# INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

IC/88/64

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

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# A NOTE ON THE BFV-BRST OPERATOR QUANTIZATION METHOD *

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## ABSTRACT

The BFV-BRST operator quantization method is applied to massive, abelian (Yang-Mills) theory which has only second class constraints. A nilpotent BFV-BRST-charge is derived and used to define a unitarizing hamiltonian. Unphysical degrees of freedom can be eliminated either in a canonical gauge or in a relativistic one. In the latter gauge this is a general feature (at least locally) of the BFV-BRST quantization of the systems with irreducible constraints.

MIRAMARE - TRIESTE
March 1988

* Submitted for publication.

One of the most important milestones in our understanding of the underlying geometry of superstrings is to formulate a manifestly supersymmetric and covariant field theory. By working out the Neveu-Schwarz and Ramond sectors separately and combining them with the GSO projection[1] , one can define a covariant superstring field theory[2]. But it is not manifestly supersymmetric. To avoid this, one has to start from a manifestly covariant and supersymmetric string action. For the time being the unique superstring action which has these properties is due to Green and Schwarz[3] . But a covariant quantization of this model shows some difficulties[4].

The BRST-charge resulting from first quantization is an essential ingredient of a covariant string field theory[5]. In [6] a covariant quantization of the Green-Schwarz action is performed in the harmonic superspace and also a BRST-charge is given. Unfortunately the large number of harmonic variables seems to obstruct the use of this BRST-charge to formulate a field theory.

Understanding the properties of the superparticle[7] will help to elucidate features of the superstring. In [8] this model is quantized covariantly by introducing some new degrees of freedom (for some other approachs see [9]). In this approach first and second class constraints can be separated covariantly but the first class constraints are linearly dependent. Unfortunately in this lagrangian formalism it is obscure how to construct a nilpotent charge which acts like a BRST- charge.

The path integral quantization of a hamiltonian system of any rank, with linearly dependent (reducible) or independent (irreducible) first class constraints can be achieved by making use of the BFV- BRST quantization method[10]. In this case a nilpotent operator which will be called the BFV-BRST-charge, is easily obtained even before gauge fixing. Operator quantization of a hamiltonian system of irreducible, first class constraints is given in [11]. This is extended to the case where the system is defined in terms of some irreducible second class constraints in [12] and to the system of first and second class irreducible constraints in [13], without making use of Dirac brackets, which is equivalent to imposing the vanishing of second class constraints strongly (in [14] it is noted that this formalism can be used to quantize the superparticle). Once one has understood the above cases, probably it will be possible to enlarge the formalism to embrace also the reducible constraints. In this connection applying this method to massive, abelian (Yang-Mills) theory, which has only second class irreducible constraints, seems useful for understanding how it works. In [15] the usual lagrangian quantization is given and in [16] BRST quantization (in lagrangian formalism) of the non-abelian case is performed.

We recall briefly the BFV - BRST operator quantization. Suppose that a lagrangian is given , the usual procedure of classical mechanics yields a hamiltonian system of some linearly independent constraints[17]. Once these are known one defines quantum commutator relations without bothering about the classes of the constraints. In the case of having only second class constraints, by making use of the fact that the matrix of the commutators of the constraints is invertiable, one adds some dynamical coordinates to get new constraints which are effectively first class. Now constructing the BFV - BRST charge is an easy task by making use of the method which is developed for the hamiltonian sytems of first class constraints. One can use this nilpotent charge to define a unitarizing hamiltonian which now has as the canonical part a hamiltonian which is invariant under the transformations which are generated by the new first class constraints. This hamiltonian which gives also the correct order of the anticommuting operators can be used to define the generating functional of the Green functions in the path integral approach by replacing the commutators with the Poisson brackets with the well known overall factors.

For our example we will show that in a canonical gauge the contribution to the scattering amplitudes is only from the transversal modes and in a relativistic gauge this follows from a general result : up to a canonical transformation, the hamiltonian of a system of bosonic first class constraints is invariant under $Osp(1,1|2)^m$ ( m = number of constraints). Thus each pair of Grassmannian ghost coordinates eliminate two bosonic degrees of freedom by the Parisi-Sourlas mechanism [18]. Then the correct counting of the degrees of freedom is achieved (the use of the Parisi-Sourlas mechanism to get rid of the unphysical degrees of freedom in the BFV - BRST quantization was first pointed out in [19]).

The Proca lagrangian which describes massive, abelian theory (we will not explicitly write the coordinate dependence of fields unless necessary) is



$$L = \int dx \ \{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu \} \tag{1}$$

where $g_{\mu\nu} = (1, -1, \ldots, -1)$, $\mu = 0, 1, \ldots, d-1$ and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad ,$$

leads to the following canonical momenta ($i = 1, \ldots, d-1$)

$$P_0 = 0$$

$$P^i = \frac{\delta L}{\delta(\partial_0 A_i)} = -F^{0i} \quad .$$

Hence the canonical hamiltonian reads

$$H_0 = \int dx \ \{ \frac{1}{2} P_i P_i - P_i \partial_i A_0 - \frac{1}{2} m^2 A_\mu A^\mu + \frac{1}{4} F_{ij} F_{ij} + u \ P_0 \} \ . \tag{2}$$

where u are some yet unspecified coefficents. Poisson brackets of the canonical variables are defined as (we deal only with the equal time Poisson brackets and commutators)

$$[A_\mu(x) , P_\nu(y)] = g_{\mu\nu} \delta(\mathbf{x} - \mathbf{y}).$$

$P_o(x)$ are primary constraints and $dP_o(x)/dt \approx 0$ yields the secondary constraints

$$K(x) = \{ P_0(x) , H_0 \} = -\partial_i P_i + m^2 A_0 \ .$$

There are no more constraints as $dK(x)/dt \approx 0$ leads to

$$u = \partial^i A^i \ .$$

Once the canonical hamiltonian and the constraints are found, we define the following commutators

$$[A_\mu(x) , P_\nu(y)] = i\hbar \, g_{\mu\nu} \delta(\mathbf{x} - \mathbf{y}) ,$$

which can be used to calculate the algebra of the constraints:

$$[P_0(x) , P_0(y)] = 0 ,$$

$$[K(x) , K(y)] = 0 , \tag{3}$$

$$[K(x) , P_0(y)] = i\hbar \, m^2 \delta(\mathbf{x} - \mathbf{y}) .$$

Therefore the constraints are second class.

We introduce the ghost fields

$\bar{\wp}_a , C_a \quad : \quad \varepsilon(\bar{\wp}_a) = \varepsilon(C_a) = 1 \quad ; \quad gh(\bar{\wp}_a) = gh(C_a) = 1 \quad ; a, b = 1,2$ ,

where $\varepsilon$ and gh indicate the Grassmann parity and the ghost numbers respectively. $C_a$ and $\bar{\wp}_a$ are, respectively, hermitian and antihermitian. They satisfy the following commutation relations (due to not having gauge symmetry, i.e. first class constraints, introducing these ghost fields is not justified at this level but in the following we will see that infact this is the correct procedure)

$$[ C_a(x) , \bar{\wp}^b(y) ] = i\hbar \delta_a^b \delta(\bar{x} - \bar{y}) \; .$$

Because of having second class constraints the fermionic charge $\Omega$ which one constructs for the case where there are only first class constraints is

$$\Omega = \int dx \{ P_0 C^1 + K C^2 \} + \int dx_1 ... dy_{n+1} \sum \frac{1}{(n+1)!n!} \bar{\wp}_{a_1} ... \bar{\wp}_{a_n} U^{a_1..a_n}_{b_1..b_{n+1}} C^{b_1} ..... C^{b_{n+1}}$$

and cannot be nilpotent. One can construct a nilpotent operator by the help of some new fermionic operators

$$\Omega_a = V^a_b C^b + \sum \frac{1}{(n+1)!n!} \int d x_1 ... d y_{n+1} \bar{\wp}_{a_1} ... \bar{\wp}_{a_n} V^{a a_1..a_n}_{b_1..b_{n+1}} C^{b_1} .... C^{b_{n+1}}$$

which are defined to satisfy

$$\{ \Omega , \Omega \} = \int d x d y \, \Omega^a (x) \, \omega_{ab} (x,y) \, \Omega^b (y)$$

$$\{ \Omega^a , \Omega \} = 0 \qquad (4)$$

$$\{ \Omega^a , \Omega^b \} = 0$$

where $\omega_{ab}$ is an arbitrary invertible, antisymmetric, c-number matrix which we choose, for convenience, as

$$\omega_{ab} (x,y) = \delta(x - y) \, \varepsilon_{ab} .$$

One can show that the following operators satisfy (4).

$$\Omega = \int d x [P_o C^1 + K C^2] \; ,$$

$$\Omega^1 = \frac{m}{\sqrt{2}} [ C^1 + C^2 ] \; , \qquad (5)$$



$$\Omega^2 = \frac{m}{\sqrt{2}} [ C^1 - C^2 ] \, .$$

Introduction of bosonic, zero ghost number, hermitian operators $\Phi_a(x)$, which satisfy

$$[\Phi_a(x), \Phi_b(y)] = - i\hbar \, \delta(x - y) \varepsilon_{ab}$$

makes it possible to build a fermionic charge

$$\Omega' = \int d x [ P_0 C^1 + K C^2 + \frac{m}{\sqrt{2}} \Phi_2 (C^1 + C^2) - \frac{m}{\sqrt{2}} \Phi_1 (C^1 - C^2) ] \qquad (6)$$

which is nilpotent

$$(\Omega')^2 = 0 \, . \qquad (7)$$

This can be written in a more familiar form

$$\Omega' = \int d x T_a C^a \qquad (8)$$

where

$$T_1 = P_0 - \frac{m}{\sqrt{2}} ( \Phi_1 - \Phi_2 )$$

$$\qquad (9)$$

$$T_2 = K + \frac{m}{\sqrt{2}} ( \Phi_1 + \Phi_2 )$$

which are effectively first class constraints

$$[ T_a , T_b ] = 0 \, . \qquad (10)$$

Now our task is to construct a unitarizing hamiltonian. To this end we introduce some other fermionic operators which satisfy

$$\{ \Omega^a (x) , \bar{\Omega}^b (y) \} = i\hbar \, \delta^{ab} \, \delta( x - y) \, ,$$

$$\{\bar{\Omega}_a , \bar{\Omega}_b \} = 0 \qquad (11)$$

$$\bar{\Omega}_a \, \Omega^a = \bar{\wp}_a \, C^a \; .$$

These operators can easily be written by using (5) as

$$\bar{\Omega}_1 = \frac{1}{\sqrt{2} \, m} ( \bar{\wp}_1 + \bar{\wp}_2 )$$

$$\bar{\Omega}_2 = \frac{1}{\sqrt{2}\, m} ( \bar{\wp}_1 - \bar{\wp}_2 ) .$$

In terms of these operators the hermitian operator H which satisfies

$$[ H , \Omega' ] = 0 \qquad (12)$$

is given as

$$H = H_0 + (i\hbar)^{-2} \int d\mathbf{x} [H_0 , \{\Omega , \bar{\Omega}_a\} ] \Phi^a + (i\hbar)^{-4} \int d\mathbf{x} d\mathbf{y} \left[ [H_0 , \{\Omega , \bar{\Omega}_a (y)\} ] , \{\Omega , \bar{\Omega}_b (x)\} \right] \Phi^b(x) \Phi^a(y)$$

$$= H_0 + \int d\mathbf{x} \left( m^{-1} K \eta - m^{-1} (\partial_i \partial_i P_0 ) \zeta + \frac{1}{2} \eta^2 + \frac{1}{2} (\partial_i \zeta) (\partial_i \zeta) \right) \qquad (13)$$

where

$$\zeta = \frac{1}{\sqrt{2}} ( \Phi_1 - \Phi_2 ) , \quad \eta = \frac{1}{\sqrt{2}} ( \Phi_1 + \Phi_2 )$$

so they satisfy

$$[\eta (x) , \zeta (y) ] = i\hbar\, \delta(\mathbf{x} - \mathbf{y}).$$

As is well known there exists also a gauge invariant formulation of massive, abelian theory due to Stueckelberg. The lagrangian is given in terms of Stueckelberg field $\vartheta$ as

$$L = \int d\mathbf{x} \left\{ - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 (A_\mu - m^{-1} \partial_\mu \vartheta )^2 \right\} .$$

Definition of the canonical momenta leads to the following constraints and the canonical hamiltonian

$$G_1 = P_0 + m \vartheta ,$$

$$G_2 = - \partial_i P_i + m^2 A_0 + m\, p_\vartheta$$

$$H_0 = \int d\mathbf{x} \left\{ \frac{1}{2} P_i P_i - P_i \partial_i A_0 - \frac{1}{2} m^2 A_\mu A^\mu + \frac{1}{4} F_{ij} F_{ij} + m \vartheta\, \partial_i A_i + \right.$$

$$\left. + \frac{1}{2} p_\vartheta^2 + \frac{1}{2} (\partial_i \vartheta)^2 + u\, (P_0 - m \vartheta ) \right\} .$$

If we set $p_\vartheta = \eta$, $\vartheta = \zeta$ the constraints are then the same as (9) and the canonical hamiltonian is equivalent (weakly) to (13) in the gauge $u = \partial_i A_i$. So that the BFV scheme of treating the secondary constraints is a systematic way of finding the Stueckelberg fields.



Now due to having a system of first class constraints we enlarge the phase space by hermitian lagrange multipliers $\lambda_a$ and their canonical partners $\pi_a$.

$$[ \lambda_a(x) , \pi_b (y)] = i\hbar\, \delta_{ab} \delta(\mathbf{x} - \mathbf{y}) \; ; \; \varepsilon(\lambda_a) = \varepsilon(\pi_a) = 0 \; ; \; gh( \lambda_a) = gh( \pi_a ) = 0.$$

Thus we need to add also the ghost fields

$$\bar{C}_a , \wp_a \quad : \quad \varepsilon(\bar{C}_a ) = \varepsilon(\wp_a ) = 1 \quad ; - gh(\bar{C}_a ) = gh(\wp_a ) = 1 ;$$

$$\{\wp_a(x) , \bar{C}^b(y) \} = i\hbar\, \delta_a^b\, \delta(\mathbf{x} - \mathbf{y}) , \quad (\bar{C}_a)^\dagger = C_a \quad (\wp_a)^\dagger = \wp_a .$$

Nilpotent BFV-BRST-charge now reads

$$\Omega'' = \Omega' + \int \pi_a \wp^a \, d\mathbf{x} . \qquad (14)$$

Subsidiary conditions which define the physical states are

$$\Omega'' |phys.\rangle = 0 \qquad |phys.\rangle \neq \Omega'' | ...... \rangle . \qquad (15)$$

Without loss of generality we may suppose that on shell all of the fields which are already introduced satisfy Klein - Gordon equations with mass m (infact this means that we work in a specific gauge but gauge independence of the system guarantees the generality of the results). Due to the abelian nature of the effectively first class constraints $T_a$ , after a Fourier analyze of the fields in terms of plane wave solutions of Klein - Gordon equations, one can show that (15) yields[21]

$$T_a^{(+)} |phys.\rangle = 0 \qquad (16)$$

$$\pi_a^{(+)} |phys.\rangle = 0 \qquad (17)$$

where $|phys.\rangle$ does not depend on the ghost fields and (+) indicates the positive frequency part of the Fourier expansion. (17) eliminates $\lambda_a$ and $\pi_a$ so that the physical states are the functions of the fields which have the following Fourier expansions

$$A_\mu(x) = \int d^d k \sum_{\lambda=1}^{4} \{ \varepsilon_\mu^{(\lambda)}\, a_{(\lambda)} (k)\, e^{-ik.x} + \varepsilon_\mu^{*(\lambda)}\, a_{(\lambda)}^\dagger (k)\, e^{ik.x} \}$$

$$P_\mu(x) = -i \int d^d k \sum_{\lambda=1}^{4} \{ \varepsilon_\mu^{(\lambda)}\, a_{(\lambda)} (k)\, e^{-ik.x} - \varepsilon_\mu^{*(\lambda)}\, a_{(\lambda)}^\dagger (k)\, e^{ik.x} \}$$

$$\zeta(x) = i\int d^d\kappa \{ b(k)\, e^{-ik.x} - b^\dagger(k)\, e^{ik.x} \}$$

$$\eta(x) = -\int d^d\kappa \{ b(k)\, e^{-ik.x} + b^\dagger(k)\, e^{ik.x} \} \quad .$$

where $\epsilon_\mu(\lambda)$ are polarization vectors which we choose as $\epsilon_\mu(\lambda) = \delta_\mu{}^\lambda$ and

$$d^d\kappa = \frac{d^dk}{\sqrt{2}\,(2\pi)^{d-1}}\ \delta(k^2 - m^2)\,\theta(k_0).$$

$a, b$ ($a^\dagger, b^\dagger$) are the usual annihilation (creation) operators which act on the ground state $|0\rangle$ as

$$a_{(\lambda)}|0\rangle = 0 \qquad b\,|0\rangle = 0$$

In the frame where $k_i = 0$ (16) leads to

$$[\, a_{(0)} - \frac{m}{\sqrt{2}}\, b \,]\ |phys.\rangle = 0.$$

which means that the introduction of the canonical pair ($\zeta$, $\eta$) is equivalent to introducing a longitudional photon which eliminates the scalar one[15]. So that the physical subspace is spanned by the states which are created by $a^\dagger_{(1)}, \ldots, a^\dagger_{(d-1)}$.

[The same result can be obtained without making explicit reference to the equations of motion: perform a canonical transformation to get $\mathfrak{P}_a = T_a$ where $\mathfrak{P}_a$ are the momenta conjugate to new coordinates $\mathfrak{X}_a$. So that a state in a functional representation is $\Psi = \Psi(\mathfrak{X}_a, A_i, C_a, \wp_a, \lambda_a)$ . Now (14) reads as

$\{ \mathfrak{P}_a(x)\, C_a(x) + \pi_a(x)\, \wp_a(x) \}\, \Psi_{phys.} = 0$ at each x and the usual procedure[21] leads to the result that $\Psi_{phys.} = \Psi_{phys.}(A_i)$]

The unitarizing hamiltonian, H', which satisfies

$$[\Omega'', H'] = 0,$$

is given by

$$H' = H + (i\hbar)^{-1} \{ \Psi, \Omega'' \}, \tag{18}$$

where $\Psi$ is gauge fixing fermion which can be taken as follows



$$\Psi = \int d\,x \{ \bar{\wp}_a \lambda^a + \bar{C}_a \chi^a \}.$$

$\chi_a$ are gauge fixing functionals so that $[\chi_a, T_b]$ should be invertible.

A canonical gauge fixing is possible without any restriction on the boundary conditions of the ghost fields[21]. A canonical gauge fixing is

$$\chi_a = \frac{d\lambda_a}{dt} + \tilde{\chi}_a$$

where $\tilde{\chi}_a$ doesn't depend on $\lambda_a$ and satisfies

$$[\tilde{\chi}_a(x), T_b(y)] = -i\hbar\, \delta_{ab}\, \delta(\mathbf{x}-\mathbf{y}).$$

Under these conditions uniterizing hamiltonian reads

$$H' = H + \int d\,\mathbf{x}\, \{ T_a \lambda^\alpha + \bar{\wp}^a \wp_a + \bar{C}_a C^a + \tilde{\chi}_a \pi^a - (d\bar{C}^a/dt)\, \wp_a + \pi^\alpha d\lambda_a/dt \}.$$

By replacing the quantum commutators with the Poisson brackets we may use this hamiltonian in the phase space path integral. For instance the generating functional of the Green functions of the physical states is given by

$$Z(J, G) = N \int \prod_x \prod_\alpha \prod_z dP_z dQ_z \exp\{(i/\hbar) \int d^dx\, [\, P_z\, dQ_z/dt - H'' + J_i A^i + G_i P^i \,]\}$$

where

$$Q_z = \{ A_\mu, \eta, \lambda_a, C^a, \bar{C}_a \} ; \quad P_z = \{ P_\mu, \zeta, \pi^a, \bar{\wp}_a, \wp^a \}.$$

Ghost fields decouple and $\lambda_a, \pi_a$ are Lagrange multipliers. Integration over the latter fields leads to $T_a = \tilde{\chi}_a = 0$. For example the following choice of the gauge fixing function

$$\tilde{\chi}_1 = (1/\sqrt{m})\, \eta \quad , \quad \tilde{\chi}_2 = (1/\sqrt{m})\, \zeta \ ,$$

yields

$$Z(J,G) = N' \int \prod_x \prod_\alpha dP_i\, dA_i\, \exp[\int d^dx\, [\, P^i dA_i/dt - P_i P_i/2 - (m^2/2) A_i A_i - (1/2m^2)(\partial_i P_i)^2 - (1/4) F_{ij} F_{ij} + J_i A^i + G_i P^i \,]$$

so that the theory is unitary.

Now we will show that in the case of a relativistic gauge fixing the cancellation of bosonic degrees of freedom with the fermionic ones is (at least locally) a general property of BFV -BRST quantization when a system is described by some linearly independent first class constraints.

Consider a system of a hamiltonian $H_0$ and some linearly independent, bosonic first class constraints $T_a$ ($a=1, \ldots, m$) which are functions of the original phase space variables $q_i, p_i$ ($i=1, 2, \ldots, n$) satisfying

$$[T_a, T_b] = i\hbar U_{ab}{}^c T_c,$$

$$[T_a, H_0] = i\hbar V_a{}^b T_b.$$

When $U_{ab}{}^c \neq 0$, $V_a{}^b \neq 0$ by a canonical transformation (at least locally) we may replace $T_a$ and $H_0$ with $T'_a$ and $H'_0$ which now satisfy [20]

$$[T'_a, T'_b] = 0,$$

$$[T'_a, H'_0] = 0.$$

We extend the phase space by adding bosonic canonical pairs $(\lambda^a, \pi_a)$ and fermionic ghost pairs $(C^a, \bar{\wp}_a)$, $(\bar{C}_a, \wp^a)$, as above. The unitarizing hamiltonian is defined as

$$H = H'_0 + (i\hbar)^{-1} [\Omega', \Psi],$$

where

$$\Omega' = T'_a C^a + \pi_a \wp^a,$$

$$\Psi = \bar{\wp}_a \lambda^a + \bar{C}_a \chi^a,$$

as before; but now we choose a relativistic gauge where $\chi^a$ is independent of $\lambda^a$. When the gauge fixing function is chosen such that

$$[\chi^a, T'_b] = -i\hbar \delta^a{}_b$$

it is possible to construct the following Osp(1,1|2) group[22] generators for each a[19] ($\hbar = 1$),



$$J_{+\theta} = -T'_a C^a + \pi_a \wp^a, \quad J_{+\bar{\theta}} = i(T'_a \bar{C}^a + \pi_a \bar{\wp}^a),$$

$$J_{-\theta} = \lambda^a C^a + \chi_a \wp^a, \quad J_{-\bar{\theta}} = -i(\lambda^a \bar{C}^a + \chi_a \bar{\wp}^a),$$

$$J_{+-} = -2(\pi_a \lambda^a + T'_a \chi_a),$$

$$J_{\theta\bar{\theta}} = (1/2)(\bar{C}^a \bar{\wp}^a + C^a \wp^a), \quad J_{\theta\theta} = i\bar{C}^a \wp^a, \quad J_{\bar{\theta}\bar{\theta}} = iC^a \bar{\wp}^a.$$

(without summation over a).

Now H' is invariant under Osp(1,1|2) ⊗. . . . . ⊗Osp(1,1|2) -(m-times), if we choose the gauge fixing function such that it leaves $H'_0$ invariant:

$$[\chi_a, H'_0] = 0. \tag{19}$$

Now in all the Green functions which result from the hamiltonian H' one may use the Parisi-Sourlas mechanism[18] to count the contribution of a fermionic coordinate to the total number of degrees of freedom as (-1) and a bosonic one as (+1). This counting yields

$$\#\text{degrees of freedom} = \#q_i + \#\lambda_a - \# C_a - \#\bar{C}_a$$
$$= n - m.$$

Then also in massive, abelian theory a relativistic gauge fixing condition, which also satisfies (19), the contribution to the path integral is only due to the physical degrees of freedom.

In the non-abelian case solutions to (4) are not easy to find. As we have shown the Stueckelberg formalism leads directly to the effective first class constraints of the BFV scheme in the abelian case. There exists a similar formalism also in the non-abelian case (see [23] and the references there in) which we can use, so that we don't need to solve (4).

ACKNOWLEDGMENTS

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. He would also like to thank E.S. Fradkin and G. Thompson for helpful discussions.

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