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Mahmood Mian

I. Ahmad

and

M.Z. Rahman Khan

INTERNATIONAL ATOMIC ENERGY AGENCY

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Mahmood Mian
Department of Physics, Guru Nanak Dev University, Amritsar, India,

I. Ahmad **
International Centre for Theoretical Physics, Trieste, Italy

and

M.Z. Rahman Khan
Department of Physics, Aligarh Muslim University, Aligarh, India.

ABSTRACT

Working within the framework of the folding model, we analyze the Λ binding energy data of light hypernuclei with effective two-body ΛN plus three-body ΛNN interaction. The two-body density for the core nucleus required for evaluating the three-body force contribution is obtained in terms of the centre of mass pair correlation. It is found that except for $^5_\Lambda$He the data are fairly well explained. The three-body force seems to account for the density dependence of the effective two-body ΛN interaction proposed earlier.

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** On leave of absence from: Department of Physics, Aligarh Muslim University, Aligarh - 202001, India.

A few years ago we proposed a phenomenological method for analyzing the ground state Λ binding energy data[1]. The essential feature of the method is to apply the folding model to generate Λ-nucleus potential in terms of a density dependent effective two-body ΛN interaction involving three free parameters. Using realistic densities for the core nuclei we have found that the Λ-nucleus potential so generated reproduces the Λ binding energy data of hypernuclei of baryon number $B \geqslant 5$ very well[1,2]. This approach also provides a very reliable estimate for the Λ binding energy in nuclear matter[1].

Theoretical considerations suggest presence of a three-body force component in the interaction potential of a Λ particle with a nucleus[3,4]. Therefore, it is not unlikely that the density dependence in our effective two-body interaction is mostly due to the three-body component of the Λ-nucleus interaction. If so, it should be possible to reproduce the Λ binding energy data within the folding model framework with an appropriate mixture of density independent two-body ΛN and the three-body ΛNN interactions. In this note we show that this is largely the case. More explicitly we demonstrate that the folding model with an appropriate combination of the two-body and the three-body interactions accounts for the Λ binding energy data reasonably well.

When the three-body force is present, the expression for the total (spin and i-spin averaged) potential for the interaction of a Λ particle described by the coordinate $\vec{r}$ with a core nucleus of mass number A may be written as

$$V(\vec{r};\ \vec{r}_1,\vec{r}_2 \ldots \vec{r}_A) = \sum_{i=1}^{A} V_{\Lambda N}(\vec{r}-\vec{r}_i) + \sum_{i<j} V_{\Lambda NN}(\vec{r};\vec{r}_i,\vec{r}_j), \qquad (1)$$

where $\vec{r}_i$ are the nucleon coordinates and $V_{\Lambda N}$ and $V_{\Lambda NN}$ are the two-body and three body potentials, respectively.

The folding model Λ-nucleus potential is obtained by taking the expectation value of eq.(1) with respect to the ground state wavefunction of the core nucleus. This gives the following expression for the spin and i-spin averaged Λ- nucleus potential:

$$V_{\Lambda A}(r) = V_2(r) + V_3(r) \quad , \tag{2}$$

where

$$V_2(r) = A \int V_{\Lambda N}(\vec{r}-\vec{r}')\, \rho(\vec{r}')\, d\vec{r}' \tag{3}$$

and

$$V_3(r) = \frac{A(A-1)}{2} \int V_{\Lambda NN}(\vec{r};\vec{r}_1,\vec{r}_2)\, \rho^{(2)}(\vec{r}_1,\vec{r}_2)\, d\vec{r}_1\, d\vec{r}_2 \, . \tag{4}$$

The quantities $\rho(r)$ and $\rho^{(2)}(\vec{r}_1,\vec{r}_2)$ in eqs.(3) and (4) are respectively the one-body and two-body densities of the core nucleus.

In this work we assume that the two-body interaction is of the gaussian form and write $V_2(r)$ as

$$V_2(r) = -A\,\overline{V}_c \int \frac{e^{-\frac{(\vec{r}-\vec{r}')^2}{d^2}}}{(\pi d^2)^{3/2}}\, \rho(\vec{r}')\, d\vec{r}' \, , \tag{5}$$

where $\overline{V}_c$ is the volume integral of $V_{\Lambda N}(r)$ and d is the range parameter.

As regards to the three-body interaction we again assume a gaussian form:

$$V_{\Lambda NN} = t_3\, e^{-\sum_{i=1}^{2} \alpha (\vec{r}-\vec{r}_i)^2} \, , \tag{6}$$

where t_3 and α are the strength and the range parameters respectively. Substitution of eq.(6) into eq.(4) gives

$$V_3(r) = \frac{A(A-1)}{2}\, t_3 \int e^{-\sum_{i=1}^{2} \alpha (\vec{r}-\vec{r}_i)^2}\, \rho^{(2)}(\vec{r}_1,\vec{r}_2)\, d\vec{r}_1\, d\vec{r}_2 \, . \tag{7}$$

Here, it may be pointed out that the densities $\rho(r)$ and $\rho^{(2)}(\vec{r}_1,\vec{r}_2)$ appearing in eqs.(5) and (7) are normalized to unity. They are the intrinsic nucleon densities and the coordinates in their arguments refer to the centre of mass (c.m.) of the core nucleus.

For a given parameter set for the ΛN and ΛNN interactions, evaluation of $V_2(r)$ and $V_3(r)$ requires knowledge of the densities $\rho(r)$ and $\rho^{(2)}(\vec{r}_1,\vec{r}_2)$. For light nuclei which concern us, the one-body density $\rho(r)$ may be obtained from the charge density of the core nucleus (after correcting for proton finite size) assuming proton and neutron densities to be the same at least for N=Z nuclei or under any other reasonable assumption as discussed in Ref.[1]. Thus evaluation of $V_2(r)$ from eq.(5) is straightforward. But the same cannot be said with regard to $V_3(r)$, since hardly any experimental information about $\rho^{(2)}(\vec{r}_1,\vec{r}_2)$ is available at present. Therefore some theoretical considerations are necessary to obtain $\rho^{(2)}(\vec{r}_1,\vec{r}_2)$ in order to be able to evaluate $V_3(r)$.

The two-body density occuring in eqs.(4) and (7) is defined as

$$\rho^{(2)}(\vec{r}_1,\vec{r}_2) = \frac{1}{A(A-1)} \sum_{i \neq j} (\Phi_0 |\, \delta(\vec{r}_1-\vec{r}_i')\, \delta(\vec{r}_2-\vec{r}_j')\, | \Phi_0) , \tag{8}$$

where Φ_0 is the intrinsic ground state wavefunction and the prime on the coordinates implies that they are measured from the centre of mass of the nucleus. The density $\rho^{(2)}(\vec{r}_1,\vec{r}_2)$ is related to the nucleon pair correlation function $C_2(\vec{r}_1, \vec{r}_2)$ through the equation[5]:

$$\rho^{(2)}(\vec{r}_1,\vec{r}_2) = C_2(\vec{r}_1,\vec{r}_2) + \rho(\vec{r}_1)\rho(\vec{r}_2) , \tag{9}$$

where $\rho(\vec{r})$ is the intrinsic one-body density and is given by

$$\rho(\vec{r}) = \frac{1}{A} \sum_{i=1}^{A} (\Phi_0 |\, \delta(\vec{r}-\vec{r}_i')\, | \Phi_0) . \tag{10}$$

Studies show that the ground state pair correlation function in nuclei is contributed by several factors. The notable ones are the c.m. correlation, Pauli correlation and the dynamical short range correlation. These pair correlations have been studied extensively and their forms have been obtained using reliable model wavefunctions[5-7].

The c.m. correlation has its origin in the fact that a realistic ground state wavefunction of a nucleus respects the translational invariance i.e. the nucleons in the nucleus move in such a way that their centre of mass maintains a uniform momentum. Feshbach et al.[5] have discussed the question of the c.m. pair correlation at some length and have also derived an expression for the c.m. pair correlation function in a model independent way in some limiting situations. Somewhat recently one of us and Auger[7] have derived a simple expression for the c.m. pair correlation function under the assumption that the antisymmeterized oscillator shell model wavefunctions provide a reasonably good approximation to the realistic wavefunctions at least as far as the separation of the nucleon internal motion and the c.m. motions are concerned. Their expression for the c.m. pair correlation function reads as

$$C_{cm}(\vec{r}_1,\vec{r}_2) = -\frac{\vec{r}_1\cdot\vec{r}_2}{2A\alpha_s^2}\left(\frac{1}{r_1}\frac{d\rho}{dr_1}\right)\left(\frac{1}{r_2}\frac{d\rho}{dr_2}\right), \tag{11}$$

where α_s is the oscillator constant. It must be emphasized that $\rho(r)$ appearing in eq.(11) is the intrinsic one-body density (the one that appears in eq.(5)) and *not* the oscillator shell model density.

As regards to Pauli and the dynamical short range correlations, the former has its origin in the antisymmetry of the realistic nuclear wavefunctions while the latter is due to the hard core in the NN interaction. The effects of both these correlations have been studied in the context of high energy p-nucleus scattering involving large momentum transfers and have been found to be small[6,7] (Pauli correlation is small only for light nuclei which concern us). Therefore, it seems reasonable to ignore them in the present study. In other words we will assume that the total pair correlation function in light nuclei is approximately equal to the c.m. pair correlation function as given by eq.(11).

Substituting expression(11) for C_2 in eq.(9) and inserting the resulting expression for $\rho^{(2)}(\vec{r}_1,\vec{r}_2)$ in eq.(7) it is easy to see that

$$V_3(r) = t_3\,\frac{A(A-1)}{2}\,(I_c + I_s), \tag{12}$$

where

$$I_c = -\frac{2\pi^2 e^{-\beta r^2}}{A\alpha_s^2}\left[\int_0^\infty r'^3 f(r')\left\{\frac{2}{\beta r r'}-\frac{1}{(\beta r r')^2}\right\}\left(e^{\beta r r'}-e^{-\beta r r'}\right)dr'\right]^2 \tag{13}$$

with

$$f(r) = e^{-\alpha r^2}\left(\frac{1}{r}\frac{d\rho}{dr}\right)$$

and

$$I_s = \left[\frac{2\pi e^{-\alpha r^2}}{\beta r}\int_0^\infty r' h(r')\left(e^{\beta r r'}-e^{-\beta r r'}\right)\right]^2 , \tag{14}$$

with

$$h(r) = e^{-\alpha r^2}\rho(r)$$

Here $\beta = 2\alpha$.

The integrals in the expressions for I_c and I_s are evaluated numerically to obtain the potential $V_3(r)$.

We now briefly describe the calculation and present our results. To evaluate $V_3(r)$ as given by eq.(12) we take $\alpha = 0.234\ fm^{-2}$ which corresponds to the exchange of one pion with each of the two nucleons. The two-body potential term $V_2(r)$ is evaluated for three different values of the range parameter d: (i) d=0.65 fm [this is equal to the range parameter for the charge distribution in proton]. Thus for this value of d $V_2(r) = -\bar{V}_0 A\ \rho_{ch}(r)$, where ρ_{ch} is the charge distribution (normalized to unity) of the core nucleus].(ii) d = 0.7 fm corresponding to 3π exchange for the gaussian shape, and (iii) d = 1.04 fm corresponding to 2π exchange again for the gaussian shape. Fixing the range parameters and not treating them as free greatly facilitates the numerical fitting work.

Having fixed the range parameters the strength parameters $\overline{V}_0$ and t_3 are determined by χ^2 fitting of the B_Λ values for $^{11}_{\Lambda}B$, $^{13}_{\Lambda}C$ and $^{15}_{\Lambda}N$ hypernuclei. The reason for choosing these hypernuclei is that they have N = Z core and hence it is safe to assume that the neutron and proton density distributions in their cases are same. We do not include $^{5}_{\Lambda}He$ and $^{7}_{\Lambda}Li$ as they create problems in the fitting. The $^{7}_{\Lambda}Li$ hypernucleus is known to be better described in the α-d framework[2]. For core nuclei ^{10}B, ^{12}C and ^{14}C of the hypernuclei chosen for the fitting we use the oscillator model (point) densities with the values for rms radii as given in Refs.[8-10] respectively. The best fit parameters for different d values are given below and the fitted binding energies are given in Table I.

Set a :	d = 0.65 fm	$\overline{V}_0$ =254.37 MeV fm^3
	t_3 = 1.35 MeV	χ^2 =16.6 (total)
Set b :	d = 0.7 fm	$\overline{V}_0$ =276.68 MeV fm^3
	t_3 = 1.80 MeV	χ^2 =15.0 (total)
Set c :	d = 1.04 fm	$\overline{V}_0$ =315.77 MeV fm^3
	t_3 = 2.02 MeV	χ^2 =16.0 (total)

It may be seen from the table that the data are fairly well reproduced and that the contribution of the three-body force is repulsive. The strengths of the three-body force corresponding to the different values for d range from 1.35 to 2.02 MeV.

We now use the parameter set a to calculate B_Λ values for other p shell hypernuclei. The results with the other two parameter sets are almost similar. The calculated binding energies are shown in Table II. The ground state densities used in the calculation have been taken from the references mentioned in the Table. Some details regarding the choice of the densities may be found in Ref.[1]. It is seen that except for $^{7}_{\Lambda}Li$

the data are fairly reproduced. The reason for somewhat unsatisfactory results for $^{7}_{\Lambda}Li$ has already been mentioned. The reasons for the failure of the present calculation for $^{5}_{\Lambda}He$ are not very clear to us but for one point. In this investigation we have totally ignored the Pauli correlation which is present in all the cases considered here except in ^4He. Now, if Pauli correlation has some effect, then it gets simulated to some extent in the fitting process and hence the parameter values which are obtained by fitting the B_Λ values of heavier hypernuclei are not suitable for calculating B_Λ for $^{5}_{\Lambda}He$. This point needs further investigation.

The fair success of the folding model with a mixture of the two-body ΛN and the three-body ΛNN interactions implies that the density dependence in the effective two-body ΛN interaction found earlier[1] is largely due to the three-body ΛNN interaction. It also highlights the importance of the c.m. pair correlation in lighter nuclei.

The fact that the quality of results obtained here is not so good as the one obtained with the density dependent effective ΛN interaction[1] indicates that apart from the three-body force there are some other smaller effects that contribute to the density dependence of the effective ΛN interaction. However, in order to reach a firm conclusion in this respect, it is highly desired that the present study be refined by using the meson theoretic forms for the three-body ΛNN force and incorporating the neglected Pauli correlation.

As a final remark it may be added that a detailed comparision of the present work with those of the earlier authors[3,4,13] is rather difficult. This is mainly because we have used a different theoretical framework for analyzing the data. Apart from the application of the folding model, the emphasis in our work is on using experimental nucleon densities for the core nuclei to obtain the Λ- nucleus interaction potential.

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TABLE I

RESULTS OF χ^2 FITTING OF Λ-BINDING ENERGIES

Hyper-nucleus and ($B_\Lambda \pm \delta B_\Lambda$)	Theoretical B_Λ for Set a		Theoretical B_Λ for Set b		Theoretical B_Λ for Set c	
	B_Λ with ΛNN force	B_Λ without ΛNN force	B_Λ with ΛNN force	B_Λ without ΛNN force	B_Λ with ΛNN force	B_Λ without ΛNN force
${}^{11}_{\Lambda}B$ (10.24±0.05)	9.87	13.76	10.23	15.44	10.23	15.97
${}^{13}_{\Lambda}C$ (11.69±0.12)	11.97	17.76	11.93	19.54	11.92	20.37
${}^{15}_{\Lambda}N$ (13.59±0.15)	13.16	20.74	13.09	23.29	13.06	24.46

TABLE II

CALCULATED B_Λ VALUES WITH THE POTENTIAL PARAMETER SET a

Hyper-nucleus	Experimental B_Λ(MeV)	B_Λ(MeV) With ΛNN force	B_Λ(MeV) Without ΛNN force	Symbol of the core nucleus the density of which is used along with the reference number	
${}^{10}_{\Lambda}Be$	9.11±0.22	8.67	11.69	${}^{9}Be$	Ref.[11]
${}^{12}_{\Lambda}B$	11.37±0.06	11.17	16.03	${}^{11}B$	Ref.[8]
${}^{12}_{\Lambda}C$	10.76±0.19	10.50	15.13	${}^{12}C$	Ref.[9]
${}^{14}_{\Lambda}C$	12.17±0.33	11.98	18.31	${}^{14}N$	Ref.[10]
${}^{9}_{\Lambda}B$	7.88±0.15	6.44	8.54	${}^{10}B$	Ref.[8]
${}^{10}_{\Lambda}B$	8.89±0.12	8.17	11.09	${}^{10}B$	Ref.[8]
${}^{7}_{\Lambda}Li$	5.58±0.03	4.13	4.93	${}^{6}Li$	Ref.[12]
${}^{8}_{\Lambda}Li$	6.80±0.03	6.72	9.43	${}^{7}Li$	Ref.[12]