ANALYTIC STOCHASTIC REGULARIZATION: GAUGE AND SUPERSYMMETRIC THEORIES*

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ABSTRACT

Analytic stochastic regularization for gauge and supersymmetric theories is considered. Gauge invariance in spinor and scalar QCD is verified to break down by an explicit one loop computation of the two, three and four point vertex function of the gluon field. As a result, non gauge invariant counterterms must be added. However, in the supersymmetric multiplets there is a cancellation rendering the counterterms gauge invariant. The calculation is considered at one loop order.

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1. INTRODUCTION

Quantization of non abelian gauge theories is a rather involved problem because local symmetries are very difficult to be mantained. For example dimensional regularization preserves gauge symmetry while breaks supersymmetry and this scheme can not be considered when supersymmetric theories are considered. Besides, non perturbatively there are the Gribov ambiguites[1] which prevent a clear gauge specification. The issue here, concerns a proposal of a new regularization scheme based on the Langevin equation[2] with a non Markovian process[3]. This procedure is not related to space-time in such a way that can circumvent the above mentioned difficulties, and some authors claim that indeed, such a regularization scheme is able to preserve all symmetries of the Lagrangian, including gauge symmetry and supersymmetry. Actually some results confirm partially this claim. It has been shown that QCD vertex functions with zero external momenta vanish, providing no mass counterterm[4]. At one loop level, two dimensional scalar QCD also confirms this fact providing that all the counterterms are gauge invariant. However this is not the case when we consider four dimensional gauge theories[5]. In this case non gauge invariant counterterms containing derivatives of the gauge field must be added. The polarization tensor and the three and four vertex function of scalar QCD will be presented and besides the usual transverse gauge invariant terms a divergent part of the form $A^{\mu}\partial^{2}A_{\mu}$ is found. As we shall see this non gauge invariant counterterm is quite innocuous in the abelian case, but constitutes a breaking of gauge symmetry in the non-abelian case.

In spinor QCD case already at one loop level the $A^{\mu}\partial^2 A_{\mu}$ counterterm appears again and non gauge invariant counterterms have to be added, spoiling the splicit gauge independence.

Finally when QCD is coupled to supersymmetric matter fields, namely two bosonic and one fermionic field with the same charge, the matter contribution to that dangerous counterterm vanishes.[6] Moreover, if we add fermions in the adjoint representation, a cancellation of gauge dependent counterterms arising from the gauge field self energy occurs.

The scheme is therefore gauge independent, at least to one loop order, for supersymmetric gauge theories.

2. STOCHASTIC QUANTIZATION AND REGULARIZATION

The Langevin equation for an arbitrary field $\varphi(x,t)$ and its corresponding noise $\eta(x,t)$ is given by:

$$\frac{\partial \varphi_\eta(x,t)}{\partial t} = - \frac{\delta S}{\delta \varphi_\eta(x,t)} + \eta(x,t) \tag{2.1}$$

where the t-variable is the so called fifth time, S the classical action. The randomic noise $\eta(x,t)$ has two point function given by:

$$\langle \eta_i(x,t)\, \eta_j(x',t') \rangle = 2\,\delta_{ij}\, \delta(x-x')\, \delta(t-t') \tag{2.2}$$

and Wicks decomposition resolves any higher function. This corresponds to a lagrangean which is the square of the noise. Therefore it is called a white noise. Usual averages for functions of the field $\varphi(x,t)$ are taken with respect to η.

$$\langle F[\varphi(x,t)] \rangle_\eta = \int \mathcal{D}\eta\, F[\varphi_\eta(x,t)]\, e^{-\int_0^\infty dt \int d^4y\, \eta_i(y,t)\, \eta_i(y,t)} \tag{2.3}$$

and the field theory is defined defining Green functions as the stationary $(t \to \infty)$ limit of the statistical average:

$$\langle T \varphi_{i_1}(x_1) \ldots \varphi_{i_n}(x_n) \rangle = \lim_{t \to \infty} \langle \varphi_{i_1}(x_1,t) \ldots \varphi_{i_n}(x_n,t) \rangle \tag{2.4}$$

Stochastic quantization was originally presented for bosonic model and only recently developed for fermions.[7] The idea is generalize the original Langevin equation introducing a kernel K_{ij} in such a way that the equation (2.1) becames:

$$\frac{\partial \varphi_i(x,t)}{\partial t} = -\int d^D_y \; K_{ij}(x,y) \frac{\delta S}{\delta \varphi_j(y,t)} + \eta_i(x,t) \tag{2.5}$$

where now the classical noise $\eta_i(x,t)$ has as two point function

$$\langle \eta_i(x,t)\, \eta_j(x',t') \rangle = 2\, K_{ij}(x,x')\, \delta(t-t') \tag{2.6}$$

Considering free fermion with the classical Euclidean action

$$S(\psi,\overline{\psi}) = -i \int d^4x \; \overline{\psi}(x) (\partial + im) \psi(x) \tag{2.7}$$

the kernel is given by

$$K_{\alpha\beta}(x,y) = \left(i\, \bar{\partial}_x + m \right)_{\alpha\beta} \delta(x-y) \tag{2.8}$$

and the free fermionic Langevin equation is

$$\frac{\partial \psi(x,t)}{\partial t} = (\partial^2 + m^2)\, \psi(x,t) + \vartheta(x,t) \tag{2.9}$$

where $\vartheta(x,t)$ is a Grassmann white noise.

In perturbation theory we can obtain the Feynman rules iterating the Langevin equations above. This has been frequently done in the literature[8]. Thus it is possible to write down the Feynaman diagrams, which turn out to present divergences, as in the usual perturbation theory. Stochastic regularization prescribes a non white noise, by means of the introduction of a non Markovian element in the stochastic process. The noise two point function will be given by

$$\langle \eta_i(x,t)\,\eta_j(x',t')\rangle = \delta_{ij}\,\delta(x-x')\,f_\varepsilon(t-t') \tag{2.10}$$

where

$$\lim_{\varepsilon\to 0} f_\varepsilon(t-t') = 2\,\delta(t-t')$$

A choice of f_ε very similar to analytic regularization is

$$f_\varepsilon(t) = \varepsilon\,|t|^{\varepsilon-1} \tag{2.11}$$

which is adopted, the Green functions being meromorphic functions of ε with poles on the real axis.

The regularized crossed propagator, given by the expression

$$\mathcal{D}(x-x';t,t') = \int_0^t d\tau \int_0^{t'} d\tau' \int dy \int dy'\; G(x-y;t-\tau)\, G(x'-y';t'-\tau')\, \langle \eta(y,\tau)\,\eta(y',\tau')\rangle \tag{2.12}$$

where $\mathcal{G}$ the retarded Green function of the Langevin equation, may be computed. In the case of a scalar field it is given by

$$\tilde{D}^{\varepsilon}(p;t,t') = \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{e^{-i\omega(t-t')}}{(p^2+m^2)+\omega^2} \hat{f}_{\varepsilon} |\omega|^{-\varepsilon} \tag{2.13}$$

while for fermions we obtain

$$\Delta_{\alpha\beta}(k;t,t') = \hat{f}_{\varepsilon} \frac{(k-m)_{\alpha\beta}}{(k^2+m^2)^{1+\varepsilon}} \int \frac{d\omega}{\pi} \frac{e^{-i\omega(k^2+m^2)(t-t')}}{1+\omega^2} |\omega|^{-\varepsilon} \tag{2.14}$$

where

$$\hat{f}_{\varepsilon} = \varepsilon\, \Gamma(\varepsilon) \sin\frac{\pi}{2}(1-\varepsilon)$$

If we are performing a one loop computation (it is the case, in this paper) we need only

$$\tilde{D}^{\varepsilon}(p;t,t') = \frac{\hat{f}_{\varepsilon}}{(p^2+m^2)^{1+\varepsilon}} \left\{ e^{-(p^2+m^2)(t-t')} - \varepsilon \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \frac{\ln\omega}{1+\omega^2} e^{-i\omega(p^2+m^2)(t-t')} + \mathcal{O}(\varepsilon^2) \right\} \tag{2.15}$$

As a sample computation, we calculate the polarization tensor in 2-dimensional QED. The computation is facilitated making an expansion on the external momenta. The diagrams are shown in fig. ( ).The divergent pieces turn out to cancel, and for the finite part contribution at first order in the external momenta expansion we have

$$\Pi_{\mu\nu} = \frac{1}{12\pi m^2}\left(\frac{p_\mu p_\nu}{p^2} - \delta_{\mu\nu}\right) \qquad (2.16)$$

which is transverse. Hence the scheme gives gauge invariant results. However, there will be problems in four dimensions due to new divergences ocurring in diagram (Ia).

3. SCALAR AND SPINOR QCD IN FOUR DIMENSIONS: A BREAK OF GAUGE INVARIANCE

The action to be studied is given by

$$\mathcal{L} = -\frac{1}{4} \mathrm{tr}\, F_{\mu\nu} F^{\mu\nu} + \overline{D_\mu \varphi_i}\, D_\mu \varphi_i + m^2 \bar{\varphi}_i \varphi_i$$
$$- \bar{\Psi}_a (i \not{D} - M) \Psi_a \tag{3.1}$$

Different flavour indices for bosons and fermions have been introduced. We treat bosonic and fermionic contributions separately.

The Langevin equations are

$$\begin{aligned}
\dot{A}_\mu &= D_\rho F_{\rho\mu} - ie\, \bar{\varphi} D_\mu \varphi + \eta_\mu \\
\dot{\varphi} &= D_\mu D_\mu \varphi - m^2 \varphi + \eta \\
\dot{\varphi}^* &= D_\mu D_\mu \varphi^* - m^2 \varphi^* + \eta^* \\
\dot{\Psi} &= -(\not{D} - iM)(\not{D} + iM)\Psi + \Theta \\
\dot{\bar{\Psi}} &= - \bar{\Psi} (\overleftarrow{\not{D}}' - iM)^T (\overleftarrow{\not{D}}' + iM)^T + \Theta
\end{aligned} \tag{3.2}$$

where

$$\begin{aligned}
D'_\mu &= -\partial_\mu + ie A_\mu \\
D_\mu &= \partial_\mu + ie A_\mu
\end{aligned}$$

The choice of the kernel for fermions is that proposed by Ishikawa[9]. However, at one loop order we will not need the gauge field contribution to the noise. Thus we use

$$\langle \Theta_\alpha(x,t)\,\bar\Theta_\beta(x',t')\rangle = (\not k + M)_{\alpha\beta}\,\delta^4(x-x')\,f_\varepsilon(t-t')$$

$$\langle \eta_\mu(x,t)\,\eta_\nu(x',t')\rangle = \delta_{\mu\nu}\,\delta(x-x')\,\delta(t-t')$$

$$\langle \eta(x,t)\,\eta(x',t')\rangle = \delta(x-x')\,\delta(t-t') \qquad (3.3)$$

If one uses dimensional regularization $(f_\varepsilon \to \delta\ ;\ \delta(x-x') \to \delta^{4-2\varepsilon}(x-x'))$ the diagrams may be grouped in such a way that gauge invariance is respected, and the counterterm structure is

$$\left(Z_1^B + Z_1^F\right) F_{\mu\nu}^a\, F_{\mu\nu}^a \qquad (3.4)$$

where

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + ie\, Z_2\, [A_\mu, A_\nu]$$

Z_1^B and Z_1^F are respectively the bosonic and fermionic contributions (we are always at one loop level).

However, the result of the calculation using analytic stochastic regularization is

$$\left(Z_1^B + Z_1^F\right) F_{\mu\nu}^a F_{\mu\nu}^a + \left(Z_3^B + Z_3^F\right) A_\mu \partial^2 A_\mu \qquad (3.5)$$

We separate the divergent part doing an expansion in the external momenta. Moreover only external gluon lines and internal matter field contributions are computed. Therefore the gauge-ghost system need not enter the computation. This is enough since gauge invariance must hold separately for these two systems, namely matter and gauge/ghost (we could consider, as an example the number of flavors big enough).

We have the following contributions:

1. graphs with two external gluon lines, fig. 1.

For bosons we have

$$1aB = \frac{\delta_{\mu\nu}}{64\pi^2 p^2 \varepsilon} + \text{finite terms}$$

$$1bB = \frac{1}{(4\pi)^2\varepsilon}\left[-\frac{2m^2\delta_{\mu\nu}}{(p^2)^2} - \frac{5\delta_{\mu\nu}}{6p^2} + \frac{p_\mu p_\nu}{3(p^2)^2}\right]$$

$$1cB = \frac{2m^2\delta_{\mu\nu}}{(4\pi)^2(p^2)^2\varepsilon}$$

Thus, for bosons

$$1B = \frac{-\delta_{\mu\nu}}{64\pi^2 p^2\varepsilon} - \frac{1}{3(4\pi)^2 p^2\varepsilon}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \tag{3.6}$$

While for fermions

$$1aF = \frac{-\delta_{\mu\nu}}{32\, p^2\pi^2\varepsilon}$$

$$1bF = \frac{7\delta_{\mu\nu}}{48\, p^2\pi^2\varepsilon} - \frac{p_\mu p_\nu}{12\,\pi^2(p^2)^2\varepsilon}$$

Thus

$$1F = \frac{1}{32\, p^2\pi^2\varepsilon} + \frac{1}{12\pi^2 p^2\varepsilon}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \tag{3.7}$$

The remaining pieces of the counterterms come from the diagrams of the type depicted in fig. 2. These give the same result as dimensional regularization. The only difference in the hole computation is a factor of 1/2 coming in diagram 1a. Thus we have

$$Z_1^B = \frac{n_B}{(4\pi)^2 12\varepsilon} \quad ; \quad Z_1^F = \frac{-n_F}{(4\pi)^2 3\varepsilon}$$

$$Z_2^B = \frac{-n_B}{(4\pi)^2 8\varepsilon} \quad ; \quad Z_2^F = \frac{n_F}{(4\pi)^2 4\varepsilon} \tag{3.8}$$

where n_B and n_F is the number of bosonic and fermionic flavors.

A perturbative way to fix the calculation obtaining a gauge invariant result, is to consider different ε's for each crossed line, inspired in Speer's work,[10] to get rid of the factor 1/2, which in such a computation would come, in diagrams of the type of 1a as a pole in $1/(\varepsilon_1+\varepsilon_2)$. Non perturbatively there is no way, unless

$$Z_2^B + Z_2^F \doteq 0 \tag{3.9}$$

This requires $n_B = 2n_F$. Therefore, for supersymmetric matter multiplet, the schemes works! One can verify that the same is true for the supersymmetric gauge multiplet, where there is a cancellation between the vector gauge field contribution (2 degrees of freedom) with the Majoraua Gaugino.

As a conclusion, we do have problems in general with the scheme, but there is a hope in supersymmetric models.

Fig. Caption: dashed lines are fermions, while continuous lines are scalars. Wavy lines are gauge fields.

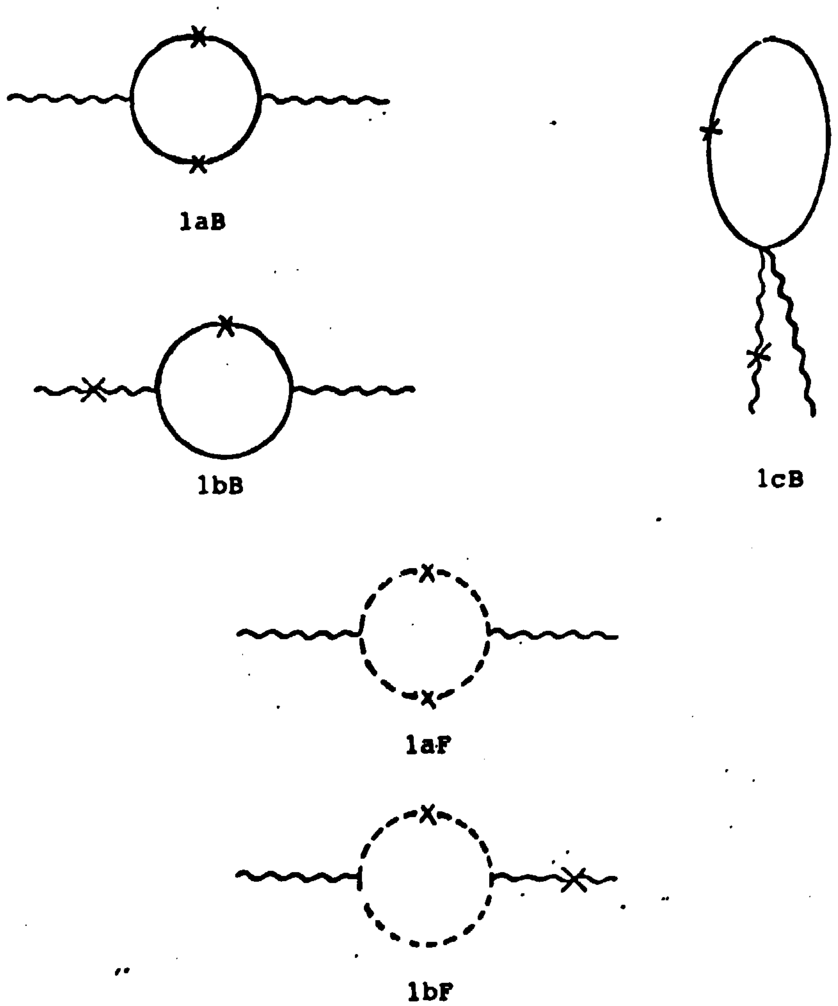

Fig. 1

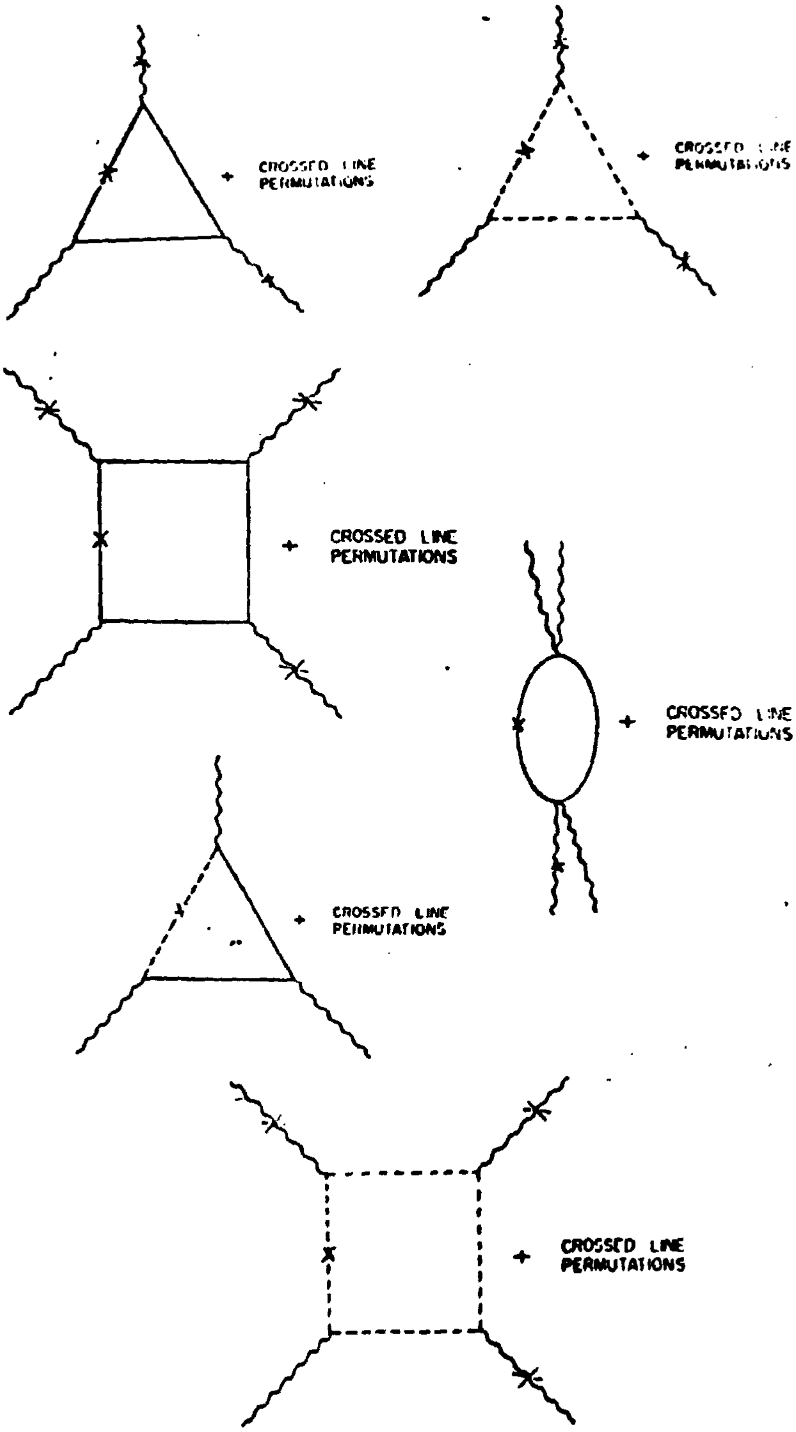
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Fig. 2

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