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THERMOSTATIC PROPERTIES OF SEMI-INFINITE POLARIZED NUCLEAR MATTER *

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ABSTRACT

The surface and curvature properties of semi-infinite polarized nuclear matter (SPNM) are calculated using an expansion for the Fermi integrals up to T^2. A density matrix expansion is obtained for a modified form of Seyler-Blanchard interaction. New parameters that characterize the surface and curvature properties of SPNM are introduced. The level density parameter is extracted from the low temperature expansion of the free energy and compared with previous calculations. A reasonable agreement is obtained for the parameters calculated before.

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1. INTRODUCTION

One of the most exciting features of finite and semi-infinite quantum systems is their surfaces [1,2]. The surface region of a nucleus greatly influences its binding energy, level spectra and reactions, i.e. to estimate the bulk surface properties.

One of the widely used models for the study of these finite and semi-infinite systems is the liquid drop model (LDM). In the liquid drop model, the binding energy of a nucleus is written as a sum of volume, surface, curvature and Coulomb terms. The volume term of polarized nuclear matter (PNM) was studied before using Seyler-Blanchard[3] and effective Skyrme interactions[4]. Surface and curvature energies are important in studying heavy ion collisions and in astrophysical applications[5]. Surface and curvature energies are also instrumental in determining the distribution of sizes of fragments which are emitted from the expanding collision region for temperatures less than the critical temperature T_c; at which the liquid-gas phase transition occurs[6,7]. These are crucial in calculating fission barrier heights and the shapes of saddle point configurations[8]. Also, in describing neutron stars[9] and supernovae[10-12], it is important to determine the sizes of nuclei, electron capture rates[13] and level densities[14], which are all sensitive to surface and curvature energies.

The thermodynamical properties of nuclei have been studied theoretically by several authors[15] using Hartree-Fock (HF)[15-17] or extended Thomas-Fermi (ETF)[4,18,19] methods at finite temperature. These calculations did not take into account the effect of temperature on the surface and curvature properties. The surface symmetry energy is slightly increased with excitation energy[20-23] and it is a relevant quantity in the studying of nuclear fission. That is why the knowledge of its variation with temperature has to be considered.

For the surface energy term of LDM, the semi-infinite slab model originally suggested by Bethe[24] and developed later by Swiatecki[25] was used to explore the sensitivity of the underlying N-N interactions. The study of the surface properties of semi-infinite nuclear matter

(SNM) is usually tested by two methods. In the first method, the surface energy is calculated by extracting from the total energy of SNM a reference energy which represents the bulk contribution. Two ways are usually used to define the bulk contribution[26] and consequently there are two definitions for the surface energy in this method. These two definitions are equivalent only for symmetric SNM at zero temperature[22,26,27]. In one of them, the reference energy term is minimized to determine the equilibrium configuration of the system[22]. This minimization results in a set of Euler-Lagrange equations. The solution of these highly non-linear coupled differential equations constitutes the exact variational method. The Euler-Lagrange equations are solved, in another way, by assuming a specific form of the density (taken usually as the Wood-Saxon form) and minimize the energy with respect to the parameters of the form assumed. The resulting set of equations can be solved to find the density distribution. This restricted variational method has been shown to be a good approximation by several authors[2,28-31].

The second method consists essentially in performing a T^2 approximation for the free energy and next a leptodermous expansion in powers of $A^{1/3}$ to account for nuclear geometry[2,20,21,30,32,33].

Dividing the energy of the system into a volume and surface terms means that the curvature energy is either included by rescaling the surface energy term[34] or neglecting it[34,35]. It is straightforward to show[30,31] that the curvature energy may be simply extracted from the SNM surface energy. Chu et al.[36] extracted the curvature energy term from semi-classical calculations of finite nuclei. For realistic nuclei, the contribution of the curvature energy to the total energy is not insignificant and there is no reason to ignore it[37]. A model calculation has been done[31,35,36,38] by extracting the curvature energy from the surface energy of SNM. Farine[39] used the soluble model of SNM to derive a closed expression for the curvature energy as a function of the surface profile asymmetry. Brack et al.[37] performed very accurate semi-classical variational calculations for the curvature energy term. Several

attempts to carry out HF calculations for the curvature energy using the leptodermous expansion resulted in unacceptably large values with the same interaction[40-42]. There is a large variation in the curvature and the curvature symmetry energy among the results for different interactions[31].

Studying the temperature effect on the bulk properties of SNM leads to a sensitive quantity which is the level density parameter. It is a good quantity to test the theoretical calculations. The level density has been introduced by Weisskopf[43] and Bethe[24,44] by applying the method of statistical mechanics. These calculations have been restricted at excitations relatively low to those encountered in heavy ion collisions[45]. Several approaches have been introduced to incorporate high excitations using HF[15-17,46-49], TF[18,50] and ETF[33,51,52]. The nuclear level density parameter extracted from experimental data leads to a value of $a = 0.125\ A\,MeV^{-1}$ [8,53] while the calculated level density parameter gives $a_{v_0} = 0.055$ [15,20,33]; for Skyrme forces including volume effects only. This difference reflects the importance of including surface, curvature and symmetry corrections in calculating the nuclear level density.

The aim of our work is to test the interaction used before[3] and to study the effect of symmetry excess parameters on the surface and curvature properties of SNM. We also estimate their temperature dependence by discussing the effect of these parameters on the level density parameter.

In our model, we start with a two body interaction (extended Seyler-Blanchard[3]) to calculate the energy of semi-infinite polarized nuclear matter (SPNM), then we expand the density matrix in the relative coordinate up to second order to account for the gradient term of the potential. We use the extended Thomas-Fermi form of the kinetic energy to write the total energy of the system. At low temperature we perform an expansion in T^2 approximation of the thermodynamical quantities to obtain the free energy. Using the Wood-Saxon form of the density we expand the free energy in powers of $A^{1/3}$, where an analytical expressions for the surface and curvature properties as a function of the potential parameters can be obtained up to second order in the symmetry excess parameter and temperature. We use these formulae to analyse the effect of temperature and symmetry excess parameters on the surface and curvature properties of semi-infinite polarized nuclear matter (SPNM). Our results are compared with the other calculations. New quantities are introduced and discussed.

II. THE MODEL AND CALCULATIONS

The direct two body matrix element between a pair of nucleons in states i and j is given by:

$$\langle ij|V|ij\rangle = \int \rho_i(\vec{r}_1)\ \rho_j(\vec{r}_2)\ V(r,s)\ d\vec{r}\, d\vec{s} \qquad (1)$$

where $\vec{r} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$

is the center of mass coordinate,

and $\vec{s} = (\vec{r}_1 - \vec{r}_2)$

is the relative coordinate.

Expanding ρ_i in powers of s up to second order[28]:

$$\rho_i(r_1) = \rho_i(r) + \frac{1}{2}\vec{s}.\nabla\rho_i(r) + \frac{1}{8}\vec{s}\,\vec{s} : \nabla\nabla\rho_i(r) \qquad (2)$$

Substituting into equation (1) and integrating by parts over r, we get:

$$\langle ij|V|ij\rangle = \int \rho_i(r)\ \rho_j(r)\ V(r,s)\ d\vec{r}\, d\vec{s} - \frac{1}{6}\int \nabla\rho_i(r).\nabla\rho_j(r)\ V(r,s)\ d\vec{r}\ s^2\ d\vec{s} \qquad (3)$$

In our model, we use the two body interaction of Seyler-Blanchard[3,54,55], which is a Yukawa plus a momentum dependent term. This interaction can be written as:

$$V(r,s) = -C_{\ell,u}\ \frac{e^{-s/a}}{s/a}\ (1 - \frac{p^2}{b^2}) = V(r)\ V(s) \qquad (4)$$

where

$$V(r) = C_{\ell u}\ (1 - \frac{p^2}{b^2}) \qquad (5)$$

and

$$V(s) = e^{-s/a}\ /\ (s/a) \qquad (6)$$

The potential energy of polarized NM, P.E., (the exchange term is neglected[3]) is:

$$\text{P.E.} = -2\pi a^3 C_{\ell\ell} \int (\rho_{n\uparrow}^2+\rho_{n\downarrow}^2+\rho_{p\uparrow}^2+\rho_{p\downarrow}^2)d\vec{r}$$

$$+ \frac{12\pi a^3 C_{\ell\ell}}{5b^2}\ (\frac{3\pi^2}{2})^{2/3} \int (\rho_{n\uparrow}^{8/3}+\rho_{n\downarrow}^{8/3}+\rho_{p\uparrow}^{8/3}+\rho_{p\downarrow}^{8/3})\ d\vec{r}$$

$$- \frac{12\pi a^5 C_{\ell\ell}}{5b^2}\ (\frac{3\pi^2}{2})^{2/3} \int (\nabla\rho_{n\uparrow})^2\ \rho_{n\uparrow}^{2/3} + (\nabla\rho_{n\downarrow})^2\ \rho_{n\downarrow}^{2/3} + (\nabla\rho_{p\uparrow})^2\ \rho_{p\uparrow}^{2/3} + (\nabla\rho_{p\downarrow})^2\ \rho_{p\downarrow}^{2/3}\ d\vec{r}$$

$$+ 2\pi a^5 C_{\ell\ell} \int (\nabla\rho_{n\uparrow})^2 + (\nabla\rho_{n\downarrow})^2 + (\nabla\rho_{p\uparrow})^2 + (\nabla\rho_{p\downarrow})^2\ d\vec{r}$$

$$- 4\pi a^3 C_{\ell u} \int (\rho_{n\uparrow}\rho_{n\downarrow} + \rho_{p\uparrow}\rho_{p\downarrow})\ d\vec{r}$$

$$+ \frac{12\pi a^3 C_{\ell u}}{5b^2}\ (\frac{3\pi^2}{2})^{2/3} \int (\rho_{\uparrow}^{5/3}\rho_{n\downarrow} + \rho_{n\downarrow}^{5/3}\ \rho_{n\uparrow} + \rho_{p\uparrow}^{5/3}\ \rho_{p\downarrow} + \rho_{p\downarrow}^{5/3}\rho_{p\uparrow})\, d\vec{r}$$

$$+ 4\pi a^5 C_{\ell u} \int (\nabla\rho_{n\uparrow}\nabla\rho_{n\downarrow} + \nabla\rho_{p\uparrow}\nabla\rho_{p\downarrow})\ d\vec{r}$$

$$- \frac{12\pi a^5 C_{\ell u}}{5b^2}\ (\frac{3\pi^2}{2})^{2/3} \int (\nabla\rho_{n\uparrow}\nabla\rho_{n\downarrow}\rho_{n\uparrow}^{2/3} + \nabla\rho_{n\uparrow}\nabla\rho_{n\downarrow}\rho_{n\downarrow}^{2/3} + \nabla\rho_{p\uparrow}\nabla\rho_{p\downarrow}\rho_{p\uparrow}^{2/3} + \nabla\rho_{p\uparrow}\nabla\rho_{p\downarrow}\rho_{p\downarrow}^{2/3})\ d\vec{r}$$

$$+ \frac{12\pi a^3 C_{u\ell}}{5b^2}\ (\frac{3\pi^2}{2})^{2/3} \int (\rho_{p\uparrow}\rho_{n\uparrow}^{5/3} + \rho_{p\uparrow}^{5/3}\rho_{n\uparrow} + \rho_{n\downarrow}^{5/3}\rho_{p\downarrow} + \rho_{p\downarrow}^{5/3}\rho_{n\downarrow})\, d\vec{r}$$

$$+ 4\pi a^3 C_{u\ell} \int (\nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow} + \nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow})\ d\vec{r}$$

$$+ 4\pi a^5 C_{u\ell} \int (\nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow} + \nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow}) \, d\vec{r}$$

$$- \frac{12\pi a^5 C_{u\ell}}{5b^2} \left(\frac{3\pi^2}{2}\right)^{2/3} \int (\nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow}\rho_{n\uparrow}^{2/3} + \nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow}\rho_{p\uparrow}^{2/3} + \nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow}\rho_{n\downarrow}^{2/3} + \nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow}\rho_{p\downarrow}^{2/3}) \, d\vec{r}$$

$$- 4\pi a^3 C_{uu} \int (\rho_{n\uparrow}\rho_{p\uparrow} + \rho_{n\downarrow}\rho_{p\downarrow}) \, d\vec{r}$$

$$+ \frac{12\pi a^3 C_{uu}}{5b^2} \left(\frac{3\pi^2}{2}\right)^{2/3} \int (\rho_{n\uparrow}^{5/3}\rho_{p\uparrow} + \rho_{n\uparrow}\rho_{p\uparrow}^{5/3} + \rho_{n\downarrow}^{5/3}\rho_{p\downarrow} + \rho_{n\downarrow}\rho_{p\downarrow}^{5/3}) \, d\vec{r}$$

$$+ 4\pi a^5 C_{uu} \int (\nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow} + \nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow}) \, d\vec{r}$$

$$- \frac{12\pi a^5 C_{uu}}{5b^2} \left(\frac{3\pi^2}{2}\right)^{2/3} \int (\nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow}\rho_{n\uparrow}^{2/3} + \nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow}\rho_{p\uparrow}^{2/3} + \nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow}\rho_{n\downarrow}^{2/3} + \nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow}\rho_{p\downarrow}^{2/3}) \, d\vec{r} \qquad (7)$$

For the kinetic energy, we used the extended TF formula. This formula was introduced to account for the variation of density, i.e. the kinetic energy, near the surface by including gradient-type corrections. The kinetic energy density in the extended TF (ETF) is given by:

$$K.E. = \frac{\hbar^2}{2m} \int \left[\alpha\rho_{\tau,s}^{5/3} + \beta \frac{(\nabla\rho_{\tau,s})^2}{\rho_{\tau,s}} + \gamma\nabla^2\rho_{\tau,s}\right] d\vec{r} \quad , \qquad (8)$$

where

$$\alpha = (3/5)\ (3\pi^2/2)^{2/3} \ .$$

The values of β and γ are taken to be[36,56-58]:

$$\beta = 1/36 \qquad \text{and} \qquad \gamma = 1/3 \ . \qquad (9)$$

The nuclear density is calculated using the restricted variational method. We take for this purpose the Wood-Saxon form for the density. This choise has been shown to give reliable results for both the surface energy and the diffuseness parameter[2,28,29,31]. The parameters of the Wood-Saxon function; namely the diffuseness parameter (d) and the half-value radius (R), in our model for SPNM are put in a general form; namely $d_{\tau,s}$ and $R_{\tau,s}$ respectively. However, the diffuseness parameter for neutrons and protons are nearly equal[31,59] (exactly equal for small neutron excesses) as can be demonstrated by considering the two coupled Euler-Lagrange equations[59]. Thus, for simplicity we put $d_{\tau,s} = d$.

The Wood-Saxon form, under the above simplifications, for the SPNM becomes:

$$\rho_{\tau,s} = \frac{\rho_{o\tau,s}}{1 + \exp[(r-R_{\tau,s})/d]} \qquad (10)$$

where $\rho_{o\tau,s}$ is the asymptotic density deep inside the system. The relation between $\rho_{o\tau,s}$ and the density ρ_o of NM is given by[22]:

$$\begin{aligned} \rho_{\tau,s}(-\infty) = \rho_{on\uparrow} &= \tfrac{1}{4}\rho_o(1+X+Y+Z) \ , \\ \rho_{on\downarrow} &= \tfrac{1}{4}\rho_o(1+X-Y-Z) \ , \\ \rho_{op\uparrow} &= \tfrac{1}{4}\rho_o(1-X+Y-Z) \quad \text{and} \\ \rho_{op\downarrow} &= \tfrac{1}{4}\rho_o(1-X-Y+Z) \end{aligned} \qquad (11)$$

where X,Y and Z are the excess parameters defined by[3]:

$$X = \frac{N\uparrow + N\downarrow - Z\uparrow - Z\downarrow}{A} \quad ,$$

$$Y = \frac{N\uparrow - N\downarrow + Z\uparrow - Z\downarrow}{A} \quad \text{and} \tag{12}$$

$$Z = \frac{N\uparrow - N\downarrow - Z\uparrow + Z\downarrow}{A}$$

The relation between the number of particles and their densities is:

$$\int \rho_{n\uparrow}\, d\vec{r} = N\uparrow \quad , \quad \int \rho_{n\downarrow}\, d\vec{r} = N\downarrow \quad ,$$
$$\int \rho_{p\uparrow}\, d\vec{r} = Z\uparrow \quad \text{and} \quad \int \rho_{p\downarrow}\, d\vec{r} = Z\downarrow \tag{13}$$

These integrations are solved analytically (see Appendix A) to give

$$\int \rho_{\tau,s}\, d\vec{r} = \frac{4\pi}{3}\, \rho_{o\tau,s}\, (R^3_{\tau,s} + \pi^2 d^2 R_{\tau,s}) \tag{14}$$

Solving for $R_{\tau,s}$ we get

$$R_{n\uparrow} = R_{n\downarrow} = R_{p\uparrow} = R_{p\downarrow} = R = (3A/4\pi\rho_o)^{1/3} + \frac{\pi^2 d^2}{3}\left(\frac{4\pi\rho_o}{3A}\right)^{1/3} \tag{15}$$

This means that, in our calculations, the half-value radius does not depend on the spin or the isotopic spin of the nucleons. This gives a zero neutron skin which is the assumption of the liquid drop model and the compressible liquid drop model[34]. This assumption has been shown experimentally[60] to be good for ordinary nuclei. However, one might expect neutron-rich neutron star matter nuclei to have a neutron skin, and for these nuclei this may not be a good assumption.

With these simplifications we can see that the total energy, contains terms like $\int \rho^q\, d\vec{r}$ and $\int \rho^p \rho^q\, d\vec{r}$. Also, the derivatives of the Wood-Saxon form can be expressed in terms of powers of Wood-Saxon.

Following the method adopted by Srivastava[61], the integrals $\int \rho^q\, d\vec{r}$ can be approximated in the form:

$$\int \rho^q_{\tau,s}\, d\vec{r} = \frac{4\pi}{3}\, \rho_{o\tau,s}\, [R^3 - 3A_1(q)\, R^2 d + 6A_2(q)\, R\, d^2 - 6A_3(q)\, d^3] \tag{16}$$

where the coefficients $A_n(q)$ are given by:

$$A_n(q) = \frac{1}{(n-1)!} \int_0^\infty [1 - (1+e^{-x})^{-q} + (-1)^n (1+e^x)^{-q}]\, x^{n-1}\, dx \tag{17}$$

Using the same technique, we found (see Appendix A):

$$\int \rho^q_{\tau,s} \rho^p_{\tau',s'}\, d\vec{r} = \frac{4\pi}{3}\, \rho_{o\tau,s}\, \rho_{o\tau',s'}\, [R^3 - 3A_1(p+q)\, R^2 d + 6A_2(p+q)\, R\, d^2 + 6A_3(p+q)\, d^3] \quad . \tag{18}$$

Applying these simplifications, we can calculate the total energy E and correspondingly the free energy F (= E - ST , where S is the entropy). The expression for S in case of polarize NM is taken as in Ref. (3).

We can express the total free energy as a sum of a volume, surface and curvature terms. Each term is a function of the potential parameters, the density at saturation and the surface diffuseness parameter. The resulting expression is written as:

$$F = F_v\, A + F_s\, A^{2/3} + F_c\, A^{1/3} \quad , \tag{19}$$

where

$$F_v = F_{vo} + F_{vx}\, X^2 + F_{vy}\, Y^2 + F_{vz}\, Z^2 \tag{20}$$

$$F_s = F_{so} + F_{sx}\, X^2 + F_{sy}\, Y^2 + F_{sz}\, Z^2 \tag{21}$$

$$F_c = F_{co} + F_{cx}\, X^2 + F_{cy}\, Y^2 + F_{cz}\, Z^2 \tag{22}$$

and

$$F_{vo} = \frac{3\pi a^3}{5b^2} C \rho_o^{5/3} \left(\frac{3\pi^2}{2}\right)^{2/3} - \frac{3\hbar^2}{10m} \rho_o^{2/3} \left(\frac{3\pi^2}{2}\right)^{2/3} - \frac{\pi^2 m T^2}{2\hbar^2 \rho_o^{2/3}} \left(\frac{2}{3\pi^2}\right)^{2/3} \quad (23)$$

$$F_{vi} = \frac{2\pi a^3}{3b^2} C_i \rho_o^{5/3} - \frac{\pi a^3}{2} C_i \rho_o + \frac{\hbar^2}{6m} \rho_o^{2/3} \left(\frac{3\pi^2}{2}\right)^{2/3} - \frac{\pi^2 m T^2}{18\hbar^2 \rho_o^{2/3}} \left(\frac{2}{3\pi^2}\right)^{2/3} \quad (24)$$

$$F_{so} = 2\pi^2 a^3 \rho_o^{4/3} d C \left(\frac{3}{4\pi}\right)^{2/3} - \frac{12\pi^2 a^3 d C}{5b^2} \rho_o^2 A_1(8/3)(9/8)^{2/3} - \frac{2\pi^2 a^5}{d} C \left(\frac{3}{4\pi}\right)^{2/3} \left(A_1(2)+A_1(4)-2A_1(3)\right) \rho_o^{5/3} + \frac{12\pi^2 a^5 C}{b} \rho_o^2 \left(\frac{9\pi}{8}\right)^{2/3} \left(A_1(8/3)+A_1(14/3)-2A_1(11/3)\right) - \frac{6\hbar^2 d}{5m} \rho_o \left(\frac{9\pi}{8}\right)^{2/3} A_1(5/3) - \frac{2\hbar^2 \pi \beta}{md} \rho_o^{1/3} \left(\frac{3}{4\pi}\right)^{2/3} \left(A_1(3)-2\right) + \frac{2mT^2 d}{\hbar^2 \rho_o^{1/3}} A_1(1/3)(1/2)^{2/3} \quad (25)$$

$$F_{si} = 2\pi^2 a^3 d \rho_o^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} C_i - \frac{2\pi^2 a^5}{d} C_i \rho_o^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} \left(A_1(2)+A_1(4)-2A_1(3)\right) - \frac{8\pi^2 a^3 d}{3b^2} C_i \rho_o^2 \left(\frac{9\pi}{8}\right)^{2/3} A_1(8/3) - \frac{8\pi^2 a^5}{3b^2 d} C_i \rho_o^2 \left(\frac{9\pi}{8}\right)^{2/3} \left(A_1(8/3)+A_1(14/3)-2A_1(11/3)\right) - \frac{2\hbar^2 \pi d}{3m} \rho_o \left(\frac{9\pi}{8}\right)^{2/3} A_1(5/3) - \frac{2\pi m T^2 d}{9\hbar^2 \rho^{1/3}} A_1(1/3)(1/2)^{2/3} \quad (26)$$

$$F_{co} = \frac{4\pi^3 a^3}{5b^2} \rho_o^{7/3} (1/2)^{1/3} \left(6A_2(8/3)\, d^2 - \pi^2 A\, d^2 - 6a^2 A_2(8/3) + A_2(14/3) - 2A_2(11/3)\right) C - \frac{2\pi^2 \hbar^2 d^2}{5m} \rho_o^{4/3} (1/2)^{1/3} \left(\pi^2 - 6A_2(5/3)\right) + \frac{2\pi\hbar^2}{m} \beta \rho_o^{2/3} \left(\frac{3}{4\pi}\right)^{1/3} + \frac{2\pi^2 m T^2 d^2}{3\hbar^2} \left(\frac{1}{3\pi^2}\right)^{1/3} \left(\pi^2 - 6A_2(1/3)\right) \quad (27)$$

and

$$F_{ci} = \frac{8\pi^3 a^3}{5b^2} \rho_o^{7/3} \left(6d^2 A_2(8/3) - \pi^2 d^2 - 6a^2 A_2(8/3) + A_2(14/3) - 2A_2(11/3)\right) C_i - \frac{2\pi^2 \hbar^2 d^2}{3\hbar^2} \rho_o^{4/3} (1/2)^{1/3} \left(\pi^2 - 6A_2(1/3)\right) - \frac{2\pi^2 m T^2 d^2}{27\hbar^2} \left(\frac{1}{3\pi^2}\right)^{1/3} \left(\pi^2 - 6A_2(1/3)\right) , \quad (28)$$

i runs over x,y and z where

$$C = C_{\ell\ell} + C_{\ell u} + C_{u\ell} + C_{uu}$$

$$C_x = 2\,C_{\ell\ell} + 2\,C_{\ell u} - C_{u\ell} - C_{uu}$$

$$C_{x'} = C_{\ell\ell} + C_{\ell u} - C_{u\ell} - C_{uu}$$

$$C_y = 2\,C_{\ell\ell} - C_{\ell u} + 2\,C_{u\ell} - C_{uu}$$

$$C_{y'} = C_{\ell\ell} - C_{\ell u} + C_{u\ell} - C_{uu}$$

$$C_z = 2\,C_{\ell\ell} - C_{\ell u} - C_{u\ell} + 2\,C_{uu}$$

$$C_{z'} = C_{\ell\ell} - C_{\ell u} - C_{u\ell} + C_{uu} \quad .$$

The surface diffuseness parameter d is obtained (as a function of the symmetry parameters and temperature) by minimization of the free energy with respect to d, thus $\partial F/\partial d = 0$. Since the volume free energy is independent of d and the curvature energy depends on d quadratically, we have

$$\frac{\partial F}{\partial d} = \frac{\partial F_s}{\partial d} = 0 \quad ,$$

If we write

$$F_s = P(\rho_o)\, d + N(\rho_o)/d$$

then,

$$d = \left(N(\rho_o) / P(\rho_o)\right)^{1/2} \quad , \tag{29}$$

where the functions $N(\rho_o)$ and $P(\rho_o)$ are given in appendix B.

The density in the above equations is the equilibrium density for zero temperature symmetric NM. The equilibrium density for zero temperature NM with a neutron excess was deduced by Weiss and Cameron[62], Bethe[63] and by Dworzecka[64]. Hassan et al.[4,65] generalized the non-zero temperature and polarized NM. In the SPNM, to get the equilibrium density as a function of the symmetry excess parameters and temperature, we minimize the free energy with respect to ρ_o. Since A is finitely large in the case of SPNM, we have[21]

$$\frac{\partial F}{\partial \rho_o} \approx \frac{\partial F_v}{\partial \rho_o} \tag{30}$$

Following the strategy adopted by Hassan et al.[4], we get ρ_o up to second order in X,Y,Z and T as:

$$\rho_o = \rho_{on}\left[1 - \frac{9\rho_{on}}{K}\left(\frac{\partial F_x}{\partial \rho_o}\Big|_{\rho_{on}} X^2 + \frac{\partial F_y}{\partial \rho_o}\Big|_{\rho_{on}} Y^2 + \frac{\partial F_z}{\partial \rho_o}\Big|_{\rho_{on}} Z^2 + \frac{\pi^2 m T^2}{3\rho^{5/3}}\left(\frac{2}{3\pi^2}\right)^{2/3}\right)\right] \quad . \tag{31}$$

Using equation (24), we get

$$\rho_o = \rho_{on}\left[1 - \rho_t\, T^2 - (\rho_x - \rho^1\, T^2)\, X^2 - (\rho_y - \rho^1\, T^2)\, Y^2 - (\rho_z - \rho^1\, T^2)\, Z^2\right] \tag{32}$$

To calculate the surface and curvature properties at the equilibrium density ρ_{on}, we have to rewrite the function $N(\rho_o)$ and $P(\rho_o)$ in terms of ρ_{on} (see Appendix B).

The surface and curvature properties as well as the diffuseness parameter have been calculated at the equilibrium density of symmetric NM. The results up to second order in X,Y,Z and T are on the form:

$$d = d_o + \alpha_o T^2 + (d_x + \alpha_x T^2)\, X^2 + (d_y + \alpha_y T^2)\, Y^2 + (d_z + \alpha_z T^2)\, Z^2 \tag{33}$$

$$F_s = F_{so} + a_{so} T^2 + (F_{sx} + a_{sx} T^2)\, X^2 + (F_{sy} + a_{sy} T^2)\, Y^2 + (F_{sz} + a_{sz} T^2)\, Z^2 \tag{34}$$

and

$$F_c = F_{co} + a_{co} T^2 + (F_{cx} + a_{cx} T^2)\, X^2 + (F_{cy} + a_{cy} T^2)\, Y^2 + (F_{cz} + a_{cz} T^2)\, Z^2 \tag{35}$$

Analytical expressions are obtained for all the parameters appearing in equations (33-35) (see Appendix C).

It is straighforward to deduce the level density parameter from the dependence of the total free energy as well as the total energy through the low-temperature expansion [33,49] and using the TF or ETF formula (for kinetic energy). This gives volume (a_v), surface (a_s) and curvature (a_c) level density parameters. The results are:

$$a_i = a_{io} + a_{ix} X^2 + a_{iy} Y^2 + a_{iz} Z^2 \qquad i = v,s,c \tag{36}$$

where a_{vo} is obtained from equation (23) as:

$$a_{vo} = \frac{\pi^2 m}{3\hbar^2 \rho_o^{2/3}} \left(\frac{2}{3\pi^2}\right)^{2/3} \tag{37}$$

and

$$a_{vx} = a_{vy} = a_{vz} = -\frac{1}{9} a_{vo} \tag{38}$$

III. RESULTS AND DISCUSSION

In a previous work[3] we have studied the bulk properties of polarized nuclear matter (PNM), focusing our attention on the equation of state. Here, we restrict ourselves to the study of the surface and curvature properties of SPNM using the extended form of Seyler-Blanchard interaction introduced before[3]. Our extended potential has a set of fitting parameters which gives a negligible value for the exchange term of the potential. We use this set of parameters in our calculations since we took into account only the direct term.

For the kinetic energy density, we used the ETF formula which contains gradient and Laplacian terms to account for variation of the density near the surface. We found that the Laplacian term is equal to zero. This is due to the fact that we neglect the neutron skin thickness. For the value of β we used $\beta = 1/36$ [36,56-58], and we found that a large value for β gives a large value for the surface energy[21,31,66].

The results below are carried out for different forms of ρ_o namely; $\rho_o = \rho_{on}$ (PW1), $\rho_o = \rho_{on}(1 - \rho_T T^2)$ (PW2), and the full expasion form of ρ_o (equation 32) (PW3).

Table (1) gives the coefficients of the density expansion (equation 32) together with other calculations. We notice that there is a fair agreement between the values of the coefficients ρ_y and ρ_z with that of Hassan et al.[4] but there is a difference in the values of the coefficient ρ_x. It has been shown[62] that these coefficients are strongly correlated with the values of E_x, E_y and E_z. In our case, we have $E_x = 33.4$ MeV which differs from that used by Hassan et al. ($E_x = 28.0$ MeV). But the values of E_y and E_z are nearly the same. This is reflected in the agreement for ρ_y and ρ_z in both calculations.

The half-value radius R can be easily deduced from the relation $R = (3A/4\pi\rho_o)^{1/3}$, keeping only the $A^{1/3}$ term in the expansion of R as a first approximation[8]. Using equation (32) we get:

$$R = (3A/4\pi\rho_{on})^{1/3} \left[1 + \frac{1}{3}\rho_t T^2 + \frac{1}{3}(\rho_x - \rho^1 T^2)\, X^2 + \frac{1}{3}(\rho_y - \rho^1 T^2)\, Y^2 + \frac{1}{3}(\rho_z - \rho^1 T^2)\, Z^2\right]$$

From this equation, we notice that the half-value of radius R increases with the increasing of X,Y,Z and T. The coefficients $\rho^1 T^2$ has a small effect in the range of temperature considered.

Table (2) gives the coefficients of the expansion of the diffuseness parameter d, equation (34). The value of d_o is slightly less than the empirical value (d_o = 0.5 fm [28]). This result can be modified by using Fermi distribution to a power for the density[20,31]. The parameter a_o has different values corresponding to the application of different forms of ρ_o. It is also sensitive to the two-body interaction used[21].

To the best of our knowledge, the values d_x, d_y, d_z, a_x, a_y and a_z have not been reproduced before. The effect of d_i is to decrease the diffuseness parameter in case of PW3[8]. An opposite effect is obtained for the two cases PW1 and PW2. This is in agreement with the other calculations[31,67]. The effect of a_i is to decrease (increase) the diffuseness parameter in case of PW2 (PW1 and PW3). The parameters of the surface energy, equation (35), are listed in Table (3). The values of F_{so} are in reasonable agreement with the previous calculations [60,69]. The value of a_{so} is very sensitive to the form of the density used (PW1, PW2 and PW3). It varies largely with the type of the force used. We notice from Table (3) that the values found in the literature for the surface symmetry energy F_{sx}, ranges between -25 and -70 MeV [51,53] and our value of F_{sx} lies in this range. The surface free energy for symmetric unpolarized system (X = Y = Z = 0) vanishes at a temperature which varies from 10.6 MeV up to 12.1 MeV for PW1 and PW2. This temperature is of a critical type because at that temperature the symmetry free energies have very large negative values and the diffuseness parameter increases to a very large positive value. This critical temperature may be seen from the fact that, with the increasing of the temperature the system undergoes a phase transition from liquid to gas phase. This phase

transition starts at a critical temperature (T_{C1} = 9 MeV) [3] and continues until a layer of the gas phase covers the surface of the system at a critical temperature (T_C = 12 MeV) at which the surface energy vanishes[3]. A similar critical temperature was obtained by Stocker[18,50] for symmetric unpolarized NM and it was also found in HF calculations[49,16,17]; there exists a limiting temperature (T_C = 8 - 10 MeV) beyond which the nucleus becomes unstable. At this temperature, no solution for HF equations can be obtained. The same kind of instability, which was found by Bonche et al.[17], was also found by Suraud[70], using a semi-classical approximation for hot nuclei (T_C = 8 MeV). The parameters F_{xy} and F_{sz} have similar effect on F_s as F_{sx}. Namely, F_s decreases with the increasing of X,Y and Z. The parameters a_{si} differ for different forms of ρ_o. The value of a_{sx} is larger than that obtained in Ref. (20), which was obtained numerically.

Table (4) gives the coefficients of the expansion of the curvature free energy (equation 35). The value of F_{co} agrees with the known theoretical value (F_{co} = 10 MeV)[71]. If we take into account the compression of the bulk (- $2F_{so}/K$) [72], one obtains a smaller value (F_{co} = 5.64 MeV). This value is in agreement with the findings of the recent analysis of nuclear masses and fission barriers heights[73,74]. The curvature symmetry energy F_{cx} agrees with that of Kolehmainen et al.[31]. Their values are extracted from the thermodynamic potential. More calculations are still necessary to determine the precise value of F_{cx}.

The surface and curvature parameters a_{si} and a_{ci}, discussed before are essentially those which appear in the level density parameter formula (equation 36). The volume level density parameters (equation 38) are given in Table (5) for PW3. The results for PW1 and PW2 are the same as those of PW3 for a_{vo} but slightly differ for a_{v1}. The value of a_{vo} is in favour of the value [8,75-78] $a = (1/8.8)\ A\ MeV^{-1}$. The values of a_{vi} are directly related to a_{vo}. In order to have a closer judgement on the level density parameter a ($a = a_v A + a_s A^{2/3} + a_c A^{1/3}$) it is shown in Fig. (1) together with previous results as well as the experimental data [71]. We notice that the agreement between our results and the experimental values is reasonable. It has been stressed before[33] that the value of the level density parameter depends on the value of the effective mass and we obtain better results when the value of m^* is near to that of bare mass m. In our case $m^*/m = 1$.

IV. SUMMARY AND CONCLUSION

In the present paper, we have studied the thermal properties of SPNM using an energy density formalism (EDF). This EDF is obtained from the density matrix expansion introduced by Moszkowski[28] to account for gradient terms in the EDF of SPNM. We assumed that our system is leptodermous, i.e. the ratio of the surface thickness to the size is a small quantity and used as an expansion parameter. Consequently, its energy can be expanded as polynomical in $A^{1/3}$. We expand the Fermi integrals in terms of temperature up to T^2. We used a restricted variational approach for the density profile. The surface diffuseness is taken as a variational quantity. Analytical expressions are obtained for the surface and curvature energies as a function of the excess parameters and temperature. These analytical expressions enable us to specify, in a simple way, the relevant ingredients of the nuclear Hamiltonian determining the thermal properties of the liquid drop model parameters. It is known that the surface symmetry energy and the curvature energy[31,66] play an important role in the determination of the fission barrier heights and in the description of the dipole-resonance strength, so it is necessary to determine their dependence on the temperature. This dependence is shown to be significant in our calculations. A simple analytical expression for the level density parameter has been obtained. The agreement between the calculated level density parameter and the experimental values reflects the fact that our formulae for the temperature dependence of the surface and curvature properties are reasonable.

Several new parameters were introduced in this work. We believe that their values need to be tested for different interations. This will be done in the near future.

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APPENDIX A

CALCULATION OF INTEGRALS INVOLVING TWO MULTIPLIED WOOD-SAXON FUNCTIONS TO A POWER:

The Wood-Saxon form of the density is given by:

$$\rho(r) / \rho_o = \{1 + \exp[(r-R)/d]\}^{-1} \qquad (A.1)$$

In our model calculations, we need to calculate integrals of the form

$$I(p,q) = \int_0^\infty \rho^p(r).\rho^q(r)\ d\vec{r} \quad ,$$

where

$$\rho^p(r) = \rho_o^p / \{1 + \exp[(r-R_1)/d_1]\}^p$$

and

$$\rho^q(r) = \rho_o^q / \{1 + \exp[(r-R_2)/d_2]\}^q \quad .$$

In our calculations we set:

$$R_1 = R_2 = R \qquad \text{and} \qquad d_1 = d_2 = d \quad .$$

Equation (A.1) can be evaluated by noting that

$$I = 4\pi \int_0^R \rho^q(r) . \rho^p(r)\ r^2\ dr + 4\pi \int_R^\infty \rho^q(r).\rho^p(r)\ r^2\ dr$$

$$= \qquad I_1 \qquad + \qquad I_2 \quad .$$

For I_1, $r < R$ and thus we define

$$-x = (r-R)/d$$

and we get,

$$I_1 = 4\pi d\ \rho_o^p\ \rho_o^q \int_0^{R/d} (1+e^{-x})^{-q} \cdot (1+e^{-x})^{-p}\ (R-dx)^2\ dx$$

$$= 4\pi d\ \rho_o^q\ \rho_o^p \int_0^{R/d} (1+e^{-x})^{-(p+q)}\ (R-dx)^2\ dx$$

$$(1+e^{-x})^{-(p+q)} = 1 + \sum_{m=1}^{\infty} (-1)^m \frac{(p+q).(p+q+1).....(p+q+m-1).e^{-mx}}{m!}$$

$$= 1 + \sum_{m=1}^{\infty} A_m(p+q)\ e^{-mx}$$

$$\therefore \quad I_1 = 4\pi d\ \rho_o^p\ \rho_o^q \int_0^{R/d} \left(1 + \sum_{m=1}^{\infty} A_m(p+q).e^{-mx}\right).(R-dx)^2\ dx$$

$$= 4\pi d\ \rho_o^p\ \rho_o^q \left\{\frac{R^3}{3d} + \sum_{m=1}^{\infty} A_m(p+q).e^{R/d}.\left(\frac{R^2}{m} - \frac{2Rd}{m^2} + \frac{2d^2}{m^3} - \frac{2d^2}{m^3} e^{-mR/d}\right)\right\}$$

(A.2)

For I_2, r > R and therefore we define

$$y = (r-R)/d$$

$$I_2 = 4\pi d\ \rho_o^p\ \rho_o^q \int_0^{\infty} (1+e^{y})^{-(p+q)}\ (R+dy)^2\ dy$$

$$= 4\pi d\ \rho_o^p\ \rho_o^q \int_0^{\infty} e^{-(p+q)y}\ (1+e^{-y})^{-(p+q)}\ (R+yd)^2\ dy$$

$$= 4\pi d\ \rho_o^p\ \rho_o^q \int_0^{\infty} e^{-(p+q)y}.\left\{1 + \sum_{m=1}^{\infty} A_m(p+q).e^{-my}\right\}\ (R+yd)^2.dy$$

$$\therefore \quad I_2 = 4\pi d\ \rho_o^p\ \rho_o^q \left\{ \int_0^{\infty} e^{-(p+q)y}\ (R+yd)^2.dy + \sum_{m=1}^{\infty} A_m(p+q) \int_0^{\infty} e^{-(p+q+m)y}\ (R+yd)^2.dy \right\}$$

(A.3)

Thus,

$$I = 4\pi\ \rho_o^p\ \rho_o^q \left\{\frac{R^3}{3} + 2d^3\ \frac{-R^2}{2d^2} A_1(p+q) + \frac{R}{d} A_2(p+q) - A_3(p+q) + \sum_{m=1}^{\infty} (-1)^{m-1} \frac{(p+q).(p+q+1).....(p+q+m-1)}{m!} \cdot \frac{e^{-mR/d}}{m^3}\right\}$$

(A.4)

with

$$A_n(p+q) = \frac{1}{(n-1)!} \int_0^{\infty} \left\{1 - (1+e^{-x})^{-(p+q)} + (-1)^n (1+e^{x})^{-(p+q)}\right\} \cdot x^{n-1}\ dx$$

If R > d the second term of I (in equation (A.4)) can be neglected thus, we can write for I

$$I(p,q) = \frac{4\pi}{3} \rho_o^p\ \rho_o^q \left\{R^3 - 3R^2 d\ A_1(p+q) + 6\ R\ d^2\ A_2(p+q) - 6d^3\ A_3(p+q)\right\}$$

(A.5)

APPENDIX B

The functions $N(\rho_o)$ and $P(\rho_o)$ are given by:

$$N(\rho_o) = N_o + N_x X^2 + N_y Y^2 + N_z Z^2 \qquad (B.1)$$

and

$$P(\rho_o) = P_o + P_x X^2 + P_y Y^2 + P_z Z^2 \quad , \qquad (B.2)$$

where

$$N_o = \frac{12\pi^2 a^5}{5b^2} C \rho_o^2 \left(\frac{9\pi}{8}\right)^{2/3} \left(A_1(8/3)+A_1(14/3)-2A_1(11/3)\right)$$

$$- 2\pi^2 a^5 C \rho_o^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} \left(A_1(2)+A_1(4)-2A_1(3)\right)$$

$$- 2\hbar^2 \pi \beta \rho_o^{1/3} \left(\frac{3}{4\pi}\right)^{2/3} \left(A_1(3) - 2\right)$$

$$N_1 = \frac{8\pi^2 a^5}{3b^2} C_1 \rho_o^2 \left(\frac{9\pi}{8}\right)^{2/3} \left(A_1(8/3)+A_1(14/3)-2A_1(11/3)\right)$$

$$- 2\pi^2 a^5 \rho_o^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} C_1 \left(A_1(2)+A_1(4)-2A_1(3)\right)$$

$$P_o = 2\pi^2 a^3 C \rho_o^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} - \frac{12\pi^2 a^3}{5b^2} C \rho_o^2 \left(\frac{9\pi}{8}\right)^{2/3} A_1(8/3)$$

$$- \frac{6\hbar^2}{5m} \rho_o \left(\frac{9\pi}{8}\right)^{2/3} + \frac{2\pi m T^2}{\hbar^2 \rho^{1/3}} (1/2)^{2/3} A_1(1/3)$$

and

$$P_1 = 2\pi^2 a^3 C_1 \rho_o^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} - \frac{8\pi^2 a^3}{3b^2} C_1 \rho_o^2 \left(\frac{9\pi}{8}\right)^{2/3}$$

$$- \frac{2h^2 \pi \rho_o}{3m} \left(\frac{9\pi}{8}\right)^{2/3} A_1(5/3) + \frac{2\pi m T^2}{\hbar^2 \rho_o^{1/3}} (1/2)^{2/3} A_1(1/3)$$

These functions are written at the equilibrium density, using the density expansion (equation 32), we get:

$$N(\rho_{on}) = N^o(\rho_{on}) + N^1(\rho_{on}) T^2 \qquad (B.3)$$

and

$$P(\rho_{on}) = P^o(\rho_{on}) + P^1(\rho_{on}) T^2 \quad , \qquad (B.4)$$

where the functions $N(\rho_{on})$ and $P(\rho_{on})$ are expanded in X,Y and Z up to second order. The result is:

$$F^{o,1} = F_o^{o,1} + F_x^{o,1} X^2 + F_y^{o,1} Y^2 + F_z^{o,1} Z^2 \quad , \qquad (B.5)$$

where

$$N_o^o = \frac{12\pi^2 a^5}{5b^2} C \rho_{on}^2 \left(\frac{9\pi}{8}\right)^{2/3} \left(A_1(8/3)+A_1(14/3)-2A_1(11/3)\right)$$

$$- 2\pi^2 a^5 C \rho_{on}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} \left(A_1(2)+A_1(4)-2A_1(3)\right)$$

$$- \frac{2\pi^2 \beta}{m} \rho_{on}^{1/3} \left(\frac{3}{4\pi}\right)^{2/3} \left(A(3) - 2\right) \qquad (B.6)$$

$$N_1^o = - \frac{24\pi^2 a^5}{5b^2} C \rho_1 \rho_{on}^2 \left(\frac{9\pi}{8}\right)^{2/3} \left(A_1(8/3)+A_1(14/3)-2A_1(11/3)\right)$$

$$+ \frac{8\pi^2 a^5}{3b^2} C \rho_1 \rho_{on}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} \left(A_1(2)+A_1(4)-2A_1(3)\right)$$

$$+ \frac{2\hbar^2 \pi \beta}{3m} \rho_1 \rho_{on}^{1/3} \left(\frac{3}{4\pi}\right)^{2/3} \left(A_1(3)-2\right)$$

$$+ \frac{8\pi^2 a^5}{3b^2} C_1 \rho_{on}^2 \left(A_1(8/3)+A_1(14/3)-2A_1(11/3)\right)$$

$$- 2\pi^2 a^5 \rho_{on}^{4/3} \left(\frac{3}{4}\right)^{2/3} C_1 \left(A_1(2)+A_1(4)-2A_1(3)\right) \qquad (B.7)$$

$$N_o^1 = -\frac{24\pi^2 a^5}{5b^2} C \rho_{on}^2 \rho_t \left(\frac{9\pi}{8}\right)^{2/3} \left(A_1(8/3)+A_1(14/3)-2A_1(11/3)\right)$$

$$+\frac{8\pi^2 a^5}{3b^2} C \rho_{on}^{4/3} \rho_t \left(\frac{3}{4\pi}\right)^{2/3} \left(A_1(2)+A_1(4)-2A_1(3)\right)$$

$$+\frac{2\hbar^2\pi\beta}{3m} \rho_t \rho_{on}^{1/3} \left(\frac{3}{4\pi}\right)^{2/3} \left(A_1(3)-2\right) \qquad \text{(B.8)}$$

$$N_i^1 = -\frac{16\pi^2 a^2}{3b^2} C_i \rho_t \rho_{on}^2 \left(\frac{9\pi}{8}\right)^{2/3} \left(A_1(8/3)+A_1(14/3)-2A_1(11/3)\right)$$

$$+\frac{8\pi^2 a^5}{3} \rho_t \rho_{on}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} \left(A\ (2)+A\ (4)-2A\ (3)\right) \qquad \text{(B.9)}$$

$$P_o^o = 2\pi^2 a^3 C \rho_{on}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} \quad \frac{12\pi^2 a^3}{5b^2} C \rho_{on}^2 \left(\frac{9\pi}{8}\right)^{2/3} A_1(8/3)$$

$$-\frac{6\pi\hbar^2}{5m} \rho_{on} \left(\frac{9\pi}{8}\right)^{2/3} \quad A_1(5/3) \qquad \text{(B.10)}$$

$$P_i^o = -\frac{8\pi^2 a^3 C}{3} \rho_i \rho_{on}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} - \frac{8\pi^2 a^3}{3b^2} \rho_{on}^2 C_i A_1(8/3) \left(\frac{9\pi}{8}\right)^{2/3}$$

$$+\frac{24\pi^2 a^3 C}{5b^2} \rho_i \rho_{on}^2 \left(\frac{9\pi}{8}\right)^{2/3} A_1(8/3) + 2\pi^2 a^3 \rho_{on}^{4/3} C_i \left(\frac{3}{4\pi}\right)^{2/3}$$

$$+\frac{6\pi\hbar^2}{5m} \rho_i \rho_{on} \left(\frac{9\pi}{8}\right)^{2/3} A_1(5/3) \qquad \text{(B.11)}$$

$$P_o^1 = -\frac{8\pi^2 a^3}{3} C \rho_t \rho_{on}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3}$$

$$+\frac{24\pi^2 a^3}{5b^2} C \rho_t \rho_{on}^2 \left(\frac{9\pi}{8}\right)^{2/3} A_1(8/3)$$

$$+\frac{6\pi\hbar^2}{5m} \rho_t \rho_{on} \left(\frac{9\pi}{8}\right)^{2/3} A_1(5/3)$$

$$+\frac{2\pi m}{\hbar^2 \rho^{1/3}} (1/2)^{2/3} A_1(1/3) \qquad \text{(B.12)}$$

and

$$P_i^1 = -\frac{8\pi^2 a^3}{3} C_i \rho_t \rho_{on}^{3/4} + \frac{16\pi^2 a^3}{5b^2} C_i \rho_t \rho_{on}^2 \left(\frac{9\pi}{8}\right)^{2/3} A_1(8/3)$$

$$-\frac{2m}{\hbar^2 \rho^{1/3}} (1/2)^{2/3} A_1(1/3) \left(\frac{1+3\rho_i}{9}\right)$$

$$+\frac{2\pi\hbar^2}{3m} \rho_t \rho_{on} \left(\frac{9\pi}{8}\right)^{2/3} A_1(5/3) \qquad \text{(B.13)}$$

APPENDIC C

We shall use equations (B.3) and (B.4) to express the surface energy and the diffuseness parameter in terms of ρ_{on}. The result up to second order in X,Y,Z and T is written in the form:

$$d = d_o + \alpha_o T^2 + (d_x+\alpha_x T^2)\, X^2 + (d_y+\alpha_y T^2)\, Y^2 + (d_z+\alpha_z T^2)\, Z^2 \tag{C.1}$$

$$F_s = F_{so} + a_{so} T^2 + (F_{sx}+a_{sx} T^2)\, X^2 + (F_{sy}+a_{sy} T^2)\, Y^2 + (F_{sz}+a_{sz} T^2)\, Z^2 \tag{C.2}$$

where

$$d_o = (N_o^o / P_o^o)^{1/2} \tag{C.3}$$

$$\alpha_o = \tfrac{1}{2}\, d_o \left[(N_o^1/N_o^o) - (P_o^1/P_o^o)\right] \tag{C.4}$$

$$\begin{aligned}\alpha_i = \tfrac{1}{2}\, d_o \{ &\tfrac{1}{2}\left[(N_i^o/N_o^o) - (P_i^o/P_o^o)\right] \cdot \left[(N_o^1/N_o^o) - (P_o^1/P_o^o)\right] \\ &+ (N_o^1/N_o^o)\left[(N_i^1/N_o^1) - (N_i^o/N_o^o)\right] \\ &- (P_o^1/P_o^o)\left[(P_i^1/P_o^1) - (P_i^o/P_o^o)\right] \}\end{aligned} \tag{C.5}$$

$$d_i = \tfrac{1}{2}\, d_o \left[(N_i^o/N_o^o) - (P_i^o/P_o^o)\right] \tag{C.6}$$

$$F_{so} = 2\, (N_o^o \cdot P_o^o)^{1/2} \tag{C.7}$$

$$a_{so} = 2\, (\alpha_o P_o^o + d_o P_o^1) \tag{C.8}$$

$$a_{si} = 2\, (\alpha_o P_i^o + d_o P_i^1 + \alpha_i P_o^o) \tag{C.9}$$

and

$$F_{si} = 2\, (d_i P_o^o + d_o P_i^o) \tag{C.10}$$

In order to obtain the similar formula for the curvature energy, we expand d^2 up to second order in X,Y,Z and T. The result is:

$$F_c = F_{co} + a_{co} T^2 + (F_{cx}+a_{cx} T^2)\, X^2 + (F_{cy}+a_{cy} T^2)\, Y^2 + (F_{cz}+a_{cz} T^2)\, Z^2 , \tag{C.11}$$

where

$$\begin{aligned}F_{co} = &\frac{6\pi^3 a^3}{5b^2}\, C\, \rho_{on}^{7/3}\, (1/2)^{1/3}\, \left(6A_2(8/3)-\pi^2\right)\, (N_o^o/P_o^o) \\ &+ \frac{3\pi^2\hbar^2}{5m}\, \rho_{on}^{4/3}\, \left(6A_2(5/3)-\pi^2\right)\, (N_o^o/P_o^o) + \frac{2\pi\hbar^2\beta}{m}\, \rho^{2/3}\, \left(\frac{3}{4\pi}\right)^{1/3} \\ &- \frac{36\pi^3 a^3}{5b^2}\, C\, \rho_{on}^{7/3}\, (1/2)^{1/3}\, \left[A_2(8/3)+A_2(14/3)-2A_2(11/3)\right]\end{aligned} \tag{C.12}$$

$$\begin{aligned}a_{co} = &\frac{6\pi^3 a^3}{5b^2}\, C\, \rho_{on}^{7/3}\, (1/2)^{1/3}\, \left(6A_2(8/3)-\pi^2\right)\, \left(d_1 - \frac{7\rho_t N_o^o}{3P_o^o}\right) \\ &+ \frac{3\pi^2\hbar^2}{5m}\, \rho_{on}^{4/3}\, \left(6A_2(5/3)-\pi^2\right)\, \left(d_1 - \frac{4\rho_t N_o^o}{3P_o^o}\right) \\ &- \frac{4\pi\hbar^2\beta}{3m}\, \rho_t\, \rho_{on}^{2/3}\, \left(\frac{3}{4\pi}\right)^{1/3} + \frac{2\pi^2 m}{3\hbar^2}\, \left(\frac{1}{3\pi^2}\right)^{1/3}\, (N_o^o/P_o^o) \\ &+ \frac{84\pi^3 a^5}{5b^2}\, C\, \rho_t\, \rho_{on}^{7/3}\, (1/2)^{1/3}\, \left[A_2(8/3)+A_2(14/3)-2A_2(11/3)\right]\end{aligned} \tag{C.13}$$

$$F_{ci} = \frac{6\pi^3 a^3}{5b^2}\ \rho_{on}^{7/3}\ (1/2)^{1/3}\ \left(6A_2(8/3)-\pi^2\right)\left[C\left(d_1 - \frac{7\rho_i N_o^o}{3P_o^o}\right)+\frac{10C_i N_o^o}{9P_o^o}\right]$$

$$+\frac{3\pi^2\hbar^2}{3m}\ \rho_{on}^{4/3}\ \left(6A_2(5/3)-\pi^2\right)\left[\frac{5N_o^o}{9P_o^o} + d_{2i} - \frac{4\rho_i N_o^o}{3P_o^o}\right]$$

$$-\frac{36\pi^3 a^5}{5b^2}\ \rho_{on}^{7/3}\ (1/2)^{1/3})\ \left[\left(A_2(8/3)+A_2(14/3)-2A_2(11/3)\right).\right.$$

$$\left.\left(\frac{10}{9}\ C_i - \frac{7}{3}\ C\ \rho_i\right)\right]$$

$$-\frac{4\pi\hbar^2}{3m}\ \rho_{on}^{2/3}\ \rho_i\ \left(\frac{3}{4\pi}\right)^{1/3} \qquad (C.14)$$

and

$$a_{ci} = \frac{6\pi^3 a^3}{5b^2}\ \rho_{on}^{7/3}\ (1/2)^{1/3}\ \left(6A_2(8/3)-\pi^2\right)\ \left[C\left(d_{3i} - \frac{7}{3}\ d_1\rho_i - \frac{7}{3}\ d_{2i}\rho_t\right)\right.$$

$$\left.+\frac{10}{9}\ C_i\left(d_1 - \frac{7\rho_t N_o^o}{3P_o^o}\right)\right]$$

$$+\frac{2\pi^2 m}{27\hbar^2}\ \left(\frac{1}{3\pi^2}\right)^{1/3}\ \left(6A_2(1/3)-\pi^2\right)\ (N_o^o/P_o^o)$$

$$+\frac{56\pi^3 a^5}{3b^2}\ C_i\ \rho_t\ \rho_{on}^{7/3}\ (1/2)^{1/3}\ \left(A_2(8/3)+A_2(14/3)-2A_2(11/3)\right)$$

$$+\frac{3\pi^2\hbar^2}{3m}\ \rho_{on}^{4/3}\ \left(6A_2(5/3)-\pi^2\right)\ \left[d_{3i} - \frac{4}{3}\ \rho_i d_1 - \frac{4}{3}\ \rho_t d_{2i}\right.$$

$$\left.+\frac{5}{9}\ \left(d_1 - \frac{4\rho_t N_o^o}{3P_o^o}\right)\right] \qquad (C.15)$$

where

$$d_i = \left[(N_o^1/N_o^o) - (P_o^1/P_o^o)\right] / (P_o^o)^2 \quad ,$$

$$d_{2i} = \left[(N_i^o/N_o^o) - (P_i^o/P_o^o)\right]/ (P_o^o)^2$$

and

$$d_{3i} = (N_o^1/P_o^o)\ \left[(N_i^o/N_o^o) - (P_i^o/P_o^o)\right] - \left[N_o^o P_o^1/(P_o^o)^2\right]\ \left[(P_i^1/P_o^o) - (N_i^1/N_o^o)\right]$$

$$+ (N_o^o/P_o^o)\ \left[(N_i^o/N_o^o) - (P_i^o/P_o^o)\right]\left[(N_o^1/N_o^o) - (P_o^1/P_o^o)\right]$$

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TABLE CAPTIONS

Table (1): The coefficients of the density expansion (equation 32). The units of these coefficients are fm^{-3}.

Table (2): The coefficients of the diffuseness parameter expansion equation (33). The units of d_i are fm and the units of α_i are $fm.MeV^{-2}$.
a) Ref. (21).

Table (3): The coefficients of the surface free energy expansion (equation 34). The units of F_{si} are MeV and the units of a_{si} are MeV^{-1}.
a) Ref. (21) , b) Ref. (66) , c) Ref. (20)
d) Ref. (31) , e) Ref. (15) , f) Ref. (67)
g) Ref. (26) , h) Ref. (29) , j) Ref. (38)
k) Ref. (39) , l) Ref. (23) , v) Ref. (30)
w) Ref. (68) and n) Ref. (18)

Table (4): The same as Table (3) but for the curvature free energy expansion (equation 35).

Table (5): The volume level density parameters (equations 36 and 37) in units of MeV^{-1}.

Table (1)

ρ_{on}	ρ_x	ρ_y	ρ_z	ρ_t	ρ^1	Ref.
0.145	1.176	1.214	1.115	0.002	-0.0002	PW3
0.144	0.09	1.62	0.84	0.012	-	4
0.225	0.49	-	-	-	-	64
0.185	0.45	-	-	-	-	62

Table (2)

Parameter / Force	d_o	d_x	d_y	d_z	$\alpha_o x10^3$	$\alpha_x x10^3$	$\alpha_y x10^3$	$\alpha_z x10^3$
PW3	0.431	-0.01	-0.01	-0.01	0.0025	0.0043	0.0024	0.0074
PW2	0.431	0.36	0.37	0.35	0.0025	0.023	0.022	0.025
PW1	0.431	0.36	0.37	0.35	0.0038	0.040	0.040	0.041
SII	0.421				0.0035			
SIII	0.398[a)]				0.0033[a)]			
SIV	0.445[a)]				0.0037[a)]			
SV	0.467[a)]				0.0038[a)]			
SVI	0.394[a)]				0.0033[a)]			
SKT	0.487[a)]				0.0043[a)]			
SEI	0.469[a)]				0.0041[a)]			

Table (3)

Parameter / Force	F_{so}	$-F_{sx}$	$-F_{sy}$	$-F_{sz}$	a_{so}	a_{sx}	a_{sy}	a_{sz}
PW3	21.5	-31.0	-30.0	-32.0	-0.145	0.320	0.259	0.419
PW2	21.5	-27.0	-26.2	-28.2	-0.145	1.018	0.979	1.08
PW1	21.5	-27.0	-26.2	-28.2	-0.191	1.616	1.6091	1.627
SII	19.65[a]	52.0[b]	-	-	-0.189[a]	-	-	-
	20.08[c]	-	-	-	-0.199[c]	0.9174[c]	-	-
	19.08[k]	-	-	-	-	-	-	-
	20.17[l]	60.0[l]	-	-	-	-	-	-
SIII	18.91[a]	29.0[b]	-	-	-0.183[a]	-	-	-
	19.29[c]	47.9[c]	-	-	-0.194[c]	0.0171[c]	-	-
	18.89[d]	33.2[d]	-	-	-0.150[e]	-	-	-
	20.37[f]	-	-	-	-	-	-	-
	18.2 [h]	39.13[h]	-	-	-0.233[n]	-	-	-
	18.8 [k]	-	-	-	-	-	-	-
	18.79[l]	35.00[l]	-	-	-	-	-	-
	24.3 [v]	-	-	-	-	-	-	-
SIV	20.28[a]	55.00[b]	-	-	-0.193[a]	-	-	-
	18.75[c]	29.6 [c]	-	-	-0.185[c]	0.0161[c]	-	-
	19.42[h]	37.29[h]	-	-	-	-	-	-
	19.6 [k]	-	-	-	-	-	-	-
	20.12[l]	64.00[l]	-	-	-	-	-	-
	24.8 [v]	-	-	-	-	-	-	-
SV	21.15[a]	74.00[b]	-	-	-0.195[a]	-	-	-
	20.15[h]	36.27[h]	-	-	-0.224[n]	-	-	-
	19.8 [k]	-	-	-	-	-	-	-
	21.21[l]	97.00[l]	-	-	-	-	-	-
SVI	18.31[a]	22.00[b]	-	-	-0.176[a]	-	-	-
	18.75[c]	23.20[c]	-	-	-0.185[c]	-	-	-
	18.3 [k]	-	-	-	-	-	-	-
	18.13[l]	25.00[l]	-	-	-0.218[n]	-	-	-

Table (3) (cont.)

Parameter / Force	F_{so}	$-F_{sx}$	$-F_{sy}$	$-F_{sz}$	a_{so}	a_{sx}	a_{sy}	a_{sz}
SKM	18.72[a]	48.2 [c]	-	-	-0.227[c]	0.0181[c]	-	-
	18.83[d]	58.9 [d]	-	-	-	-	-	-
	19.16[f]	-	-	-	-0.228[n]	-	-	-
	17.2 [k]	-	-	-	-	-	-	-
	17.34[l]	57.00[l]	-	-	-	-	-	-
	23.00[v]	-	-	-	-	-	-	-
SKM*	19.37[c]	47.9 [c]	-	-	-0.235[c]	0.0188[c]	-	-
	19.06[d]	58.0 [d]	-	-	-	-	-	-
	17.96[l]	55.00[l]	-	-	-	-	-	-
SKT	19.54[a]	53.00[b]	-	-	-	-	-	-
MDI4	18.32[h]	29.31[h]	-	-	-	-	-	-
SEI	20.1 [w]	-	-	-	-	-	-	-
SI'	17.94[d]	38.9 [d]	-	-	-	-	-	-
SKa	18.89[h]	32.87[h]	-	-	-	-	-	-
	19.57[l]	74.00[l]	-	-	-	-	-	-
P1	20.5 [w]	38.0 [w]	-	-	-	-	-	-
P2	20.8 [w]	100.0 [w]	-	-	-	-	-	-
SKM1	23.6 [v]	-	-	-	-	-	-	-
SKM (ITF)	-	-	-	-	-0.139[n]	-	-	-

Table (4)

Parameter / Force	F_{co}	$-F_{cx}$	$-F_{cy}$	$-F_{cz}$	a_{co}	a_{cx}	a_{cy}	a_{cz}
PW3	9.9	20.0	19.3	21.0	0.429	0.229	0.223	0.239
PW2	9.9	17.0	16.0	18.0	0.429	0.358	0.356	0.431
PW1	9.9	17.0	16.0	18.0	0.422	0.440	0.445	0.431
SI'	6.49[d]	27[d]	-	-	-	-	-	-
SII	14.4[k]	-	-	-	-	-	-	-
	11.6[l]	-	-	-	-	-	-	-
SIII	7.22[d]	26[d]	-	-	-			
	11.29[f]	-	-	-	0.103[n]	-	-	-
	10.67[j]	-	-	-	-	-	-	-
	10.60[k]	-	-	-	-	-	-	-
	10.00[l]	-	-	-	-	-	-	-
SIV	12.30[k]	-	-	-	-	-	-	-
	12.10[l]	-	-	-	-	-	-	-
SV	12.9 [k]	-	-	-	0.039[n]	-	-	-
	13.8 [l]	-	-	-	-	-	-	-
SVI	9.90[k]	-	-	-	0.166[n]	-	-	-
	9.30[l]	-	-	-	-	-	-	-
SKM	9.99[d]	51[d]	-	-	-	-	-	-
	15.35[f]	-	-	-	0.175[n]	-	-	-
	12.6 [k]	-	-	-	-	-	-	-
	12.9 [l]	-	-	-	-	-	-	-
SKM*	10.63[d]	52[d]	-	-	-	-	-	-
	14.50[l]	-	-	-	-	-	-	-
SKa	13.6 [l]	-	-	-	-	-	-	-
SKM (ITF)	-	-	-	-	0.267[n]			

Table (5)

Force	a_{vo}	a_{vx}	a_{vy}	a_{vz}
PW3	0.075	-0.008	-0.008	-0.008
SII	0.041[a]	-0.0035[c]	-	-
SIII	0.055[e]	-0.006 [e]	-	-
	0.054[n]	-0.0061[n]	-	-
	0.054[c]	-	-	-
SIV	0.068[c]	-0.0082[c]	-	-
SV	0.026[n]	-	-	-
SVI	0.068[n]	-	-	-
SKM	0.053[c]	-0.005 [c]	-	-
	0.053[c]	-	-	-

FIGURE CAPTION

Fig.1 The level density parameter (a) as a function of A.

Ref.(18) = SKM(m*/m = 1)

Ref.(76) = CI

Present work = PW

Ref.(33) = S VI

Ref.(33) = S III

Ref.(33) = SKM(ITF)

Ref.(33) = SCM

Ref.(33) = S V

Ref.(33) = S II

Experimental data = Ref.(71)