS Mixing**</u> Recently, Molnar et al. gave the first identification of an intruder band in ^{96}Zr with a double subshell closure gs.[20] Also, the first known extensive set of collective excitations built on an intruder state has been identified in ^{98}Zr and the characteristics of double subshell closure shown to extend over two nuclei (^{96}Zr and ^{98}Zr).[19] Recent beta decay[18] and on-going in-beam studies[14] of ^{96}Zr have shown that not only does it gs have double closed

subshell character but that a coexisting intruder band (i-band) that forms the first excited 0^+ state has members up to 8^+ in spin.[14,15] Further, Mach et al.[18] have shown that the 0^+ gs- and i-states are relatively unmixed (~96% pure). However, there is conflicting evidence concerning the mixing of band members. The enhancement of the intra- to inter-band B(E2) ratio (E2-ER) for the intruder 2^+ state (2^+(i)) suggests virtually no mixing while the unique first forbidden (UFF) beta strength to the 2^+(i) and 2^+(g) states exhibit evidence of strong mixing. Our estimate of the effects of mixing in the 2^+ levels from the UFF beta strength observed in the decay of 0^- $^{96}Y^g$ to the 2^+ levels of ^{96}Zr shows that the 2^+(i) level has ~1/3 of the unmixed 2^+(gs) configuration. However, the experimentally measured E2-ER of 115 for the 2^+ i-band suggests nearly pure states. Kusnezov et al.,[12,13] using a symmetry model with configuration mixing, have shown that the wave functions mix in an important way such that the B(E2) values are modified in a major way. Not only forbidden transitions between originally unmixed levels can occur in the mixed system but allowed transitions in the originally unmixed system can become **strongly quenched**. By taking standard values for the unmixed transition rates and the estimated configurations, we arrive at a value of 29 SPU for the 2^+(2225) to 0^+(1581) transition and a value of 0.24 SPU for the 2^+(2225) to 0^+(gs) transition. Thus strong mixing yields a correct value (121,Th/115,exp). The 6^+ and 8^+ i-band members can be identified at 3772 and 4390 keV. As no other 8^+ has been found that is close in energy, strong mixing cannnot account for the back bend, suggesting a less collective nature at high J in $^{96}Zr^*$ than $^{98}Zr^*$.

4.3.2 Monopole Strength and Intruder-State Pairing-Vibration Mixing

Henry[15] has reported a third 0^+ level in ^{96}Zr at 2695 keV, which has its most intense E0 transition of the 0^+(i) band head. We assign the 2695-keV level in

^{96}Zr and the 1437-keV level in ^{98}Zr as the two-neutron $s_{1/2}$ pairing-vibration configuration. The occurrence of a 2n-$s_{1/2}$ state is consistent with experimental expectations[16] and semimicroscopic calculations.[17] The 2n-$s_{1/2}$ state will be nearly spherical and the intruder state has been shown to have a modest deformation.[18] Thus strong mixing will lead to the large MS observed between the 2695- and 1581-keV levels. The observation of an E0 to the gs from the 2695-keV level suggests further mixing in ^{96}Zr and is accounted for by our identification of another 0^+(i) level at 2925 keV in ^{96}Zr, which is observed in both in-beam and beta decay studies as well as in the two-neutron transfer reaction studies of Flynn et al.[16] (reported at 2936±~12 keV). We have observed a gs E0 transition and have tentative evidence for a 2925- to 2695-keV level E0 transition.[14] The observed properties of the 2925-keV level lead us to assign this level as the 0^+(i) two-phonon member, which will mix with the 2n-$s_{1/2}$ level and alter the latter's properties.

<u>4.3.3 Possible Discovery of an Octupole Two-Phonon Multiplet and Its Mixing</u> The levels associated with the octupole state appear to exhibit quadrupole-octupole and double octupole excitations. For the former, the 5^- and 7^- levels suggest an effective vibrational core with an energy of circa 1200 keV. For the latter, a quartet of levels with a centroid circa 1400 keV above the 3^- octupole state are suggested to be the octupole two-phonon multiplet. Three of the members are observed in the in-beam studies[12,14] while the fourth may be observed in the beta decay of the ^{96}Y gs. The 2^+ and 4^+ members have strong E1 branches to the 3^- state while the 6^+ member has its predominant deexcitation to the 5^- octupole excitation. These intense E1 transitions, shown in Fig. 3, are indicative of strong mixing between the octupole two-phonon members and the octupole-quadrupole members.[21] Identification of other octupole-quadrupole members as well as more precise values for the E3 transition strength are required

before a complete analysis of the mixing strength can be made. Although suggested in an odd-mass nucleus,[22] this would be the first identification of members of a two-phonon octupole multiplet in an even-even nucleus.

5. Does only the n-p Interaction Produce Deformation?

As we discussed earlier (see Section 3.2), the 1g nuclei provides an almost unique opportunity to explore the question of whether the **n-p** $^{3}S_{1}$ force and its strong interaction via large overlap of orbits provide the main driving force leading to deformation. As shown in Fig. 4, if we normalize the even-even Zr nuclei to have their 0^{+}(1g) excitation as the effective gs, the 2^{+}(1g) levels exhibit a steady progression from closed shell nature to an energy near the rigid rotor value in ^{102}Zr. The concept of the 1g system providing the full deformation driving force is further supported by our recent discovery of eight excited states built on the 0^{+}(1g) state in ^{98}Zr.[19] This set of levels, termed ^{98}Zr*, shows an O(6) character, while the lighter Zr* excitations exhibit SU(5) patterns. Thus the sequence predicted by symmetry models for the smooth onset of deformation of SU(5)-->O(6)-->SU(3) (i.e., vibrational-->gamma-soft-->symmetric-rotor nuclei) for the onset of deformation in these nuclei is well confirmed. It is equally important to recognize that the deformed state in ^{100}Zr is the gs and that ^{100}Zr possesses a coexisting excited spherical state. That is, for the N=60 Zr nuclei, only the 1g system appears to be contributing to the deformed structure while the residual orbitals remain in a coexisting spherical (vibrational) configuration. Further, $^{98}Sr_{60}$, which is the N=60 isotone of ^{100}Zr, has recently been shown to have a rather large deformation of b=0.40. This must be contrasted with the b~0.3 known for the heavier even-even deformed nuclei in this mass region which <u>do not</u> have coexisting spherical states. Recent studies of $^{102}Zr_{62}$ give evidence of not having a coexisting 0^+ state present (c.f. Wohn[23] has performed a detailed study of the beta decay of 1^+ ^{102}Y at TRISTAN and finds no evidence for a spherical state). All this leads to the fact that, within the information that is presently available, the 1g system by itself appears to be more strongly deformed and a better symmetric rotor than when the entire nuclear system contributes to deformation (i.e., all orbitals are contributing). Perhaps, crudely, this can be visualized from the perspective that it is the n-p $^3S_1$ force interacting through the strong overlap of the $1g_{9/2}p$ and $1g_{7/2}n$ spin-orbit partners that is solely driving the nucleus to deformation, and that once "forced" into the condition of all nucleons contributing, a contraction to smaller deformation occurs (c.f. the concept used originally for the onset of deformation in the rare-earth nuclei as contrasted with the concepts reviewed by Federman and Pittel[8]).

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*JuBBuLLi or Jubilee: a celebration; origin: yobhel (Hebrew) meaning a ram's horn

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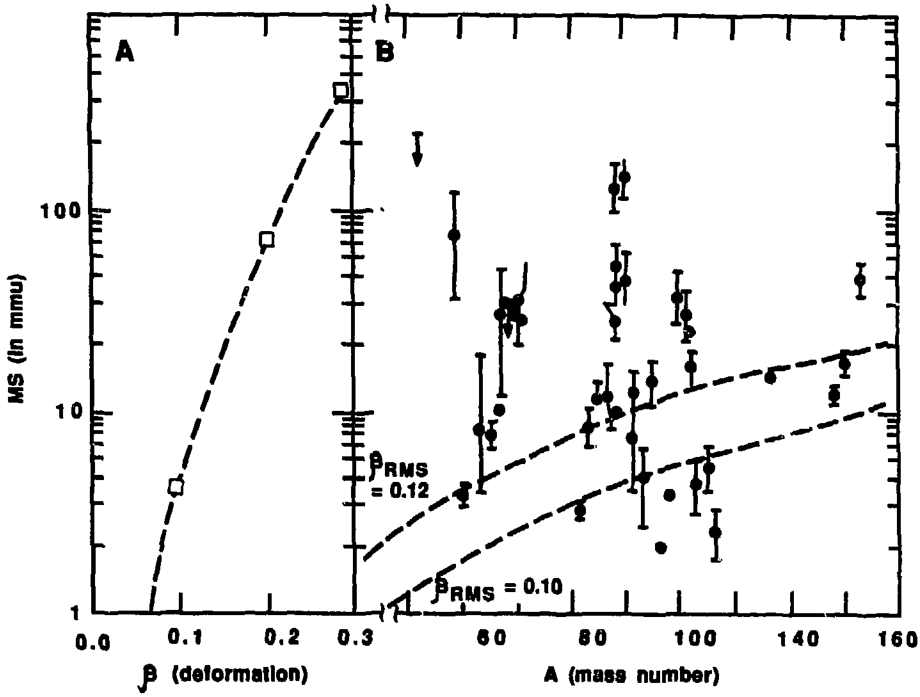

Fig.1 A) Variation of the MS with deformation for a completely mixed system.
B) MS values known in even-even nuclei in the mass range $40 \leq A \leq 160$ (dashed lines give the quadrupole phonon model estimate. See Sec. 2.2.1)

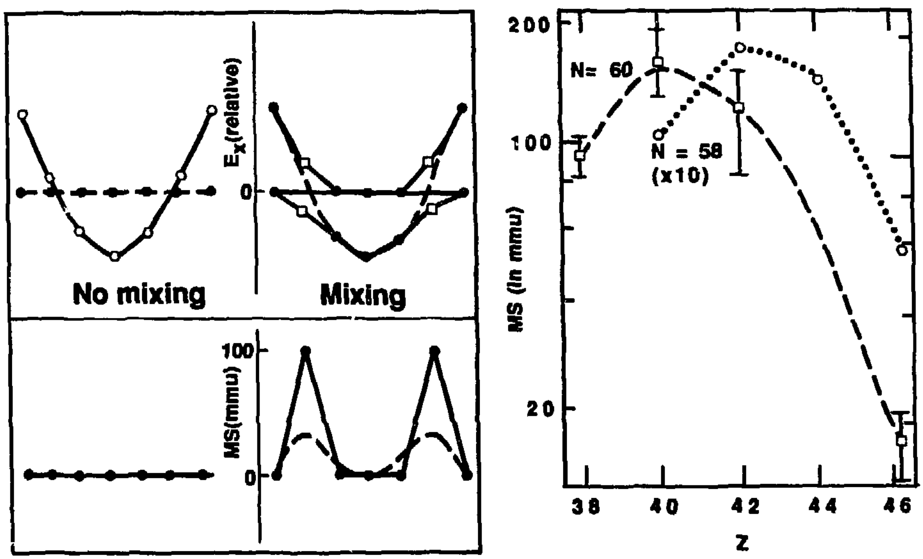

Fig.2 A) Left side: relative energy (upper) and MS (lower) between two unmixed shape coexisting systems and right side: same for completely mixed systems.
B) MS values between the first excited 0^+ level and the gs in the N=58 and N=60 even-even isotones (see text).

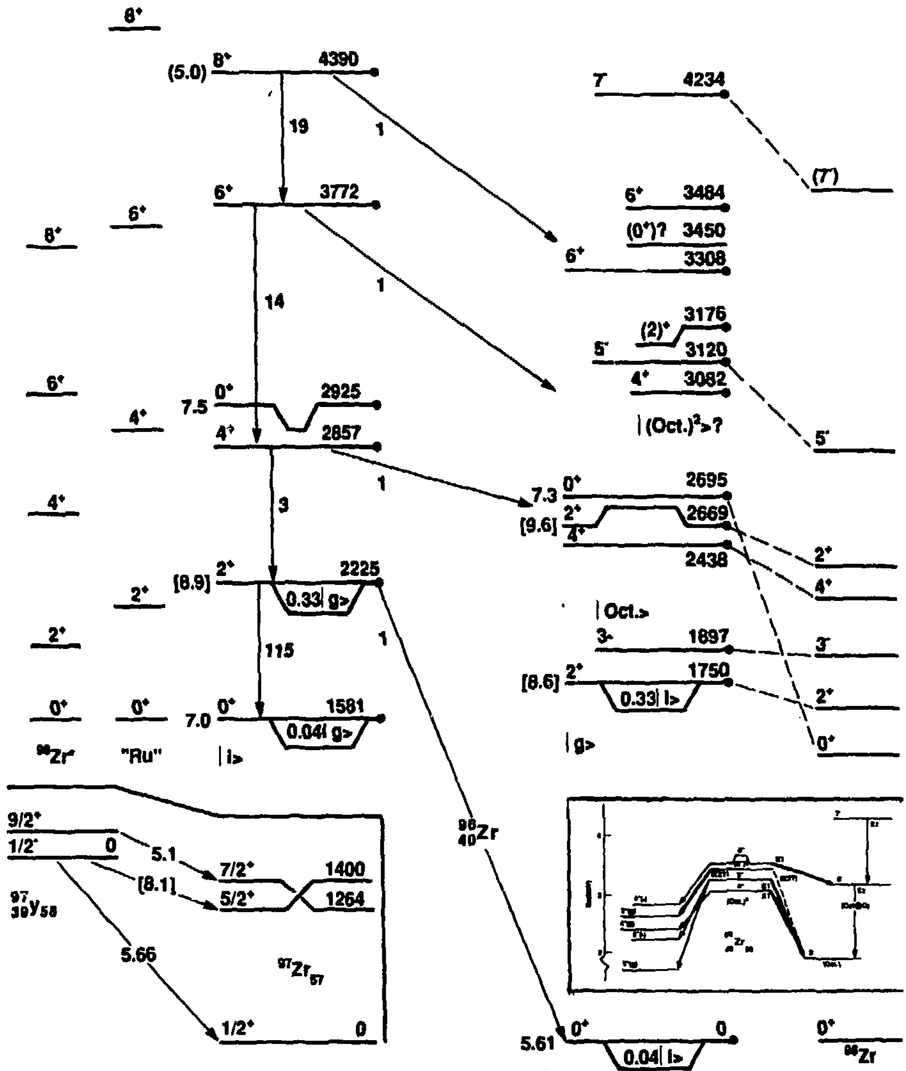

Fig. 3 Selected experimental level properties of ^{96}Zr (taken from refs. 7,14,15,18,20). The left half of the figure shows the i-band (|i>), the yrast levels in the equivalent Ru nucleus ("Ru", see ref. [19]), and the intruder state yrast levels in ^{98}Zr (^{98}Zr*). The right half of the figure shows those levels that are associated with the double subshell closure gs with those for ^{96}Zr labeled as |g> (longest lines), octupole plus quadrupole coupled (medium length lines), octupole two-phonon candidates (shortest lines) and the right hand most column showing the gs set for ^{98}Zr (note expected drop in energy of the $2s_{1/2}$-n level). Numbers to the left of each level are log ft and [log f_1t] values for beta transitions from the 0^- gs of ^{96}Y [18] while those in parenthesis are the GT-bt log ft values from the 8^+ isomer [7]. The numbers next to the arrows are the B(E2) enhancement factors relative to the inter-band transition. Dots on the right hand side of level signifies its identification in our in-beam studies [14,15] and the figure below the level gives the estimated mixed component in the level. Bottom right insert: deexcitation properties of the double octupole states. Bottom left insert: beta population of ^{97}Zr; note similarity with ^{96}Zr of i) summed UFF log f_1t, ii) log ft for gs to gs, andiii) isomer to $7/2^+$ vrs. 8^+(2808). See text and refs cited above for discussion of the various features.

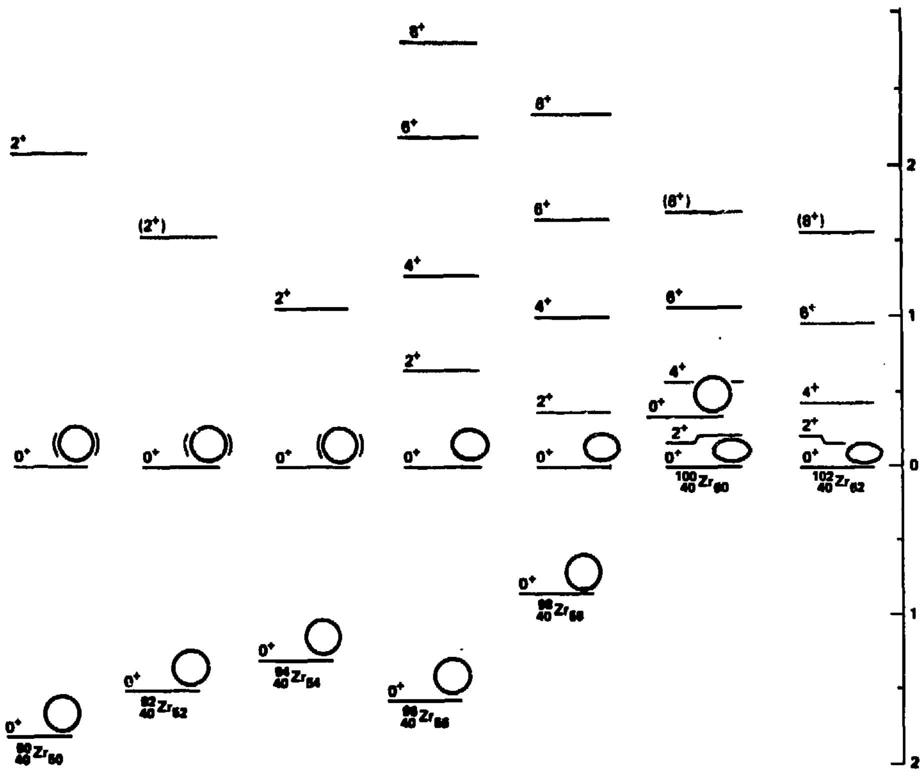

Fig.4 Systematics of gs (fp-d configuration) and intruder 0^+ and $2^+(1)$ state (g-g configuration) for the Z=40 Zr even-even isotopes normalized to the 0^+ g-g intruder state.