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LEPTONS AS SYSTEMS OF DIRAC PARTICLES *

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ABSTRACT

Charged leptons are treated as systems of three equal independent Dirac particles in an external static effective potential which has a vector and a scalar term. The potential is constructed to reproduce the experimental mass spectrum of the charged leptons. The Dirac covariant equation for three interacting particles is discussed in order to comment on the magnetic moment of leptons.

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1. Introduction. Three families of quarks and leptons differing essentially only in their masses, suggest the idea that quarks and leptons are clusters of constituents [1]. There is no direct experimental support for this idea, but in spite of that, it seems attractive from the point of view that instead of three (maybe even more) families with two quarks and two leptons each, only a few fermions are elementary particles. These constituents carry additional charge, the source of hypercolour, which confines constituents into a size smaller than 10^{-19} m. In this paper we shall not discuss the meaning of such constituent models or the justification for them. We shall rather study a simple treatment of the dynamics for one of these models, since the dynamics is very unusual due to the fact that localization is much smaller than the corresponding Compton wavelength. The behaviour of the magnetic moment is especially unusual. In Table I radii, Compton wavelengths and the ratios of these two quantities for some typical examples are presented*.

	λ_c (fm)	R (fm)	R/λ_c	MvR
Electron in hydrogen atom	2.10^{3}	10^{5}	2.10^{1}	1
Tritium nucleus (pnn)	4.10^{-1}	2	4	3
Nucleon (qqq)	1.10^{-1}	1	1	5
Electron ($\bar{T}\bar{T}\bar{T}$)	2.10^{3}	$<10^{-3}$	$<10^{-6}$	$<10^{-6}$

Table 1: Compton wavelength λ_c ($=2\pi/M$, M is the mass of the system) is compared to the size of the system R. The third column shows the inverse ratio between these two quantities. The fourth column shows the ratio between the product evR/2 (of the current (I) circulating around the axis and the corresponding area (S)) and the magnetic moment e/2M of the system. It is also, up to a constant, the ratio between the size of the system R and the de Broglie wavelength.

The expectation value of the magnetic moment operator:

$$\hat{\vec{\mu}} = -\frac{1}{2}\int \hat{\vec{J}} \times \vec{r}\, d^3r \qquad (1)$$

which is proportional to current times area, exhibits completely different behaviour at the level of quark and lepton constituents as compared to the cases we are used to handling up to now. This can be seen from the last column in Table I. While the ratio between IS=evR/2 and the magnetic moment e/2M is ~~approximately 1 in the case of an electron in~~ a hydrogen atom or in the case of a proton in a nucleus, it is very small for the constituents of leptons and quarks. We shall discuss this problem again later.

We assume in this paper the Harari $SU(3)_{HC} \times SU(3)_c \times U(1)$ model(2). In this model the charged leptons are systems of three fermions carrying 1/3 of an electric charge, spin, colour and hypercolour. We suppose that the three fermions are Dirac particles determining all the dynamics of the system and that hypergluonic fields and hypermesonic fields ($\bar{T}T$, ...) are taken into account only through the effective interaction. Hypergluonic and hypermesonic fields should be very strong at the level of lepton constituents, much stronger even than at the quark level, leading to an effective potential with scalar and vector terms. We guess, as at the quark level, that the hypergluonic field manifests scalar confining fields. We look for an effective potential which confines the system of three constituents into a volume with $R<10^{-18}$ m, leading to an energy spectrum in agreement with the charged lepton mass spectrum and having levels with spin 3/2 greatly above 20GeV. We shall study the magnetic moments and decay properties of such systems. In order to comment on the magnetic moments of leptons, we present covariant equations of three interacting Dirac particles in an external field**.

2. The equations of motion. We write the action for the system of three particles in the form:

$$S = \int d^4x \Big\{ \sum_{i=1}^{3} \bar{\Psi}_{(i)} \big(i\gamma^{\mu}_{(i)} \partial_{(i)\mu} - m_{(i)} - f_{(i)}\varphi \big) \Psi_{(i)} - g_{(i)} \bar{\Psi}_{(i)} \gamma^{\mu}_{(i)} \Psi_{(i)} A_{\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}\big(\partial_{\mu}\varphi\, \partial^{\mu}\varphi\big) \Big\}. \qquad (2)$$

The hypercolour fields which appear in the action (2) as scalar and vector fields confine the constituents into clusters.

Barut and Komy[5] present the derivation of relativistic equations from the action (2) in the case of an electromagnetic field for two particles. In this paper we shall leave as an open problem the derivation of relativistic equations from the action (2) for three particles interacting with vector and scalar potentials. We shall also not discuss the origin of scalar fields, which is not yet known even for colour interaction among quarks on the hadron level. We shall use scalar fields in order to assure bound states similar to the way quarks are confined in the potential only if scalar fields are included.

Let $\psi(x_1,x_2,x_3)$ be the three particle field, which is 4^3 component spinor and which fulfils the following three body relativistic equations in a covariant form:

$$\{(\gamma^{\mu}_{(1)} i\partial_{(1)\mu} - m_{(1)})\otimes\gamma_{(2)0}\otimes\gamma_{(3)0} + \gamma_{(1)0}\otimes(\gamma^{\mu}_{(2)} i\partial_{(2)\mu} - m_{(2)})\otimes\gamma_{(3)0} +$$
$$\gamma_{(1)0}\otimes\gamma_{(2)0}\otimes(\gamma^{\mu}_{(3)} i\partial_{(3)\mu} - m_{(3)}) -$$
$$\sum_{i<j}[V_{ij}(r_{ij}) + W_{ij}(r_{ij})]\}\psi(x_i) = 0 \qquad (3)$$

with $r_{ij} = [-(x_i-x_j)_{\mu}(x_i-x_j)^{\mu} + (x_i-x_j)_{\mu}\eta^{\mu}]$

where the two potentials representing the scalar and the vector part are written in a covariant form***.

Let us introduce the operators for relative coordinates and centre of mass coordinates by[5]:

$$P = \sum_{i=1}^{3} p_{(i)}, \quad m = m_{(1)} + m_{(2)}, \quad M = \sum_{i=1}^{3} m_{(i)},$$
$$\Pi_1 = (m_{(2)}p_{(1)} - m_{(1)}p_{(2)})/m$$
$$\Pi_2 = (m\,p_{(3)} - m_{(3)}(p_{(1)} + p_{(2)}))/M,$$
$$r_1 = x_1 - x_2, \quad r_2 = (-m_{(1)}x_1 - m_{(2)}x_2 + m x_3)/m, \quad R = \frac{1}{M}\sum_{i=1}^{3} m_{(i)}x_i, \qquad (4)$$

If we insert the expressions for $p_{(i)}$ from eq. (4) into eq. (3), we find that only centre of mass time appears in it:

$$\{-P_0 + (m_{(1)}\vec{\alpha}_{(1)} + m_{(2)}\vec{\alpha}_{(2)} + m_{(3)}\vec{\alpha}_{(3)})/M\cdot\vec{P} +$$
$$(\beta_{(1)}m_{(1)} + \beta_{(2)}m_{(2)} + \beta_{(3)}m_{(3)}) + (\vec{\alpha}_{(1)} - \vec{\alpha}_{(2)})\vec{\Pi}_1 +$$
$$(-\vec{\alpha}_{(1)}m_{(1)}/m - \vec{\alpha}_{(2)}m_{(2)}/m + \vec{\alpha}_3)\vec{\Pi}_2 + \qquad (5)$$
$$\text{interaction terms}\}\,\psi(x_i) = 0$$

While $\vec{P}$ commutes with the rest of the operator in eq. (5), $(m_{(1)}\vec{\alpha}_{(1)} + m_{(2)}\vec{\alpha}_{(2)} + m_{(3)}\vec{\alpha}_{(3)})/M\cdot\vec{P}$ does not****. The eq. (5) does not allow the separation of the centre of mass motion from the internal motion. This can easily be understood since the spin of the system depends on the internal motion.

When the internal motion manifests spin 1/2, the system should behave as a free Dirac particle if no external field is present. One can write

$$(-P_0 + \vec{\alpha}\cdot\vec{P} + \beta E_r)\,\psi^{1/2}(x_i) = 0 \qquad (6)$$

Here βE_r represents all terms in eq. (5) except the first two. Matrices $\vec{\alpha}$ and β are still $4^3 \times 4^3$ matrices and $\psi^{1/2}$ is still 4^3 spinor. If one integrates over internal degrees of freedom, the expression

$$\langle P_{int}\psi^{1/2}|\beta E_r|P_{int}\psi^{1/2}\rangle = \beta_{cm}m_{cm}$$

enters as a mass term in the Dirac equation for a spin 1/2 particle, while $\langle P_{int}\psi^{1/2}|\vec{\alpha}\cdot\vec{P}|P_{int}\psi^{1/2}\rangle = \vec{\alpha}_{cm}\cdot\vec{P}$ should behave as a momentum term. Here $\psi^{1/2}$ is the solution of eq. (6) for the spin 1/2 case and P_{int} is the projector, projecting $\psi^{1/2}$ into the space concerning relative motion.

One can expect that the system of three Dirac particles still behaves in the above discussed way as one Dirac particle if it is put in a weak external field. The structure of the system would manifest now through the anomalous magnetic moment. One can understand this as follows.

In the presence of an external abelian field A_μ, (eqs. (1) and (3)) are modified. This would be manifested in eq. (3) so that :

$$\partial^\mu_{(i)} \longrightarrow \partial^\mu_{(i)} - g_{(i)} A^\mu$$

Since, for example, $A_\mu(x_1) = A_\mu(R + \frac{m_{(2)}}{m} r_1 - \frac{m_{(3)}}{M} r_2) = A_\mu(R) + \Delta A_\mu(R, r_1, r_2)$; then for fields which are homogeneous enough on the scale of the constituents: $A_\mu(x_1) \cong A_\mu(R)$.

In eq. (5) the term $(m_{(1)}\vec{\alpha}_{(1)} + m_{(2)}\vec{\alpha}_{(2)} + m_{(3)}\vec{\alpha}_{(3)})/M \cdot \vec{P}$ would then be modified to $(m_{(1)}\vec{\alpha}_{(1)} + m_{(2)}\vec{\alpha}_{(2)} + m_{(3)}\vec{\alpha}_{(3)})/M \cdot \vec{P} - (g_{(1)}\vec{\alpha}_{(1)} + g_{(2)}\vec{\alpha}_{(2)} + g_{(3)}\vec{\alpha}_{(3)}) \cdot \vec{A}$.

In the case of three constituents with equal masses and with $g = 1/3\, e_0$ the term becomes: $\frac{1}{3}(\vec{\alpha}_1 + \vec{\alpha}_2 + \vec{\alpha}_3)(\vec{P} - e_0\vec{A})$. The time component of the external field will appear with P_0: $P_0 - e_0 A_0$.

This means that the system of Dirac particles with total angular momentum 1/2 indeed behaves in a homogeneous enough external field A^μ as one Dirac particle. However, in most cases the contribution of the internal motion to the magnetic moment of the system is not negligible. In the case of atoms the anomalous magnetic moment is of the order of 10^3 times larger than the Dirac magnetic moment. Internal motion of quarks in a nucleon contribute a few times more to the magnetic moment of the nucleus than the Dirac magnetic moment. In the case of strongly bound constituents of leptons the contribution to the Dirac magnetic moment is due to highly relativistic internal motion of the constituents for many order of magnitudes smaller than the Dirac one as we shall show in chapter 3.

In this paper we shall not make use of eq. (5) when studying the dynamics of leptons. We shall simplify the equation of motion (5) into equations of three independent particles in

an external field with a vector and a scalar part. However, we need the above derivation to comment on the magnetic moment of the system of three constituents. It will be shown in chapter 3.4 that the correct treatment of the centre of mass motion is essential to reproduce correctly the magnetic moment of the system of particles.

We present our calculations as a first step to better understanding of the dynamics of constituents when they are localized to much smaller radii than the Compton wavelengths. We shall not (among other things) treat the centre of mass motion correctly, but in spite of the fact that it can be very large, we still believe that some general features of the dynamics of three relativistic particles can be seen from such simplified calculations.

3. Equation of motions for three non-interacting Dirac particles in an external field. We shall interpret the charged lepton generations as the radial excitations of the ground state.

3.1. The single particle equation of motion. Our Hamiltonian

$$H = \sum_{i=1}^{3} H_{(i)} = \sum_{i=1}^{3} [\vec{\alpha}_{(i)} \cdot \vec{P}_{(i)} + \beta_{(i)}(m_{(i)} + W(r_i)) + V(r_i)] \qquad (7)$$

has a vector (a zeroth component of a four vector) and a scalar potential and it is radially symmetric. We need a scalar potential to assure that the system is confined. Too strong a vector potential in the absence of a scalar potential has no bound states. We follow the standard procedure $\vec{J}_i = \vec{\ell}_i + \frac{1}{2}\vec{\sigma}_i$, $K_i = \beta_i(\vec{\sigma}_i \cdot \vec{\ell}_i + 1)$, $K\psi^\mu_\varkappa = \varkappa\psi^\mu_\varkappa$ with $\varkappa = \pm(j + 1/2)$ and $m_i = m$,

$$\psi^\mu_\varkappa = \begin{bmatrix} f/r\, \chi^\mu_\varkappa \\ ig\, \vec{\sigma} \cdot \frac{\vec{r}}{r}\, \chi^\mu_\varkappa \end{bmatrix} \qquad (8)$$

From here the equations for radial functions f and g follow:

$$
\begin{aligned}
df/dr + \frac{\kappa}{r} f + (E + m + W - V) g &= 0 \\
dg/dr - \frac{\kappa}{r} g - (E - m - W + V) f &= 0
\end{aligned} \qquad (9)
$$

3.2. The antisymmetric wave function with a mixed radial symmetry. We choose the wave functions of charged leptons which are made of three T particles to be hypercolour and colour singlets. Since the total angular momentum J should be 1/2, the radial wave function with mixed symmetry should be coupled to the wave function with the momentum J=1/2 also having mixed symmetry to produce an antisymmetric wave function:

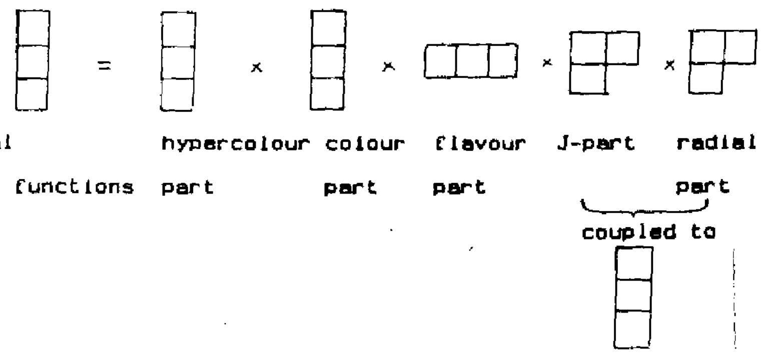

Table II: The scheme of the permutational symmetry of the wave function

To fulfil the above conditions, at least two different radial wave functions are needed. If E_i are solutions with J=1/2 of the equation (9), then we need the Hamiltonian with the following eigenvalues: $E_1 = \frac{2}{3} m_e - \frac{1}{3} m_\mu$, $E_2 = \frac{1}{3} m_\mu - \frac{1}{3} m_e$, $E_3 = m_\tau - \frac{4}{3} m_e + \frac{2}{3} m_\mu$.
We take $m_e = 0{'}51$ MeV, $m_\mu = 105$ MeV, $m_\tau = 1{'}8$ GeV, while the mass m of constituents (Eq.(9)) is taken to be zero.

3.3. Numerical search for the potentials. The desired Hamiltonian has three peculiar properties:

i) Energy differences between two neighbour states grow with radial quantum number very fast

ii) Levels with total angular momentum J=3/2 appear much higher than levels with J=1/2.

iii) The system is confined to a sphere of the radius which is much smaller than the corresponding Compton wavelength.

To simplify the problem we took m = 0 and we construct the potential from the vector part of the form:

$$
V(r) = -\frac{\lambda}{r} - V_1 \Theta[0, R_1] - V_2 \Theta[R_1, R_2] - V_3 \Theta[R_2, R_3] \qquad (10a)
$$

where V_1, V_2 and V_3 are strengths of the three step functions on the interval $[R_i, R_j]$.

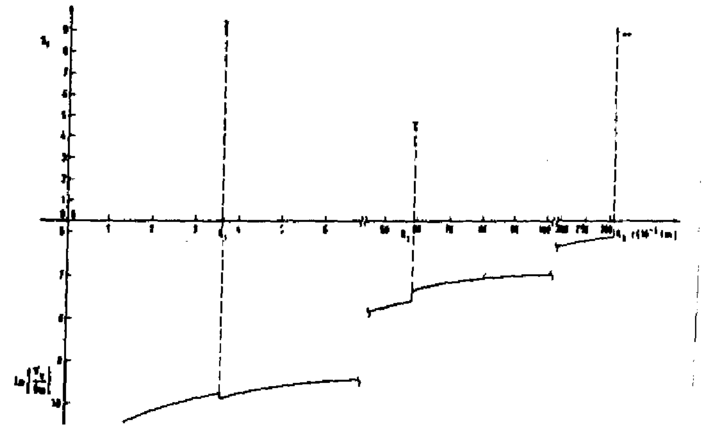

Fig. 1. The vector potential (full line) and the strengths of delta functions in the scalar potential (dashed line) used in the calculations. The chosen parameters are presented in Table III. Note that the scale for the vector potential is different than the scale for the scalar potential.

For the scalar potential we choose two delta functions at R_1 and R_2 respectively and an infinite wall at R_3:

$$
W(r) = \begin{cases} S_1 \delta(r - R_1) + S_2 \delta(r - R_2), & r < R_3 \\ \infty, & r \geq R_3 \end{cases} \qquad (11b)
$$

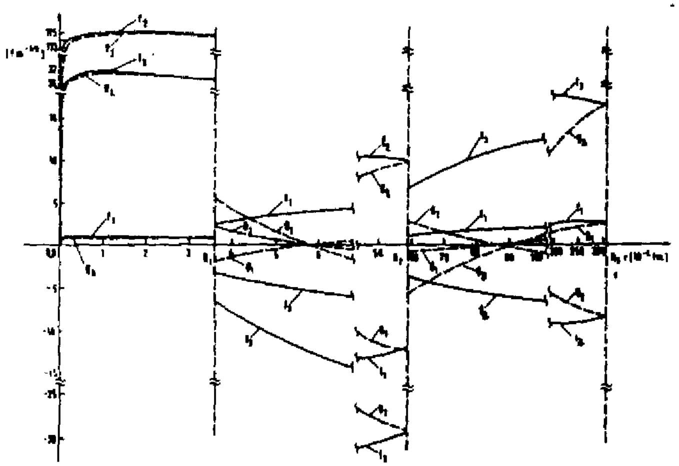

Fig. 2 The radial dependence of the upper components f_i (full lines) and of the lower components g_i (dotted lines) of the three wave functions corresponding to the energies E_i. The three vertical dashed lines indicate the positions of the two delta functions and the infinite wall, respectively. Note that only the upper components behave as in a nonrelativistic case[8].

	$\langle r\rangle$ [fm]	μ [fm e_0]
e^-	$2.3\cdot10^{-4}$	$1.1\cdot10^{-4}$
μ	$3.5\cdot10^{-4}$	$1.6\cdot10^{-4}$
τ	$8.2\cdot10^{-4}$	$3.7\cdot10^{-4}$

Table IV: The mean radii and the magnetic moments for e^-, μ and τ.

The chosen scalar and vector potentials assure that the system of three constituents is localized to the volume of $r<10^{-18}$ m and has the desired mass spectrum. A simple bag potential would, for example, localize particles to the desired volume, but would make a system very heavy with a mass about 200 GeV. We would like to point out again that the

rather complicated effective potential reflects among other things, the fact that all other degrees of freedom but particle ones are taken into account in it.

The radial dependence of the three wave functions corresponding to the ground state and the first two excited states is presented in Fig. 2.

3.4. **The magnetic moment of the system.** To evaluate the magnetic moment of the system described by the wave function from Table II and Fig.2 we have to evaluate the expectation value of the magnetic moment operator (eq.(1)) :

$$\vec{\mu} = -\frac{e_0}{6}\int\prod_i d^3r_i\ \psi^+(r_1,r_2,r_3)\begin{bmatrix}0, & \vec{r}_i\times\vec{\sigma}_i\\ \vec{r}_i\times\vec{\sigma}_i, & 0\end{bmatrix}\psi(r_1,r_2,r_3). \qquad (11)$$

We shall first discuss the magnetic moment of one particle in the state $\psi^{\mu}_{\varkappa}$, which is equal to:

$$\mu = \frac{-4\mu\varkappa e_0}{4\varkappa^2-1}\int_0^\infty g f r\, dr \qquad (12)$$

with [8]:

$$\int f g r\, dr = \frac{2\varkappa-1}{4m}\left\{1-\frac{4\varkappa}{2\varkappa-1}\int_0^\infty g^2 dr\right\},\ \text{if } V\neq0, W=0$$
$$\int f g r\, dr = \frac{2\varkappa-1}{4E}\left\{1+\frac{2}{2\varkappa-1}\int_0^\infty g^2 dr\right\},\ \text{if } V=0, W\neq0 \qquad (13)$$

For weak binding and therefore for the nonrelativistic limit E->m and $\int g^2 dr$ -> 0 so that μ for one particle is $e_0/2m$ in both cases. This is then, up to the centre of mass motion correction, the contribution of each particle to the magnetic moment of the system if in addition the prescription how to generate the total wave function from single particle wave functions is taken into account. In the case of very strong binding, $\int g^2 dr$ increases. The magnetic moment reduces from $e_0/2m$ to the value $e_0\langle r\rangle$, due to increase of the lower component g of the radial part of the wavefunction, where $\langle r\rangle$ is the average radius of the bound particle. Since $\langle r\rangle$ is in our case of the order 2.10^{-4} fm, the magnetic moment of one particle is therefore of the order of 2.10^{-4} fme_0, it is 10^{-4} μ_e. In Table IV the magnetic moments of the leptons e^-, μ and τ are presented, calculated for the wave function from eq. (11). We repeat again that the centre of mass motion was neglected therefore one cannot expect to reproduce the Dirac magnetic moment.

This result only roughly evaluates the contribution of internal motion to the magnetic moment of the system. We can understand this fact by means of the discussion in section 2. The system of three constituents with internal total angular momentum 1/2 behaves in a weak enough external field almost as one Dirac particle with the rest mass determined by the internal motion of the three particles and with an anomalous magnetic moment. The magnetic moment of the internal motion is the correction to the Dirac one. The stronger the particles are bound in the system, the smaller is the volume in which they are bound, the more relativistic is their behaviour and the smaller is their contribution to the magnetic moment of the system. On the quark level, for example, the relativistic effect is only of the order of 20%, the contribution of the internal motion to the magnetic moment of the system is few times larger than Dirac one and gives the correct order of magnitude of the total magnetic moment.

To determine the parameters of the proposed potential we used a numerical procedure, rather similar to the procedure used by Kopper and Dürr [4] in nonrelativistic dynamics.

We restrict ourselves to the case where the invariant

$$I(r;E) = \frac{-\varkappa}{r^2} - \frac{\varkappa^2}{r^2} - \frac{\varkappa}{r}\cdot\frac{-V'+W'}{E-V+m+W} - \frac{1}{4}\frac{(-V'+W')^2}{(E-V+m+W)^2} + \frac{1}{2}\left(\frac{-V'+W'}{E-V+m+W}\right)' + (E-V)^2-(m+W)^2$$

of the radial equations (9) grows with energy: $\frac{\partial I}{\partial E}(r;E) > 0, \forall r$.

It turns out that this is not a severe limitation. We define the node of the wavefunctions as a zero of the difference of the two radial components (f-g) from eq. (9). We conclude that to the eigenfunction with higher energy a higher number of nodes corresponds*****.

An appropriate choice of λ, S_i and V_i forces the three wavefunctions corresponding to states E_1, E_2 and E_3 to oscillate in the desired manner.

λ	V_1 [fm^{-1}]	V_2 [fm^{-1}]	V_3 [fm^{-1}]	S_1, R_1 [fm]	S_2, R_2 [fm]	R_3 [fm]	$\langle r\rangle_1$ [fm]	$\langle r\rangle_2$ [fm]	$\langle r\rangle_3$ [fm]
$1-10^{-4}$	200	500	200	8.30,	4.6,		$2.35\cdot10^{-8}$	$9.5\cdot10^{-4}$	$1.2\cdot10^{-3}$
				$3.6\cdot10^{-8}$	$5.9\cdot10^{-4}$	$3.06\cdot10^{-3}$			

Table III: The chosen parameters and the properties of the system.

To get more precise results one should solve eq. (5), taking into account correctly the centre of mass motion and also the correlations in the internal motion. This is the explanation why the problem with magnetic moment has never appeared to be so drastic as in the case of constituents of leptons.
To calculate the anomalous magnetic moments of a system of three charged Dirac particles - leptons, one should solve the eq. (5) in the presence of an external field. In this, also the Dirac part of the magnetic moment would be reproduced.

3.5. Electromagnetic decay of μ and τ. Due to the symmetry of the wave functions, spontaneous transitions of the type $1/2^+ \to 1/2^+$ are not allowed since the corresponding Clebsh-Gordan coefficients are zero. The transitions in the second order through the states 3/2 are small due to the high energy of those states.

Conclusion. In this paper we presented the dynamics of three equal relativistic paticles, localized in a sphere with a radius which is at least 10^{-4} smaller than the corresponding Compton wavelength of the system. We have shown that such a system, if coupled to the total angular momentum 1/2, behaves in a weak external field as a one Dirac particle system, with the mass determined by the mass and the energy of the internal motion of the constituents. In such a case the system of three constituents of charged leptons manifests the magnetic moment of an electron with g-2 affected by the internal structure. If the strength of the internal field forces the constituents to be highly relativistic particles, as it would be in the case of the constituents of leptons and quarks, the contribution to the magnetic moment of the system decreases to a value $e_0\langle r\rangle$.

In this letter we study the dynamics of the system of three constituents of leptons in a simplified model; we treat three independent particles in an effective external field taking care of all other degrees of freedom. The effective scalar and vector potentials turn out to have very peculiar shapes in order that all features of the dynamics could be reproduced by particle degrees of freedom only, and that the localization to radius 10^{-18} m could be achieved while still preserving the system's desired mass spectrum. In this model the single particle states with J=3/2 are pushed to energies higher than 6 GeV, but the next radially excited state appears to be low, at 1.9 GeV. This should for shure be corrected, together with the centre of mass motion. Although the centre of mass motion is not treated correctly, the model nevertheless illustrates the peculiar behaviour of a system of constituents localized to $r \ll \lambda_c$.

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FOOTNOTES

* We use in this paper $\hbar = c = 1$.

** All fields here are treated classically. The quantum treatment would be needed, since for example $\frac{e^2}{4\pi\varepsilon_0 r} \sim 200\,\mathrm{GeV}$ for $r = 10^{-3}$ fm.

*** Barut and Stromel [6] suggest for the particles, interacting with electromagnetic force the following expression for a vector potential:

$$V_{ij}(r_{ij}) = \gamma_{(i)\mu} \otimes \gamma_{(j)}^{\mu} \otimes \prod_{\substack{k \neq i \\ k \neq j}} \gamma_{(k)0}\, V^{s}_{ij}(r_{ij})$$

where $V^{s}_{ij}(r_{ij})$ is a scalar function of the relative distance r_{ij}. Instead of $\gamma_{(k)0}$ one could write $\gamma_{(k)\mu}\eta^{\mu}$ to point out the covariant form of eqs. (3). One can then choose $\eta^{\mu} = (1,0,0,0)$ to reproduce eqs. (3).

**** Similar equations also follow in the case when all masses in eqs. (4) would be replaced by energies:

$m_{(i)} \to \varepsilon_{(i)}$, $m \to \varepsilon$, $\varepsilon = \varepsilon_{(1)} + \varepsilon_{(2)}$, $M \to \mathcal{E} = \sum_i \varepsilon_{(i)}$ with $\varepsilon_{(i)}$ being the energy of the particle i.

Such a proposal is made in references [7].

***** In regions where the potential is a monotonic function of the radius, the number of zeros of the large component f follows the same rule as the non-relativistic radial function. The number of zeros of the small component g can exceed the number of zeros of f by 1 in such regions. This problem is discussed by M.E.Rose[9].

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