Beam Profile
Effects on
NPB Performance

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ABSTRACT

A comparison of neutral particle beam brightness for various neutral beam profiles indicates that the widely used assumption of a Gaussian profile may be misleading for collisional neutralizers. An analysis of available experimental evidence shows that lower peaks and higher tails, compared to a Gaussian beam profile, are observed out of collisional neutralizers, which implies that peak brightness is over estimated, and for a given NPB platform-to-target range, the beam current (power), dwell time or some combination of such engagement parameters would have to be altered to maintain a fixed dose on target. Based on the present analysis, this factor is nominally about 2.4 but may actually be as low as 1.8 or as high as 8. This is an important consideration in estimating NPB constellation performance in SDI engagement contexts.

I. INTRODUCTION

Several studies of the power requirements for neutral particle beam (NPB) weapon platforms are available.[1, 2, 3] These investigations indicate that the power requirements are sensitive to a number of parameters including beam divergence, dwell time, target range and lethality requirements. One assumption common to all of these studies is a Gaussian shape for the beam power profile. However, there is experimental evidence [1] that the power profile resulting from the use of foil or gas neutralizers is significantly less peaked than a Gaussian, having more particles in the wings of the distribution. This experimental evidence supports the intuitive realization that a negative ion beam

(the precise shape of the ion beam is also not well known) from the accelerator would likely experience spreading owing to collisions with the neutralizer gas or foil atoms.

A collisionally neutralized beam has a broad profile (in comparison to the Gaussian-like shape possible with the ion beam) and will therefore have a lower peak value for the same input power. A larger neutral beam power would be required if the peak brightness is to be maintained for these collisionally broadened profiles. The enhanced power requirement (with respect to that assuming a Gaussian beam profile) will be quantified in the following sections. However, the results obtained there should not be interpreted as indicating that the beam power for an operational system must necessarily be increased to make the peak brightness equal to that for the Gaussian beam. This situation exists because of the flexibility with which target assignments can be made to individual NPB platforms when these platforms are deployed in a constellation and because other parameters (e.g., dwell time) can be varied to compensate for the reduction in brightness. Preliminary engagement simulations indicate that the NPB weapon system, considered as a whole, may be robust in this respect. The results obtained herein do indicate that the specification of the proper profile is a potentially important effect and should be considered in NPB engagement analyses.

Laser neutralizers do not exhibit the same beam spreading effect encountered with collisional neutralizers. However, the temptation to assume the profile problem away with the use of a laser neutralizer should be carefully examined. This technology is relatively high risk and would rule out the near-term NPB option. In addition, the laser neutralizer will likely have very significant power requirements of its own that must be factored into the total NPB platform needs.

II. SCATTERING EXPERIMENT

This study examines the experimental (as well as theoretical) evidence that the neutral beam profile produced by a collisional neutralizer is not Gaussian in shape. Analysis of one such experiment was performed at Los Alamos National Laboratory using an H^- ion source and a gaseous neutralizer [1]. The accelerated ion beam is stripped of its extra electrons through collisions with the neutral gas at the expense of angular deflections from the beam center line. The scattering distribution was determined by counting the number of scattered particles hitting a series of horizontally arranged detector strips (only univariate data is obtained).

Assuming that the distribution is spherically symmetric, the scattering data was

described with the following density function:

$$f(x_1, x_2) \propto \frac{1}{(1 + x_1{}^2 + x_2{}^2)^\tau} \quad , \tag{1}$$

where $\tau > 1$. This is the distribution of particles on a plane normal to the beam axis which was assumed to correspond to a Pearson type distribution.[1]

Unfortunately, the results of the experiments are not entirely conclusive. Excellent fits for the data from individual experiments were obtained, but there was a significant spread in the results amongst the various runs.[4] In addition, an unknown fraction of the neutralized particles in the tails were not counted although in most cases this fraction was estimated to be greater than $70 - 80\%$. Although the data fits were constructed taking this fact into account, some uncertainty in the shape of the remote tails exists. However, the following two points are evident from the results:

- The value of τ deduced from the experiments varied between 1.12 and 1.84 with the values centered about ≈ 1.5.

- A Gaussian distribution was not indicated.

Therefore, in the next section an expression is developed for the neutralized beam distribution keeping τ as a variable. The results are then compared with the present Gaussian assumption.

III. PROFILE COMPARISON

Before developing an expression for the neutralized beam distribution, it will be useful to write the distribution used for the experiment in terms of notation commonly used in NPB analyses.[5] First, a scale parameter (with units of length) for the deflections, σ^* is introduced so that Eq.(1) is rewritten

$$f_1(x_1, x_2) \propto \frac{1}{(1 + \frac{x_1{}^2 + x_2{}^2}{\sigma^{*2}})^\tau} \quad . \tag{2}$$

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[4]Information provided by Mark E. Johnson, Los Alamos National Laboratory, Los Alamos, NM 87545, May 1987.

[5]This discussion makes some small angle assumptions which do not significantly affect the results.

Next, a transformation is made to polar coordinates using Fig. 1 to give

$$f_2(r, \phi) \equiv f_1(r \cos \phi, r \sin \phi) \propto \frac{1}{(1 + \frac{r^2}{\sigma^{*2}})^\tau} \quad . \tag{3}$$

In terms of common NPB terminology, r is then a spot radius at the target, and σ^* is a measure of the beam divergence scaled by the target range, R. Thus, refering to Fig. 2, we may finally write the distribution in terms of the NPB beam divergence $\sigma = \sigma^*/R$ and beam half angle $\theta = r/R$ (for r/R $\ll$ 1) as

$$f(\theta, \phi) \propto \frac{1}{(1 + \frac{\theta^2}{\sigma^2})^\tau} \quad . \tag{4}$$

Now the constant of proportionality (call it C_T) is determined by normalizing the distribution to unity, equivalent to normalizing for unit-radiated power if Eq.(4) is
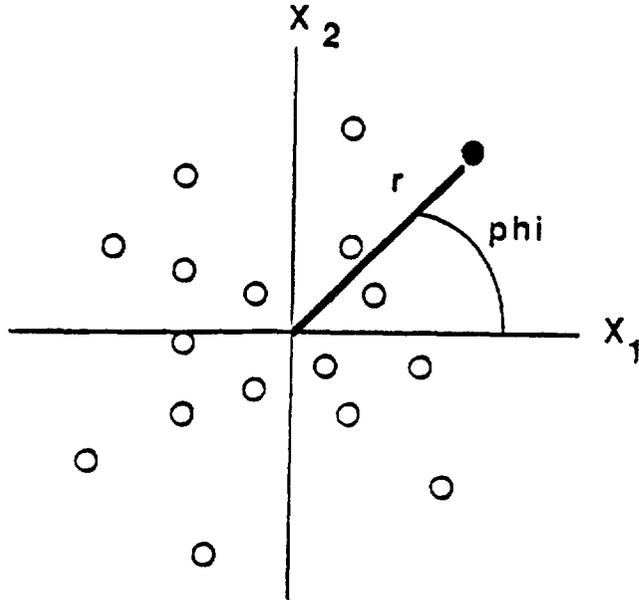

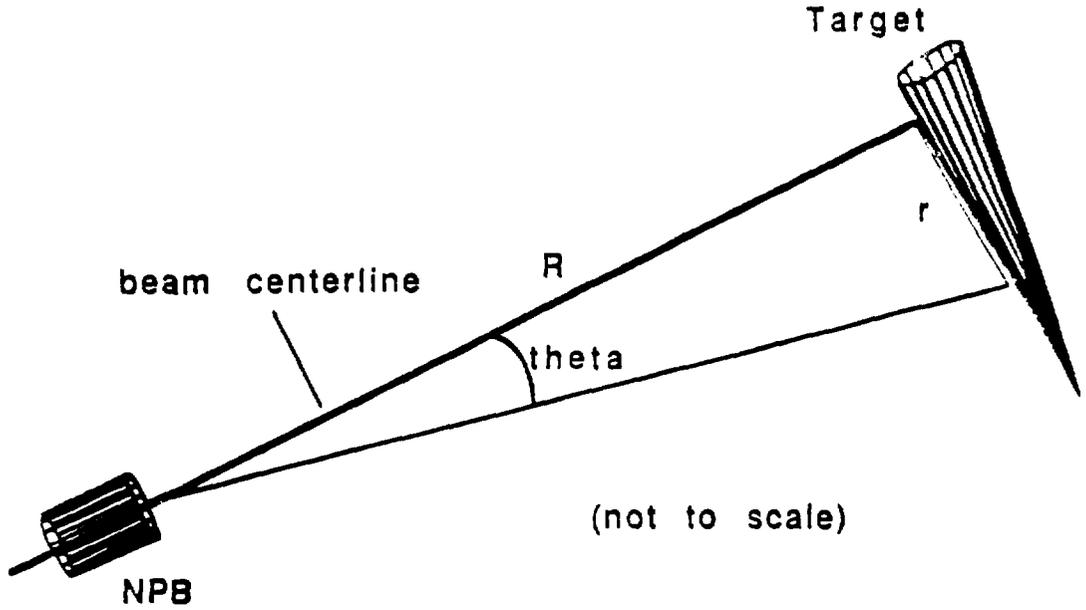Figure 1: Experiment coordinate transformation.

Figure 2: NPB divergence and beam half width.

interpreted as brightness. Thus to find C_T we solve in the spherical polar co-ordinate system shown in Fig. 3:

$$\int_0^{2\pi} d\phi \int_0^{\pi} \frac{C_T}{(1 + \frac{\theta^2}{\sigma^2})^T} \sin\theta d\theta = 1 \quad . \tag{5}$$

Then Eq.(4)becomes

$$f(\theta) = \frac{\tau - 1}{\pi\sigma^2 \left[1 + \frac{\theta^2}{\sigma^2}\right]^T} \quad . \tag{6}$$

The inequality $\tau > 1$ must hold as a consequence of the convergence requirement placed on the integral evaluated to obtain Eq.(6). This, then, is the general expression for the neutral beam distribution as a function of the distributions characteristic exponent τ and the beam parameters beam half angle and beam divergence.
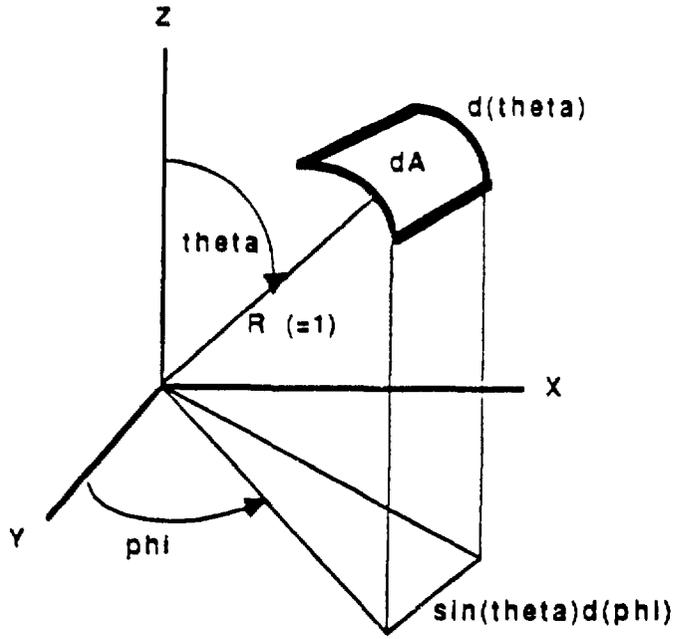
Figure 3: Coordinate system for normalization.

This distribution must be compared with an analagous expression for a Gaussian. These representations will be used to compare the power requirement sensitivity tp profile assumptions with a single engagement expression for power requirements. Rather than use Eq.(6) for $\tau \to \infty$ we will use the Gaussian form commonly seen in NPB analysis for the comparison. That is

$$g(\theta) \propto \exp \frac{-\theta^2}{2\sigma_g^2} \quad , \tag{7}$$

where σ_g is the beam divergence for the Gaussian distribution.

Again normalizing for unit radiated power, we find the normalization constant C_g with

$$\int_0^{2\pi} d\phi \int_0^\pi C_g \exp \frac{-\theta^2}{2\sigma_g^2} \sin\theta d\theta = 1 \tag{8}$$

so that the Gaussian takes the form

$$g(\theta) = \frac{\exp \frac{-\theta^2}{2\sigma_g{}^2}}{2\pi\sigma_g{}^2} \quad . \tag{9}$$

Now to compare these distributions on a consistent basis we need a relationship between σ and σ_g. Here we choose to require that the full width at half maximums (FWHMs, written as $2\beta_{0.5}$) for the two profiles are equal. This criterion is unambiguous, has precedence in antenna theory for the comparison of distributions, and has been adopted in other NPB work [2].

The Gaussian is maximum at $\theta = 0$ so that the half maximum value is $\frac{1}{4\pi\sigma_g{}^2}$. Solving

$$\frac{1}{4\pi\sigma_g{}^2} = \frac{\exp \frac{-\theta^2}{2\sigma_g{}^2}}{2\pi\sigma_g{}^2} \tag{10}$$

for the theta value at half maximum, we find the FWHM for the Gaussian distribution to be

$$2\beta_{0.5} = 2(2\ln 2)^{\frac{1}{2}}\sigma g \quad . \tag{11}$$

Following a similar procedure for the generalized distribution we find

$$2\beta_{0.5} = 2\sigma\sqrt{2^{\frac{1}{T}} - 1} \quad . \tag{12}$$

Equating these we find the relationship between σ and σ_g to be

$$\sigma^2 = \frac{2\ln 2}{2^{\frac{1}{\tau}} - 1}\sigma_g^2 \quad .$$

(13)

Fig. 4 shows a comparison plot of these distributions for $\tau = 3/2$ and $\tau = 1.1$. Note that these distributions are significantly less peaked and broader than the Gaussian. In fact, from Eqs.(6)and(9)the ratio of the Gaussian to the generalized peak brightness for equal FWHM beam widths is $\frac{C_g}{C_\tau}$

$$\frac{C_g}{C_\tau} = \frac{\ln 2}{(\tau - 1)(2^{\frac{1}{\tau}} - 1)} \quad .$$

(14)

A plot of the same three distributions having the same peak value (and equal half power points) is shown in Fig. 5. The excellent agreement in shape for theta well past the half power points is an illustration of the strength of the approach suggested by

PROFILE COMPARISON

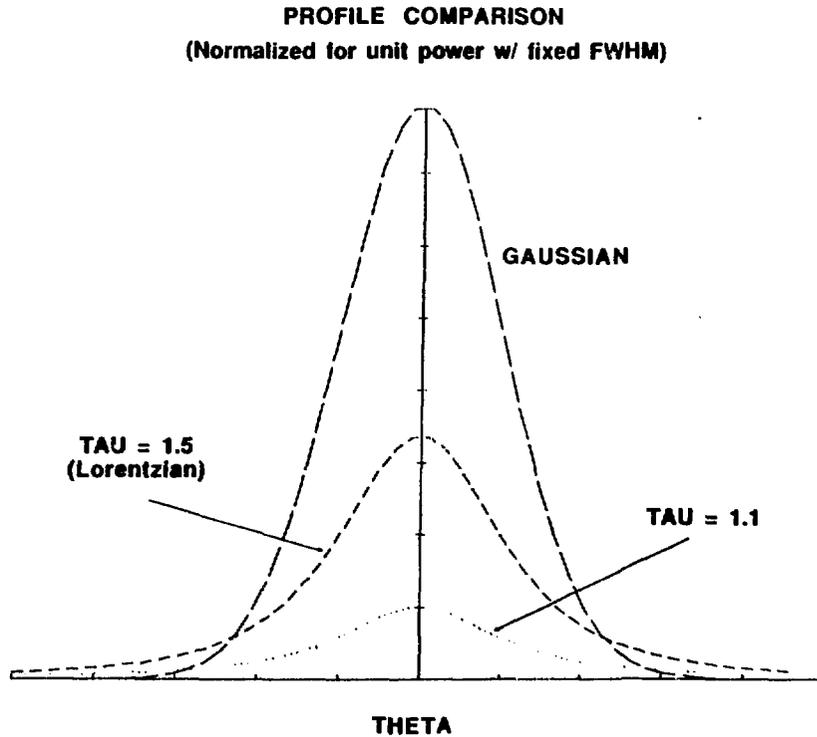**(Normalized for unit power w/ fixed FWHM)**

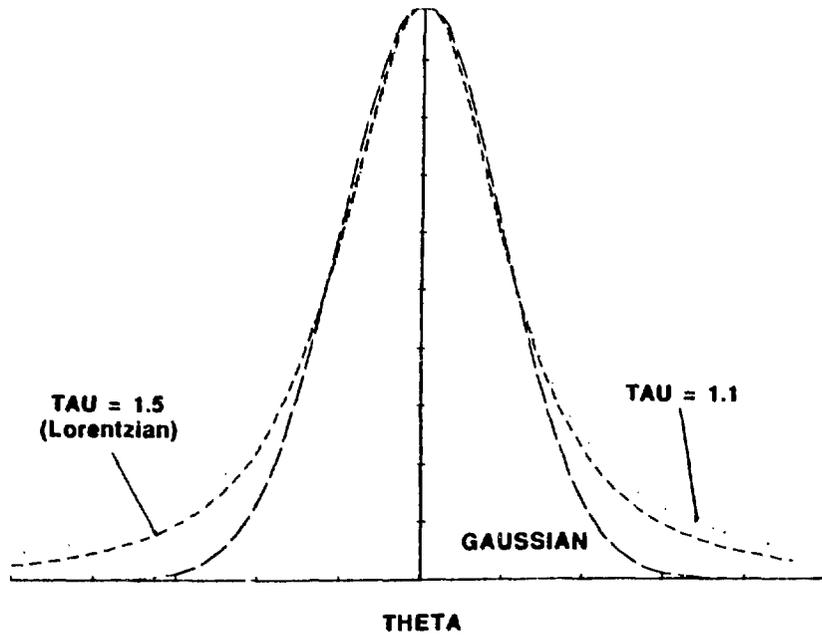Figure 4: Profile comparison normalized for unit power.

Figure 5: Profile comparison with fixed peak.

Graves [2] for an NPB beam descriptor. That is, the Gaussian shape in combination with correct values of beam half width and peak brightness is an adequate and useful description of the neutral beam if the target falls within the half power beam width. However, the beam peak and width must be determined with the correct profile and on a consistent basis.

IV. NPB WEAPON POWER

In this section, the neutral beam power for a beam with a generalized profile of the form of Eq.(1) will be compared with that for a Gaussian beam with the same FWHM beam width under the constraint that energy is to be delivered to the target at the same rate. As discussed earlier, these results should not be interpreted as indicating that the beam power for an operational system should necessarily be increased because a variety of parameters of the complete deployed NPB weapon system can be varied to achieve the same lethality. Rather, the results indicate that some performance degradation could occur relative to a system with an idealized Gaussian beam, that system analysis is in order to determine the performance of an operational system

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with peak brightness reduced in the indicated manner, and that system tradeoffs are necessary to upgrade performance to the desired level if the performance degradation is found to be unacceptable.

To begin, the power requirements as a function of the beam characteristics and lethality parameters using the generalized distribution is derived. This derivation is quite similar to that for the Gaussian beam presented previously[6] but is presented here to illustrate the assumptions involved and to form a basis of comparison.

The beam flux in W/m^2 (MKS) is simply the total power in the beam, IE, divided by the total area through which the power passes times the shaping factor. Thus

$$\Phi = \frac{IE}{4\pi R^2} K \frac{1}{\left[1 + \frac{\theta^2}{\sigma^2}\right]^\tau} \quad , \tag{15}$$

where I is the beam current, E is the beam energy and R is the range of the target. The normalization constant K is found by demanding that the integral of the flux over the surface of a sphere of radius R is equal to the total power out of the beam. Thus

$$IE = \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin\theta \ R^2 \frac{IE}{4\pi R^2} K \frac{1}{\left[1 + \frac{\theta^2}{\sigma^2}\right]^\tau} \quad . \tag{16}$$

Integrating for symmetry in ϕ and small angles θ, we find

$$K = \frac{4(\tau - 1)}{\sigma^2} \quad , \tag{17}$$

and the beam flux becomes

$$\Phi(R, \theta) = \frac{IE}{\pi R^2 \sigma^2} \frac{\tau - 1}{\left[1 + \frac{\theta^2}{\sigma^2}\right]^\tau} \quad . \tag{18}$$

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[6]Information provided by Ronald J. Adler and Paul J. Van Zytveld, Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico 87117, February 1984.

The next step is to relate the beam flux to the volumetric dose in the target. The flux is degraded upon entering the target with the derivative

$$\frac{d\Phi}{dx} = \frac{I(\tau - 1)}{\pi R^2 \sigma^2 \left[1 + \frac{\theta^2}{\sigma^2}\right]^\tau} \frac{dE}{dx} \quad , \tag{19}$$

where x is the distance of penetration into the target (x = 0 at target surface). This equation is the volumetric dose (watts/m^3 in MKS). To obtain the dose delivered to the target in J/kg (MKS), we first divide the above equation by the density of the target material to obtain the dose in power per unit mass or watts/kg in MKS. We also define a new parameter, the penetration depth, $\Delta = \rho x$ with the units kg/m^2 in MKS and obtain

$$\frac{d\Phi}{d\Delta} = \frac{I(\tau - 1)}{\pi R^2 \sigma^2 \left[1 + \frac{\theta^2}{\sigma^2}\right]^\tau} \frac{dE}{d\Delta} \quad . \tag{20}$$

Representing the dose as 'W' in its usual units of energy/mass (J/kg in MKS), we note that the left hand side of the above equation for the power per unit mass delivered to the target is the rate at which the dose W is delivered. Thus the equation is rewritten

$$\frac{dW}{dt} = \frac{I(\tau - 1)}{\pi R^2 \sigma^2 \left[1 + \frac{\theta^2}{\sigma^2}\right]^\tau} \frac{dE}{d\Delta} \quad , \tag{21}$$

and the dose delivered to the target in time t is

$$W = \int_0^t \frac{I(\tau - 1)}{\pi R^2 \sigma^2 \left[1 + \frac{\theta^2}{\sigma^2}\right]^\tau} \frac{dE}{d\Delta} dt \quad . \tag{22}$$

To do its job, the NPB must deliver a dose that is considered lethal over some area considered adequate to cover the vulnerable section of the target. The lethal dose and area depend on the kill mechanism assumed for a particular application. The above

equation indicates that the dose changes with theta or, equivalently, with the radius of the lethal spot area. Therefore, if we define a minimum lethal dose at the lethal spot radius, then all points inside the lethal area will receive a dose at or above the lethal dose. To write the equation in terms of these lethality parameters, we note $\theta \approx \frac{r_s}{R}$. Also if we choose the beam energy so that the penetration depth equals the depth of the vulnerable area in the target, then the energy loss rate $\frac{dE}{d\Delta}$ can be written as its average value $\frac{E}{\Delta}$. The result is

$$W = \frac{IE}{\pi\Delta} \int_0^t \frac{\tau - 1}{R^2\sigma^2 \left[1 + \frac{r_s^2}{R^2\sigma^2}\right]^\tau} \, dt \quad . \tag{23}$$

If the properties of the neutral beam are not changing with time, then the only time dependent variable in the integral is target range. That is, during an engagement the range will change with respect to some reference range, R_o (perhaps the distance of closest approach), as $R = R_o + v\,t$, where v is the velocity of the target. Here it will be assumed that there is only a small change in range during a given engagement and the integral evaluates trivially to

$$W = \frac{IEt(\tau - 1)}{\pi\Delta R^2\sigma^2 \left[1 + \frac{r_s^2}{R^2\sigma^2}\right]^\tau} \quad . \tag{24}$$

Rewriting this in terms of I E, we finally have the NPB beam power requirements in terms of the lethality parameters and neutral beam properties:

$$P = \frac{\pi\Delta W R^2\sigma^2 \left[1 + \frac{r_s^2}{R^2\sigma^2}\right]^\tau}{t(\tau - 1)} \quad . \tag{25}$$

Using a similar derivation for the Gaussian distribution, we find the Gaussian beam power requirement to be

$$P_g = \frac{2\pi R^2 \sigma^2 \Delta W}{t \exp \frac{-r_s^2}{2R^2 \sigma_g^2}} \quad . \tag{26}$$

The ratio of the power requirements depending on the beam distribution assumed is then

$$\frac{P}{P_g} = \frac{\sigma^2}{2(\tau-1)\sigma_g^2} \exp\left(\frac{-r_s^2}{2R^2\sigma_g^2}\right)\left[1 + \frac{r_s^2}{R^2\sigma^2}\right]^\tau \quad . \tag{27}$$

Note that the major driver here is $\sigma^2/(\tau-1)\sigma_g^2$. The dependence of this power ratio on values of spot sizes, ranges, and characteristic pattern widths generally of interest is weak. That is, the squared ratio of spot size to the product of range and characteristic width (which we define here to be the range ratio) tends to be a small number, and the two profile correction terms tend to 1. Although this result depends on the values of R_n, R and σ achievable, present projections for these parameters indicate that the range ratio is generally less than 1 and the lethal area is exposed to just the middle (peaked) section of the beam.

Equation(27)has been plotted in Fig. 6 for τ between 1.1 and 1.9 (the experimentally observed values) where the profile terms have been approximated as 1. Note that the power requirement for the more realistic broadened profiles can be as much as eight times that predicted with a Gaussian model. The experimental data appear to be centered about $\tau = 1.5$ (the Lorentzian); therefore, we choose the nominal power increase to be a factor of 2.4.

V. SUMMARY

A comparison of NPB brightness for various neutral beam profiles indicates that the widely used assumption of a Gaussian profile may be misleading for collisional neutralizers.

An analysis of available experimental evidence shows that lower peaks and higher tails, compared to a Gaussian beam profile, are observed out of collisional neutralizers, which implies that peak brightness is over estimated, and for a given NPB platform-to-target range, the beam current (power), dwell time, or some combination of such engagement parameters would have to be altered to maintain a fixed dose on target.
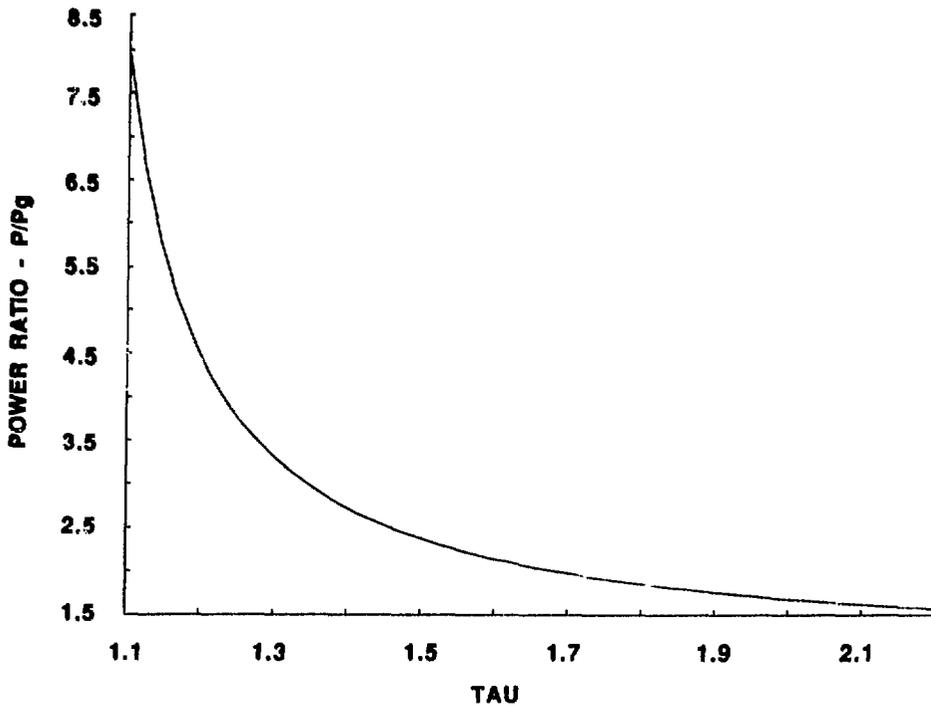
Figure 6: Power ratio versus tau.

Based on the Beckman/Johnson analysis of a Los Alamos experiment [1] and the present analysis, this factor is nominally about 2.4 but may actually be as low as 1.8 or as high as 8. This is an important consideration in estimating NPB constellation performance in Strategic Defense Initiative engagement contexts.

We therefore recommend that (1) the beam profile effect be considered in engagement analysis for NPBs, (2) additional experiments be performed to determine the relationship between the beam width and the peak brightness of the NPB for a given neutral beam current, and (3) the NPB community should adopt a common set of criteria for characterizing the beam width and peak brightness of an NPB [2].

ACKNOWLEDGMENTS

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2. Ross E. Graves, "Characterization of the Beamwidth and Brightness of a Neutral Particle Beam," Los Alamos National Laboratory memorandum S-3:87-65, April 1987.