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14. Abstract/Notes

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15. Remarks

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NONLINEAR DIFFUSION IN THE PRESENCE OF A TIME-DEPENDENT EXTERNAL ELECTRIC FIELD

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ABSTRACT

The influence of a time-dependent external electric field on the nonlinear diffusion process studied by Mendonça and Rowlands (1985) is investigated. A new solution of the diffusion equation is obtained for the case when electron-ion collisions can be neglected.

The diffusion of a plasma through a stationary neutral gas is usually described by a linear equation derived by Shimony and Cahn (1965). When the electron temperature T_e is much higher than the ion temperature T_i, the solution of the one-dimensional version of Shimony and Cahn's equation indicates that the plasma interface propagates with a maximum speed close to the ion-acoustic speed $C_s = (kT_e/m_i)^{1/2}$, where k is the Boltzmann constant and m_i is the ion mass. This limiting speed is particularly relevant in laser-produced plasmas for technological applications, viz., laser-isotope separation (Becker and Kompa, 1982; Chen and Borzileri, 1981). However, Mendonça and Rowlands (1985) have recently shown that the inclusion of the dominant nonlinear terms in the diffusion equation leads to diffusion waves in the form of solitary pulses. Such type of nonlinear diffusion waves have been experimentally identified by Sã and Mendonça (1984) in a glow discharge. Solitary diffusion waves are relevant not only as an interesting physical process (Hastings, Hazeltine, and Morrison, 1986), but also because they allow the possibility of overcoming the limiting expanding speed in practical applications.

In the derivation of the nonlinear diffusion equation, Mendonça and Rowlands (1985) have included the effect of a constant external electric field and taken into account both the collisions between charged species and between them and neutral atoms. The solitary solution of the diffusion equation is found to depend strongly on the electron-ion collision frequency. However, in some situations of practical relevance, the plasma is weakly ionized and the electron-ion collision frequency is negligible in comparison with their collision frequencies with neutral atoms. Furthermore, in applications such as in laser isotope separation, one cannot apply a strong external electric field before the plasma is formed because this can substantially detune the resonant levels. Thus, one has to consider a time-dependent external electric field to extract the plasma. In this note we extend the work of Mendonça and Rowlands (1985) including a time-dependent external electric field. We are interested in applications for weakly ionized plasmas and consequently neglect the electron-ion collisions. We obtain an analytical solution of the nonlinear diffusion equation which is substantially different from that obtained by Mendonça and Rowlands (1985).

Neglecting collisions between charged species, the nonlinear particle flux for species α is given by (Mendonça and Rowlands, 1985)

$$\vec{\Gamma}_\alpha = D_\alpha \nabla n_\alpha + \mu_\alpha n_\alpha \vec{E} - \frac{1}{\nu_{\alpha o}} \frac{\partial \vec{\Gamma}_\alpha}{\partial t} + \vec{\Gamma}_\alpha^N , \qquad (1)$$

where

$$\vec{\Gamma}_\alpha^N = -\frac{1}{\nu_{\alpha o}} \left[\frac{\vec{\Gamma}_\alpha}{n_\alpha} \nabla \cdot \vec{\Gamma}_\alpha + \vec{\Gamma}_\alpha \cdot \nabla \left(\frac{\vec{\Gamma}_\alpha}{n_\alpha} \right) \right] , \qquad (2)$$

μ_α is the particle mobility, D_α is the diffusion coefficient, n_α is the particle density, $\nu_{\alpha o}$ is the collision frequency with neutral atoms, and $\vec{E}$ is the electric field. Following the procedure of Mendonça and Rowlands (1985), we consider the electric field and the particle fluxes given respectively by $\vec{E} = \vec{E}_o + \vec{E}_{amb}$ and $\vec{\Gamma}_\alpha = \vec{\Gamma}_{\alpha o} + \vec{\Gamma}$, where $\vec{E}_o$ is the external (time-dependent) electric field, $\vec{E}_{amb}$ is the ambipolar electric field, $\vec{\Gamma}_\alpha$ is the corresponding particle flux, and $\vec{\Gamma}_{\alpha o}$ is required to satisfy the equation

$$\vec{\Gamma}_{\alpha o} + \frac{1}{\nu_{\alpha o}} \frac{\partial \vec{\Gamma}_{\alpha o}}{\partial t} = \mu_\alpha \bar{n} \vec{E}_o \;, \tag{3}$$

where $\bar{n}$ is the background equilibrium plasma density. Substituting (3) into equation (1) and using the continuity equation and the ambipolar approximation $n_e \simeq n_i \simeq n$, we obtain the nonlinear diffusion equation of Mendonça and Rowlands (1985)

$$\frac{\partial n}{\partial t} = D_a \nabla^2 n - \frac{1}{\nu_a} \frac{\partial^2 n}{\partial t^2} + F(\vec{\Gamma}) \;, \tag{4}$$

where $D_a = (\mu_e D_i - \mu_i D_e)/(\mu_e - \mu_i)$ is the ambipolar diffusion coefficient,

$$F(\vec{\Gamma}) = \frac{1}{\bar{n}\nu_a} \nabla \cdot (\vec{\Gamma}\nabla \cdot \vec{\Gamma} + \vec{\Gamma} \cdot \nabla\vec{\Gamma}) - \nabla \cdot (\vec{b}\nabla \cdot \vec{\Gamma} + \vec{b} \cdot \nabla\vec{\Gamma}) \;, \tag{5}$$

and

$$\vec{b} = \frac{\nu_{io}\mu_i \vec{\Gamma}_{eo} - \nu_{eo}\mu_e \vec{\Gamma}_{io}}{\bar{n}\nu_{io}\nu_{eo}(\mu_e - \mu_i)} \;. \tag{6}$$

Our expression for $F(\vec{\Gamma})$ corresponds to the one of Mendonça and Rowlands (1985) (their equation 12) with $a = c = 0$. However, we note that there is a term $\vec{c} \cdot \nabla n$ that is missing and there should be a minus sign in front of the entire right-hand side of their expression. Unfortunately, this wrong sign is carried throughout their paper.

We now consider the one-dimensional version of equation (4) and assume the external electric field varying linearly in time, i.e.,

$E_0 = E_{00} + E_{01}t$. Then the solution of equation (3) is given by $\Gamma_{\alpha 0} = \mu_\alpha \bar{n}(E_{00} + E_{01}t - E_{01}/\nu_{\alpha 0})$ and the expression for b (equation 6) becomes $b = b_0 + b_1 t$, where

$$b_0 = \frac{\mu_i \mu_e}{\nu_{i0}\nu_{e0}(\mu_e - \mu_i)} \left[(\nu_{i0} - \nu_{e0})E_{00} + \left(\frac{\nu_{e0}}{\nu_{i0}} - \frac{\nu_{i0}}{\nu_{e0}}\right)E_{01}\right]$$

and

$$b_1 = \frac{\mu_i \mu_e}{\nu_{i0}\nu_{e0}(\mu_e - \mu_i)} (\nu_{i0} - \nu_{e0})E_{01} .$$

Substituting the expressions for b and $F(\Gamma)$ into equation (4), we obtain the one-dimensional nonlinear diffusion equation

$$\frac{\partial n}{\partial t} = D_a \frac{\partial^2 n}{\partial x^2} - \frac{1}{\nu_a} \frac{\partial^2 n}{\partial t^2} + \frac{1}{\nu_a \bar{n}} \frac{\partial^2 \Gamma^2}{\partial x^2} - 2(b_0 + b_1 t) \frac{\partial^2 \Gamma}{\partial x^2} . \qquad (7)$$

where ν_a is the ambipolar collision frequency, $\nu_a = \nu_{e0}\nu_{i0}(\mu_e - \mu_i)/(\nu_{e0}\mu_e - \nu_{i0}\mu_i)$. A solution of equation (7) can be found by realizing that a time-dependent electric field can nonlinearly give rise to a net motion of the background plasma. This motion appears as a time-dependent homogeneous flux at some average velocity. Thus we make the ansatz

$$\Gamma(x,t) = n(z)v + \bar{n}\phi(t) , \qquad (8)$$

where $z = x - vt$, v is a constant propagation speed, and $\phi(t)$ is a function to be determined. Substituting equation (8) into equation (7), we find

$$\phi(t) = \frac{\mu_i \mu_e(\nu_{io} - \nu_{eo})}{\mu_e \nu_{eo} - \mu_i \nu_{io}} E_{o1} t \simeq \left(\frac{\nu_{io}}{\nu_{eo}} - 1\right) \mu_i E_{o1} t \tag{9}$$

and the equation for n becomes

$$n + \left(\frac{D_a}{v} - \frac{v}{\nu_a} - 2b_o + \frac{2v}{\bar{n}\nu_a}\right) \frac{dn}{dz} = \bar{n} \ , \tag{10}$$

where we have imposed the boundary condition $n \to \bar{n}$, $dn/dz \to 0$ as $z \to \infty$. Equation (9) shows that the time-dependent background flux is due to the unbalance between the electron and ion collision frequencies. In the case of linear diffusion, the external field can be completely balanced by the ambipolar field and such a flux does not appear. Equation (10) can be integrated by quadrature; the solution is given implicitly by the equation

$$\left(\frac{n}{\bar{n}} - 1\right)^{\delta} e^{\left(\frac{n}{\bar{n}} + \frac{\nu_a z}{2v}\right)} = 1 \ , \tag{11}$$

where

$$\delta = \frac{1}{2} + \frac{D_a \nu_a}{2v^2} - \frac{b_o \nu_a}{v} \ . \tag{12}$$

Substituting the expression for b_o into equation (12), we find that $\delta > 0$. Then it follows that the solution given by equation (11) corresponds to a monotonically decreasing profile from $z = 0$. The value of $n(z = 0)$ can be found only numerically. Finally, taking into account that $|\mu_i/\mu_e| \ll 1$, one can easily verify that the background flux is negligible in comparison with the electron equilibrium flux and it introduces a correction proportional to $\nu_{i0}/\nu_{e0} - 1$ in the ion flux.

In conclusion, we have found a new particular solution for the non-linear diffusion equation derived by Mendonça and Rowlands (1985), when the electron-ion collisions are neglected and including an external electric field varying linearly in time. This solution is represented by equations (8), (9), (11), and (12). We remark that our solution is different from that of Mendonça and Rowlands (1985) even for a constant external electric field, i.e., $E_{01} = 0$. In this case we have $\phi(t) = 0$ and $\Gamma = nv$; however, the density perturbation given by equation (11) does not have the form of a solitary pulse.

REFERENCES

BECKER F.S. and KOMPA K.L. (1982) Nuclear Tech. 58, 329.

CHEN H.L. and BORZILERI C. (1981) J. Chem. Phys. 74, 6063.

HASTINGS D.E., HAZELTINE R.D., and MORRISON P.J. (1986) Phys. Fluids 29, 69.

MENDONÇA J.T. and ROWLANDS G. (1985) Plasma Phys. Contr. Fusion 27, 777.

SHIMONY Z. and CAHN J.H. (1965) Phys. Fluids 8, 1704.

SÁ A.B. and MENDONÇA J.T. (1984) Proceedings of the International Conference on Plasma Physics, Lausanne, Switzerland.