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QUARK CORE STARS, QUARK STARS AND STRANGE STARS

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ABSTRACT

A recent one flavour quark matter equation of state is generalized to several flavours. It is shown that quarks undergo a first order phase transition. In addition, this equation of state depends on just one parameter in the two flavour case, two parameters in the three flavour case, and these parameters are constrained by phenomenology. This equation of state is then applied to 1^0)the hadron-quark transition in neutron stars and the determination of quark star stability, 2^0) the investigation of strange matter stability and possible strange star existence.

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1. INTRODUCTION

The possibility that a quark core might exist in some types of stars, was first suggested by Ivanenko & Kurdgelaidze in 1969 [1]. Then in 1970, Itoh [2] computed the maximum mass of quark stars, neglecting totally interactions between quarks. This mass, as in the *perfect neutron gas* of Oppenheimer & Volkoff of 1939 ([3]), was very small, $\sim 10^{-3}\, M_{\odot}$: the interactions between quarks were poorly known.

A few years later in 1976, after the M.I.T. bag model was invented, Brecher & Caporaso [4] showed that, by making use of it, higher maximum masses could be obtained for quark core stars. However, when first order corrections in the strong coupling constant were included in the quark M.I.T. equation of state, as done by Baym & Chin [5] and Chapline & Nauenberg [6], the density at which the quark phase should appear was much higher than the maximum central density reachable by stable neutron stars. So a a quark-hadron phase transition inside neutron stars could not occur and the possibility that quark core stars or quark stars might exist was well diminished. (However -see Kislinger & Morley [7] - both these papers made use of a high value of the coupling constant, obtained when fitting hadronic spectra and, as the coupling constant was held fixed, the equation of state contained large logarithms involving the density).

In parallel to this phenomenological approach, in 1975, Collins & Perry [8] demonstrated that asymptotic freedom which holds for large momentum transfers, also holds at high density, for example inside neutron stars. Then Keister & Kisslinger [9], using a perfect gas equation of state, and Chapline & Nauenberg [10] , starting from an expression of the energy density computed to first order in the -density dependant-coupling constant, concluded again that the quark-hadron phase transition would occur at too large a density, so that the possibility of having a pure quark phase inside neutron stars could be ruled out. However, these calculations were too crude for the first one and not completely self-consistent for the second one (see [11]). In 1978, it was shown independently by Kislinger & Morley [7] and Freedman & Mc Lerran [11] -and checked in a different approach by Baluni [12]- that when an expansion of the quark matter energy in the density dependant coupling constant was done, the quark-hadron phase transition was predicted to occur at densities lower than the maximum central density for a stable neutron star. It was also noticed by Freedman & Mc Lerran, that this result still hold if one added a vacuum constant to the equation of state - thus defining an improved M.I.T. bag, without divergent logarithms when a proper choice of the subtraction point was made. This was checked by Fechner & Joss [13], with and without vacuum constant, in the case of a first or a second order phase transition between hadrons and quarks, and for several nuclear-matter equations of state. They also showed that besides quark core stars, stable (pure) quark stars could even exist.

In addition to these studies, a number of other phenomenological approaches have been tried. In a paper from 1977, Bowers & al. [14], extrapolated the Walecka model ([15,16,17,18,19]) of nuclear matter to quark matter, namely they assumed that quarks exchanged massive scalar and vector particles and treated them in the Hartree, or mean field approximation. They showed that for reasonable choices of their parameters, quark core stars might exist but that quark stars would not be stable. This model was re-interpreted by Alvarez & Hakim [20,21] in the context of the

SLAC bag model: the scalar field was used to generate a first order phase transition from a state of massive particles to a state of particles of decreasing mass, the vector field was identified with gluons -massive from the beginning while in the SLAC model they acquire a mass through the Higgs mechanism. This allowed them to give an estimate of the parameters in their model, and they concluded that a quark-hadron phase transition within neutron stars was possible. Finally, starting simply from the QCD lagrangian with a Hartree-Fock approximation, Alvarez [22,23] computed another equation of state. He concluded -see also [24]- that the quark-hadron phase transition would occur at a higher density than allowed in stable neutron stars and that pure quark stars could exist but would be unstable. (In our mind, this equation of state always lies close to $p=\rho/3$ hence it is quite close to that of Keister & Kisslinger and it should not come as a surprise if the quark-hadron transition occurs at too high a density).

In addition to quark core stars and quark stars, the existence of another class of exotic dense objects has recently been proposed. In 1984, Witten [25] suggested that the true ground state of hadrons might be strange matter and not ^{56}Fe. Strange matter is bulk quark matter containing roughly equal numbers of u, d and s quarks plus a small admixture of electrons to insure charge neutrality. It is conjectured that this matter has an energy per baryon lower that that of ordinary nuclei, and is thus absolutely stable. If this is true, then what we usually think of as being neutron stars might actually be strange stars, i.e., stars made of strange matter. Contrarily to quark core stars and quark stars whose mass-radius or moment of inertia-mass relationships are quite similar to that of neutron stars, Haensel & al. [26] and Alcock & al. [27], showed that strange stars have mass-radius and moment of inertia-mass curves quite distinct from that of neutron stars -though in the range of masses typically observed, namely $1.4 M_{\odot}$, they are fairly similar. Moreover, strange stars may not have a crust and this could give rise to interesting new phenomena. (However one should keep in mind that regardless of whether or not strange stars have a crust, it is not clear how they could produce glitches: see Alcock & al. for details).

From what precedes, it is seen that *the equation of state of quark matter plays a crucial part in the possible existence of quark core stars, quark stars and strange stars, and in the determination of their macroscopic properties.* So the aim of this paper is twofold. First we will present a generalization to several flavours of a phenomenological one flavour quark matter equation of state derived previously [28,29] (section 2). This equation of state exhibits a first order phase transition from a state of massive quarks to a state of quarks of decreasing mass -thus mimicking the onset of asymptotic freedom. Also, immediately after the transition, quarks are in a collective bound state -thus suggesting that they are just leaving the inside of hadrons. So this transition may be interpreted as a hadron-quark transition. In addition this equation of state depends on just one parameter in the one or two flavour case (section 3), two parameters in the three flavour case (section 4), and these parameters can be constrained by using phenomenology. Second, we will show that the use of this equation of state in astrophysics does not present any difficulty and we will apply it to the study of quark core stars and quark stars (section 5). We will also discuss the possibility that strange matter might be more stable than ordinary matter in the context of this equation state and study strange stars -two families of which are found-in section 6. The new features of our approach, reservations and future problems are discussed in section 7.

2. EQUATION OF STATE

The equation of state that we will use to describe quark matter is a generalization to several flavours, of a one flavour quark matter equation of state derived elsewhere [28,29]. For pedagogical purposes, we first recall the results for the one flavour case, since those for the several flavour case can be derived in a similar fashion but are a little less straightforward to understand.

Because it is not known how confinement arises in QCD, we suppose that interactions between quarks can be reproduced by a phenomenological potential. For reasons already mentioned in [28], this interquark potential may be assumed to have the following Lorentz structure

$$V(r) = V_V(r)\,\gamma_\mu^{(1)}\,\gamma^{\mu(2)} - V_S(r)\,1_D^{(1)}\,1_D^{(2)} \tag{2.1}$$

where

$$V_S(r) = V_C(r)\,1_c^{(1)}\,1_c^{(2)}$$

with $V_C(r)$, a confining potential, linear for instance- which dominates at long interquark distances-

and where

$$V_V(r) = V_G(r)\,\lambda_a^{(1)}\,\lambda^{a(2)}$$

with $V_G(r)$, the one gluon-exchange potential -which dominates at short interquark distances.

(1_D designates the unity matrix in Dirac space and 1_c, the unity matrix in colour space. The superscript (1) refers to one of the quarks interacting via V(r) and (2) to the other.) The quark (Dirac) equation of motion reads

$$(i \not\partial 1_c - m 1_D 1_c)\, G(x,y) = \delta^4(x-y) 1_D 1_c + i \int d^3z G(x,z;y,z^+) V(\mid x-z \mid)_{|z_0=x_0} \quad (2.2)$$

where the notation z^+ means that the time z^{0+} is infinitesimally greater than z^0. In order to be able to solve (2.2), one can make the Hartree approximation

$$G(x,y;z,t) = G(x,z)G(y,t). \quad (2.3)$$

Then, after a Fourier transform, one gets an equation involving the two-point Green function only

$$[\not p 1_c - m 1_D 1_c - U_H] G(p) = 1_D 1_c \quad (2.4)$$

where the Hartree field U_H is given by the matrix

$$U_H = U_S^H 1_D 1_c \quad (2.5)$$

and its coefficient U_S^H is

$$U_S^H \approx -\int d^3z V_C(\mid x-z \mid) . Tr[-iG(z,z^+) 1_D 1_c] \quad (2.6)$$

In expressions (2.5-6), the one-gluon exchange potential has disappeared as is the case for any colour-dependent potential in the Hartree approximation (see [28] and references therein).

To see more clearly what the effect of our approximation (2.3) is, equation (2.4) may be rewritten as

$$[\gamma_0 p^0 - \vec{\gamma}\vec{p} - m_H]\, G(p) = 1 \tag{2.7}$$

where by definition, $m_H \equiv m + U_S^H$. From this equation, one sees that the interactions of a given quark with all the others give simply rise to a change in its mass (in this approximation).

The two-point Green function G can then be computed by standard methods [30,31,32] and used to calculate the Hartree field (2.5). One obtains the following self-consistent equation for the change of mass U_S^H, as a function of μ

$$U_S^H = -\tilde{\Gamma}(\mu).\bar{n}(U_S^H, \mu) \tag{2.8}$$

where

$$\begin{aligned} \tilde{\Gamma}(\mu) &\equiv \int d^3r[V_C(r)] \\ &= \frac{2\mu}{\bar{n}(U_S^H = -2\mu, \mu)} \end{aligned} \tag{2.9a}$$

$$\begin{aligned} \bar{n}(U_S^H, \mu) &\equiv 6\int_0^{p_F} \frac{d^3p}{(2\pi)^3} \frac{m_H}{\sqrt{\vec{p}^2 + m_H^2}} \\ &= \frac{3}{2\pi^2} m_H \left[p_F\sqrt{p_F^2 + m_H^2} - m_H^2 \log\left(\frac{p_F + \sqrt{p_F^2 + m_H^2}}{m_H}\right)\right] \end{aligned} \tag{2.9b}$$

and

$$p_F(U_S^H, \mu) = \begin{cases} \sqrt{\mu^2 - m_H^2} & \text{if } \mu \geq m_H \\ 0 & \text{otherwise} \end{cases} \tag{2.9c}$$

(in (2.9a), the equality is obtained by imposing the constraint $| U_S^H | \leq 2\mu$, otherwise pair creation occurs, see [28]. This renders U_S^H independent of the shape of the interquark potential.)

In the case of several flavour quark matter, one should replace (2.2) by

$$
\begin{aligned}
&(i \not\partial 1_c - m_q 1_D 1_c)\, G^q(x,y) = \\
&\qquad \delta^4(x-y) 1_D 1_c + i \int d^3z \sum_{q'=u,d,s,...} G^{q,q'}(x,z;y,z^+) V(| x - z |)_{|z_0 = x_0}
\end{aligned}
\tag{2.10}
$$

Then proceeding as for the one flavour case, it is easy to show that

$$
U_S^H = -\tilde{\Gamma}(\mu_L) \cdot \sum_{q=u,d,s,...} \bar{n}_q(U_S^H, \mu_q) \tag{2.11}
$$

where

$$
\begin{aligned}
\tilde{\Gamma}(\mu_L) &\equiv \int d^3r [V_C(r)] \\
&= \frac{2\mu_L}{\sum_{q=u,d,s,...} \bar{n}_q(U_S^H = -2\mu_L, \mu_q)}
\end{aligned}
\tag{2.12a}
$$

$$
\begin{aligned}
\bar{n}_q(U_S^H, \mu_q) &\equiv 6 \int_0^{p_F} \frac{d^3p}{(2\pi)^3} \frac{m_H^q}{\sqrt{\vec{p}^2 + m_H^{q\,2}}} \\
&= \frac{3}{2\pi^2} m_H^q \left[p_F^q \sqrt{p_F^{q\,2} + m_H^{q\,2}} - m_H^{q\,2} \log\left(\frac{p_F^q + \sqrt{p_F^{q\,2} + m_H^{q\,2}}}{m_H^q} \right) \right]
\end{aligned}
\tag{2.12b}
$$

and

$$
p_F^q(U_H^s, \mu_q) = \begin{cases} \sqrt{\mu_q^2 - m_H^{q\,2}} & \text{if } \mu_q \geq m_H^q \\ 0 & \text{otherwise} \end{cases} \tag{2.12c}
$$

In the above expressions, μ_L designates the smallest among the Fermi energies of the various quark flavours; pairs of this flavour are created preferably to screen the interquark potential.

It can be seen that the Hartree field U_H is the same whatever the flavour, and the effective masses $m_H^q \equiv m_q + U_S^H$, which play a similar part to running masses, will all have a similar decrease.

Once (2.11) is solved, similarly to the one flavour case, the quark contribution to the equation of state will be obtained from

$$\begin{aligned}
\epsilon_Q &= \sum_{q=u,d,s,\ldots} 3\int_0^{p_F^q} \frac{d^3p}{(2\pi)^3}\left[\sqrt{\vec{p}^2 + m_H^{q\,2}} + \frac{\vec{p}^2 + m_H^q m_q}{\sqrt{\vec{p}^2 + m_H^{q\,2}}}\right] \\
&= \frac{3}{2\pi^2}\Bigg\{p_F^{q\,3}\sqrt{\vec{p}_F^{q\,2} + m_H^{q\,2}}/2 \\
&\quad + \left(-\frac{m_H^{q\,2}}{2} + m_H^q m_q\right)\left[p_F^q\sqrt{\frac{\vec{p}_F^{q\,2}}{2} + m_H^{q\,2}/2} - \frac{m_H^{q\,2}}{2}\log\left(\frac{p_F^q + \sqrt{\vec{p}_F^{q\,2} + m_H^{q\,2}}}{m_H^q}\right)\right]\Bigg\}
\end{aligned} \tag{2.14a}$$

$$\begin{aligned}
p_Q &= \sum_{q=u,d,s,\ldots} 3\int_0^{p_F} \frac{d^3p}{(2\pi)^3}\left[\sqrt{\vec{p}^2 + m_H^{q\,2}} + \frac{-\vec{p}^2/3 - m_H^q m_q}{\sqrt{\vec{p}^2 + m_H^{q\,2}}}\right] \\
&= \frac{3}{2\pi^2}\Bigg\{p_F^{q\,3}\sqrt{\vec{p}^2 + m_H^{q\,2}}/6 \\
&\quad + \left(\frac{m_H^{q\,2}}{2} - m_H^q m_q\right)\left[p_F^q\frac{\sqrt{\vec{p}_F^{q\,2} + m_H^{q\,2}}}{2} - \frac{m_H^{q\,2}}{2}\log\left(\frac{p_F^q + \sqrt{\vec{p}_F^{q\,2} + m_H^{q\,2}}}{m_H^q}\right)\right]\Bigg\}
\end{aligned} \tag{2.14b}$$

$$\begin{aligned}
n_Q &= \sum_{q=u,d,s,\ldots} 6\int_0^{p_F^q} \frac{d^3p}{(2\pi)^3} \\
&= \sum_{q=u,d,s,\ldots} n_q
\end{aligned}$$

$$= \sum_{q=u,d,s,\ldots} \frac{p_F^{q\,3}}{\pi^2} \tag{2.14c}$$

If electrons must be included to insure charge neutrality, their contribution to the equation of state will be taken to be that of a perfect Fermi gas

$$\epsilon_e = \frac{\mu_e^4}{4\pi^2} \tag{2.15a}$$

$$p_e = \frac{\mu_e^4}{12\pi^2} \tag{2.15b}$$

$$n_e = \frac{\mu_e^3}{3\pi^2} \tag{2.15c}$$

Let us see how to apply this model in a few examples.

3. CONVERSION OF NEUTRON MATTER TO TWO FLAVOUR QUARK MATTER

In the case of neutron matter undergoing a phase transition to quark matter, weak interactions do not have time to settle, so there are only two flavours of quarks. Assuming charge neutrality (we have in mind the study of the interior of neutron stars), one gets

$$e(\frac{2}{3}n_u - \frac{1}{3}n_d) = 0 \tag{3.1}$$

Hence, denoting

$$\mu_u \equiv \mu \tag{3.2a}$$

and using expressions (2.14c) for densities and (2.12c) for Fermi momenta, we get the following relation between μ_d and μ

if $\mu \geq m_u = m_d$

$$\mu_d = [2^{2/3}\mu^2 + (1 - 2^{2/3})m_H^2]^{1/2} \tag{3.2b}$$

else

$$\mu_d \equiv \mu \tag{3.2b'}$$

One sees that there is just one independent chemical potential μ, and that μ_d is greater or equal to μ, so $u\bar{u}$ pairs will be created rather than $d\bar{d}$ pairs, i.e. in (2.11-12a), L=u.

Equation (2.11) for the change of mass U_S^H may be solved numerically as a function of the only independent chemical potential μ. In figure (1), the effective mass m_H (i.e. the initial mass plus its change), is shown as a function of the Gibbs energy per baryon $G \equiv \sum_{q=u,d,s,...} \mu_q n_q / n_Q / 3 = \mu_u + 2\mu_d$. Its general behaviour is quite similar to that of the one flavour case, so one expects that the quark plasma will undergo a first order phase transition. This transition (represented by the solid line) corresponds to a passage from a state of massive particles to a state of particles of decreasing mass -thus mimicking the onset of asymptotic freedom. In order to check that such a transition actually takes place and compute the Gibbs energy per particle at which it occurs, the pressure as a function of G is plotted in figure (2.a). The thermodynamically preferred state (i. e. G minimum for a given p) is p=0 from[1] G=0 to G=1.575, and then p starts to increase abruptly. So there is indeed a first order phase transition [33] and it happens when $G = 1.575$. In order to know at which density this corresponds, the baryon density has been depicted as a function of

[1]The fact that the pressure is zero in the interval [0,1.575] is due to the fact that we assume T=0. As a consequence, the integral (2.14b) for the pressure is null for a whole range range of chemical potentials ($\mu_q \leq m_H^q$), therefore of Gibbs energy. In reality, T≠0 and the pressure is approximately zero.

G in figure (2.b). One sees that when $G = 1.575$, the density is $n_t = 0.0085$. We do not show the energy density per baryon versus the baryon density because it is very similar to the one flavour case: immediately after the transition, the quarks are in a collective bound state, ($\epsilon_Q/n_Q \leq 3m_u$), which we interpret as a manifestation of the fact that they just start to "go out" of the baryons.

In what precedes, all the quantities have been computed in unit of the quark mass, m_u, and indeed this is the only parameter of the two flavour model. It is in fact possible to find a lower bound for this parameter: n_t should be greater or equal to the nuclear matter density, so we must have

$$n_t \times m_u^3 \geq n_{nuc.matt.} = 1.28\, 10^6\, MeV^3 \tag{3.3}$$

hence

$$m_u \geq 532.\, MeV \tag{3.4}$$

The fact that we obtain 532. MeV as a lower bound for the u-d constituent mass -usually thought to be of the order of 340. MeV- is an indication that our model is reasonable but crude.

It is also possible to obtain a higher bound for m_u, as follows. In figure (3), p(ϵ) is plotted. The behaviour of this curve is very different from the one which one would obtain by doing a perturbative calculation. On the other side, one may compute the approximate value of the baryon density, n_{pert}, at which quark matter should start to be describable with a perturbative equation of state. Ideally one would like to know n_{pert} from experiment. In practice, this quantity is not known but we can get an estimate of it by solving

$$\alpha_s(\mu^2) = \frac{6\pi}{(33 - 2\,N_f)\ln(\mu/\Lambda)} = 0.999... \qquad (3.5)$$

(This is the expression of the running coupling constant in the case of quarks of mass much smaller than μ.)

If we take Λ to be 200. MeV for instance and $N_f = 3$, the solution of (4.5) is $\mu = 402.$MeV. This corresponds to a baryon density (for non-interacting particles) $n_{pert} \sim \mu^3/3\pi^2 = 2.19\,10^6 MeV^3$. So, the following constraint should be satisfied

$$n_t \times m_u^3 \leq n_{pert} = 17.8\,10^6\; MeV^3 \qquad (3.6)$$

hence,

$$m_u \leq 636.MeV \qquad (3.7)$$

This upper bound is compatible with the lower bound (3.4); m_u must belong to the interval [532.MeV,636.MeV] and accordingly, the transition density n_t lies in the interval [$n_{nuc.matt.}$,1.71$n_{nuc.matt.}$. Note that while the nuclear matter density is (rather) well known, the calculation of n_{pert} is more unaccurate. Had we taken $\Lambda = 500.MeV$ instead of $200.MeV$, we would have obtained an upper limit for the u mass equal to 1592. MeV and an upper limit for n_t equal to 26.78 $n_{nuc.matt.}$.

As a final remark, let us recall how the formulae (2.14a-b) for p and ϵ were obtained [28]. The energy momentum tensor was computed from the effective Lagrangian describing the quarks, then results from the Hartree approximation were inserted in it and (2.14a-b) followed. It is therefore not obvious that p and ϵ would obey the thermodynamical equalities. We computed numerically the energy density

$$\epsilon_{verif} = G n_B - p. \tag{3.8}$$

On the plot of $p(\epsilon)$ (figure 3), it could not be graphically distinguished.

4. THREE FLAVOUR QUARK MATTER IN CHEMICAL EQUILIBRIUM

We have seen in the previous section that neutron matter undergoes a phase transition to two flavour quark matter. This two flavour quark matter will then, on a weak interaction timescale, transforms to three flavour quark matter and reach chemical equilibrium, via the following reactions

$$d \longleftrightarrow u + e + \bar{\nu}_e \tag{4.1a}$$

$$s \longleftrightarrow u + e + \bar{\nu}_e \tag{4.1b}$$

$$s + u \longleftrightarrow d + u \tag{4.1c}$$

This implies that

$$\mu_d = \mu_s \equiv \mu \tag{4.2a}$$

$$\mu_u + \mu_e = \mu \tag{4.2b}$$

and overall charge neutrality requires that

$$e(\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e) = 0 \tag{4.3}$$

Thus again, there is only one independent chemical potential, which we choose as being μ. In addition, one sees that $\mu_u = \mu - \mu_e$ is smaller or equal to μ, therefore

$u\bar{u}$ pairs will also be created preferably. Note that in the three flavour case, there are two parameters: $m \equiv m_u$ and $r \equiv m_s/m_u$. Since m_u and m_s are constituent masses, we expect that $r \approx 500./340. = 1.47$.

Equation (2.11) for U_S^H and (4.3) can be solved numerically simultaneously, with the input (4.2a-b). Once this is done, the thermodynamical quantities (pressure, energy density, baryon density, electronic density) can be computed by using (2.14a-c) and (2.15a-c) The behaviour of these various functions is rather similar to that of the two flavour quark matter, so we do not show them. (This does not mean that there will be another phase transition, once u and d quarks start to appear with a given density, they will be gradually depleted, the pressure needs not vary abruptly.)

5. QUARK CORE STARS AND QUARK STARS

Now that we have an equation of state, expressed as a function of m_u and r, and bounds or orders of magnitude for these parameters, we may try to apply it to stellar objects. First let us consider neutron stars with high central density, thus possibly containing a quark core.

Usually, to study such objects, one starts with two equations of state, one for the neutron phase and one for the quark phase. Then to see if a first order transition is possible, a Maxwell construction may be done to determine the transition pressure; under this pressure, the neutron equation of state must be used, above, the quark one. If a second order transition is assumed, the two equations of state are matched at some transition pressure. In our case, a first order transition is already embodied in the equation of state that we computed, so in principle we do not need a second equation of state. However, because we neglected temperature, the whole neutron (/massive quark) phase, is at zero pressure. We may circumvent this by replacing the p=0 branch of our equation of state by the Baym-Bethe-Pethick equation of state [34]. In figure 4, this has been matched at the transition density computed in section 3, to our quark equation of state. For comparison, we also represented the usual matching of the Bethe-Johnson equation of state [35] to the Baym-Bethe-Pethick equation of state. (We chose m_u =532.MeV, i.e. a quark-hadron phase transition occurring at a low density $\sim n_{nuc.matt.}$. In the case where m_u would have a higher value, the matching would simply occur at a higher density.)

In addition, in figure 4, at high density, our quark equation of state is matched smoothly with the equation of state of massless u-d quarks and massive s quarks, computed (for simplicity) to order zero in the strong coupling constant. We have taken the s mass to be 250. MeV because if $m_u^H = 0$ then $U_s^H = -m_u$ and $m_s^H = (r + U_s^H)m_u$=250.MeV (for our choice of m_u=532.MeV and r=1.47). Our equation of state provides a bridge between the equation of state for quarks confined inside hadrons at low densities, and the equation of state of non-interacting quarks at high densities.

In order to calculate the effects of the appearance of a quark phase in the deep interior of neutron stars, the matched equation of state has to be used in the solution of the so-called Oppenheimer-Volkoff system. These are the differential equations of static equilibrium in general relativity and read (see for example [36])

$$p'(r) = -G(\rho(r) + p(r))\frac{\mathcal{M}(r) + 4\pi r^3 p(r)}{r(r - 2G\mathcal{M}(r))} \tag{5.1a}$$

$$\mathcal{M}'(r) \equiv 4\pi r^2 \rho(r) \tag{5.1b}$$

$$\nu' = -\frac{2p'}{\epsilon + p} \tag{5.1c}$$

These equations have to be solved with the boundary conditions

$$\begin{cases} p(r=0) = p(\rho_c) \\ \mathcal{M}(r=0) = 0 \\ \nu(R) = 1 - 2G\mathcal{M}(R)/R \end{cases}$$

The integration of this system is stopped when a radius R is reached where the pressure becomes lower or equal to zero. A unique solution exists for each value of the central density ρ_c. In figure (5a), the mass-central density relation is presented. One sees that it has two extrema: as usual the first corresponds to neutron stars and the second to quark stars. As can be seen in figure (5b), the curve for the mass-radius relation curls counterclockwise, so these quark stars have unstable oscillation modes [37]. However, the so called neutron stars of the stable peak, consist of a very massive quark core, surrounded by a thin shell of more conventional neutron matter.

If the neutron star is slowly rotating (which is usually the case for pulsars, see e.g. [38]), they perturbe the metric only slightly and one has just one more Einstein equation to solve [39,40]

$$\frac{1}{r^4}\frac{d}{dr}(r^4 j\frac{d\bar{\omega}}{dr}) + \frac{4}{r}\frac{dj}{dr}\bar{\omega} = 0 \tag{5.2}$$

where

$$j(r) = e^{-\nu/2}\sqrt{1 - 2G\mathcal{M}(r)/r}$$

This second order equation must be integrated with the boundary following conditions

$$\begin{cases} \bar{\omega}(0) = constant \\ \bar{\omega}(R) + \frac{R}{3}\frac{d\bar{\omega}}{dr}_{|R} = \Omega \end{cases}$$

(where Ω is the angular velocity of the fluid in the star with respect to remote stars). Once this equation is solved, the moment of inertia is obtained from

$$I = \frac{R^3}{2}(1 - \frac{\bar{\omega}}{\Omega}) \tag{5.3}$$

In figure (5c), the moment of inertia-mass relation is plotted.

Because our equation of state is very stiff (see figures 3 and 4), the maximum masses and maximum moments of inertia reachable may be fairly high: $M_{max} \sim 3.35 M_\odot$ and $I_{max} \sim 9.7\,10^{45} g\,cm^2$ if $m_u = 532. MeV$. For comparison, a neutron star with a Walecka equation of state at high density, has $M_{max} \sim 2.6 M_\odot$ and $I_{max} \sim 3\,10^{45} g\,cm^2$ -see e.g. [38], [41]. The precise location of the maxima in figures 5a-c, depends sensitively on the matching with the neutron equation of state: the higher the transition density (e.g. the higher the value of m_u), the lower these quantities.

6. STRANGE MATTER STABILITY AND STRANGE STARS

As suggested by Witten [25], the true ground state of matter may be matter formed of u,d and s quarks because at T=0

$$G(u,d)_{p=0} \geq G_{nucleon,\ p=0} \tag{6.1a}$$

(ordinary matter is formed of nucleons and not of u-d quarks) but it is possible that, due to the Pauli principle,

$$G(u,d,s)_{p=0} \leq G_{nucleon,\ p=0} \tag{6.1b}$$

This hypothesis has been studied more closely in the framework of the M.I.T. bag equation of state by Fahri & Jaffe [42]. They concluded that there exist reasonable windows in the ranges of their parameters (the strange quark mass, the strong coupling constant and the vacuum constant), for which inequalities (6.1a-b) are satisfied. More precisely, as they explained, the following criteria for strange matter stability are fulfilled

$$G(u,d)_{p=0} \geq G_{Fe} + 4.MeV = 934.MeV \tag{6.2a}$$

and

$$G(u,d,s)_{p=0} \leq G_{Fe} = 930.MeV \tag{6.2b}$$

It is possible in the context of our equation of state, to see if these last inequalities can hold. We saw in section, that G(u,d)$_{p=0}$ = $1.575 \times m_u$, so that (6.1a) holds if and only if

$$m_u \geq 593.MeV \tag{6.3a}$$

As for inequality (6.2a), if r=1 for instance, G(u,d,s)$_{p=0}$ = $1.3825 m_u$, so it is satisfied if only if

$$m_u \leq 673.MeV. \tag{6.3b}$$

Other values for r lead to other upper bounds for m_u. In figure 6, we have depicted the surface in the (m_u,r) plane, for which strange matter would be more stable than ordinary matter. As can be seen, if r$\sim$ 1.47, (which is expected as mentioned in section 4), the chance that strange matter be the true ground state of hadrons is small. A similar conclusion was reached in a different approach by Bethe, Brown & Cooperstein [43]

If one assumes though for instance $r \sim 1.3$ and m_u=595.MeV, then strange matter would be more stable than ordinary matter at zero pressure. Strange stars might then exist, they could be formed either because after the neutron-quark transition, s quarks would appear in the core and this strange matter could eat the neutronic matter in the star up to its surface [25], or via some other more exotic processes as suggested in reference [27]. Their properties have been studied in the context of the M.I.T. bag equation by Haensel, Zudnik & Schaeffer [26] and by Alcock, Farhi & Olinto [27]. In figures (5a-c), we plotted for our equation of state, the results of the integration of the Oppenheimer-Volkoff system (5.1a-c) and of the equation for the moment of inertia (5.2), in a case where strange matter would be more stable than nuclear matter. Our results are in qualitative agreement with those of the authors mentioned above. $M(\rho_c)$ has a vertical asymptote, at ρ_c=5 10^{14} g cm^3 (corresponding to stars with zero pressure, i.e. for which gravity is not important) and it reaches a maximum beyond which no stable configuration can be found: the second strange star peak in figure 5a is unstable, like the quark star peak of the previous section. The mass-radius relation is different from that of neutron stars for low mass (because then gravity does not matter and $M \propto R^3$, while for low mass neutron stars, the gravitational pull is small and the star can expand far), but is fairly similar in the range of masses ($1.4M_\odot$) typically observed. The same remark holds for the moment of inertia.

7. CONCLUSION

In this paper, we generalized to several flavours, a one flavour quark matter equation of state derived recently [28,29]. It was shown that the main features of the one flavour case still hold. Precisely, as density increases, the quarks undergo a first order phase transition from a state of massive particles to a state of particles of decreasing mass -thus mimicking the onset of asymptotic freedom. Also, immediately after the transition, the quarks are in a collective bound state, thus suggesting that they just started to go out of the hadrons. As mentioned in section 5, it is not a common feature for quark equations of state to exhibit a phase transition , even though this is what one expects. One usually has to compare two different equations of state to determine if there is indeed a phase transition and at which density (see for exemple[5,6,11-14,20-24,26]). Here the answer to these questions comes out more simply.

Because of screening through pair creation, our results depend on just one parameter, the u quark constituent mass, in the two flavour case, and two parameters on the three flavour case, the u quark constituent mass and the ratio of the s mass and u mass. While parameters in usual quark matter equations of state (M.I.T. S.L.A.C., ...), are usually fitted to reproduce static properties of hadrons (mass spectrum, magnetic momenta, mean radii...), we fitted ours by using properties of the hadrons-quarks phase transition : $n_{nuc.matt.} \leq n_t \leq n_{pert}$.

The three flavour equation of state was matched with the Feynman-Metropolis-Teller plus Baym-Pethick-Sutherland plus Baym-Bethe-Pethick neutron matter equation of state at low density, and with a perturbative quark matter equation of state at high density. This matched equation of state was applied to the study of the hadrons-quarks phase transition in dense stars. It was shown that neutron stars have a big quark core and that their inertia parameters could be larger than in usual models of high density matter. How large they really are, depends on the exact transition density. However, in any case, quark stars are not stable.

This equation of state was also applied to the investigation of strange matter stability. It was shown that within the range of values expected for our two parameters, strange matter should not be more stable than ordinary matter at zero pressure (contrarily to what may be predicted within the framework of the M.I.T. bag model [42]). One may however make a choice of parameters such as to ensure strange matter stability. In that case, two families of strange stars (i.e. stars entirely made of stable strange matter), can be found. The first one would be qualitatively similar to that of [26] and [27], which were studied in the context of the M.I.T. bag equation of state. The second one corresponds to unstable objects.

In our mind, the results obtained above should be considered as indicative only. Rather than trying to get definitive numerical results, the emphasis in this paper was on *how* the one flavour equation of state could be generalized to several flavours, and *how* it could be applied to dense objects. This equation of state can be (and should be) improved (for details see [28]).

As a final remark, we would like to recall that if asymptotic freedom settles early, i.e. if $n_{pert} \sim$ some $n_{nuc.matt.}$, the M.I.T. bag equation of state may not be applicable to the study of dense objects. To be more precise, the solution of $\alpha_s(\mu^2)$=0.999..., is

$\mu = 402.\text{MeV}$ if Λ=200.MeV, which corresponds to $n_B \sim 4.81\ 10^{14}$ g cm^{-3}

$\mu = 1005.\text{MeV}$ if Λ=500.MeV, which corresponds to $n_B \sim 75.10\ 10^{14}$ g cm^{-3}.

So if Λ = 200.MeV, the perturbative regime is reached in the center of quark core stars and strange stars -not to mention pure quark stars. This would rule out the use, for these objects, of the M.I.T. bag equation of state -because it represents confined quarks- but would favor an equation of state relating the confined regime to the perturbative regime stiffly -as in our case. If $\Lambda = 500.MeV$, an equation of state like the M.I.T. one, could be used safely. At present, there is no consensus on the actual value of Λ. In addition, current estimates come fron analyses of experiments involving space-like momenta, it is not clear that these values of Λ can be used when dealing with density as a variable.

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FIGURE CAPTIONS

Fig. 1 Plot of the effective mass as a function of the Gibbs energy per particle in the case of two flavours of quarks with similar mass (dotted line). There is a phase transition from the massive quark state to the phase of quarks of decreasing mass takes place (solid line).

Fig. 2a Plot of the pressure as a function of the Gibbs energy per particle. The transition is seen to be first order and occurs when G/m_u=1.575. It is associated to the change of mass (see solid curve in figure 1).

Fig. 2b Plot of the baryonic density as a function of the Gibbs energy per particle (dotted line). When $G/m_u = 1.575$, the density jumps to $n/m_u^3 = 0.0085$ (solid line).

Fig. 3 Plot of the pressure as a function of the energy density for two quark flavours (solid line). For comparison, the M.I.T. equation of state has been plotted as well (dotted line).

Fig. 4 Plot of the pressure versus energy density for u,d,s quarks and electrons in chemical equilibrium, with m_u=532.MeV and r=1.47 (solid curve labeled Q_3). It is matched (dotted line) to a perturbative equation of state at high density and to the Feynman-Metropolis-Teller plus Baym-Pethick-Sutherland plus Baym-Bethe-Pethick equation of state at low density. For comparison, the usual matching with the Bethe-Johnson equation of state is also shown (dashed-dotted curve).

Fig. 5a Plot of the mass-central density relation for the matched equation of state for two values of m_u and m=1.47 (solid line). Plot of the mass-central density relation for a stable strange matter equation of state for m_u = 595. MeV and r=1.30 (dotted line).

Fig. 5b Plot of the mass-radius relation for the matched equation of state and for a stable strange matter equation of state (see figure 5.a for curve designation).

Fig. 5c Plot of the moment of inertia-mass relation for the matched equation of state and for a stable strange matter equation of state (see figure 5.a for curve designation).

Fig. 6 Contour in the (m_u-r) plane inside which strange matter would be more stable than ordinary matter. The numbers on the right indicate respectively the value of the hadronic electric charge per baryon and the strangeness per baryon.

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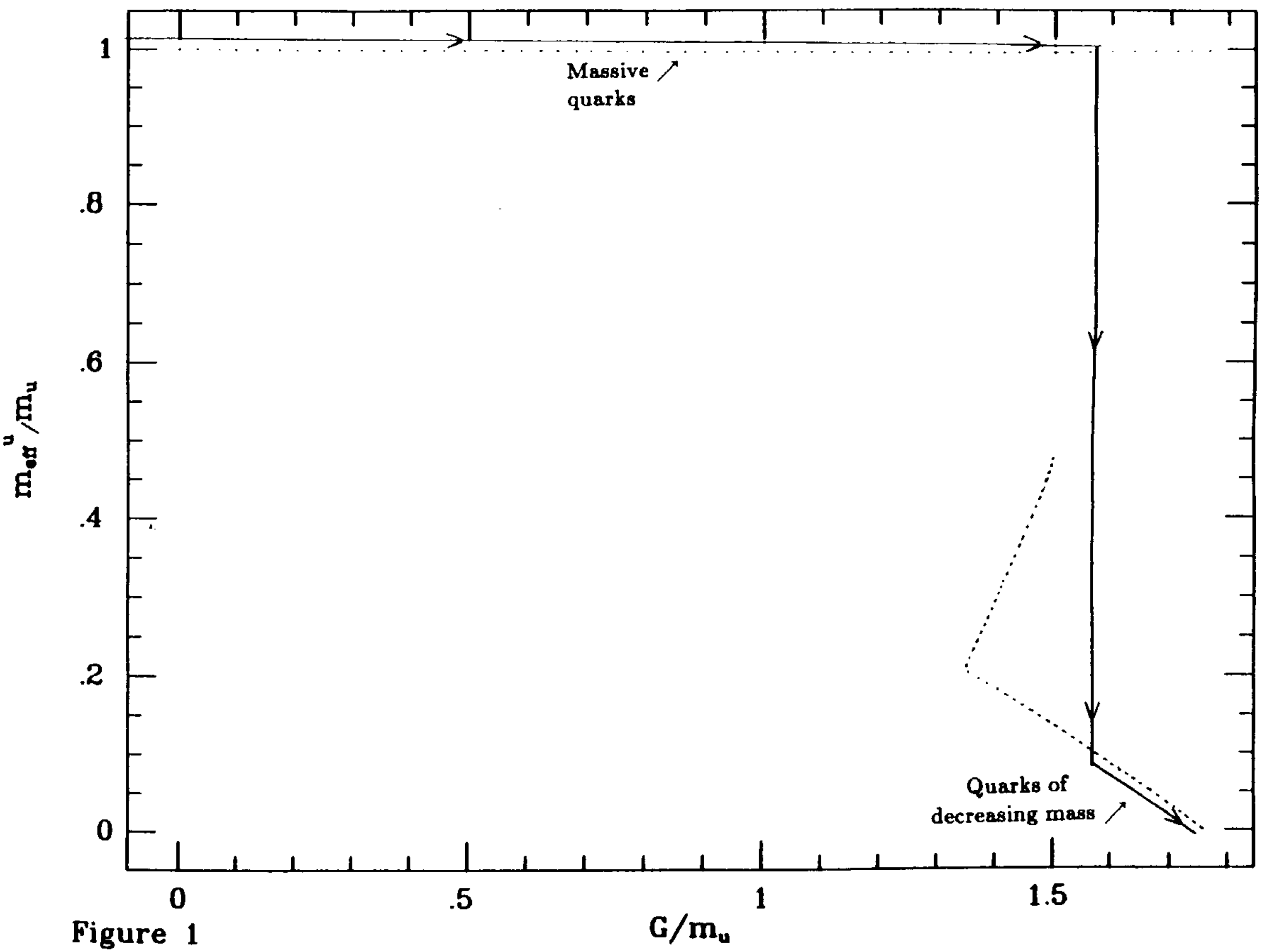
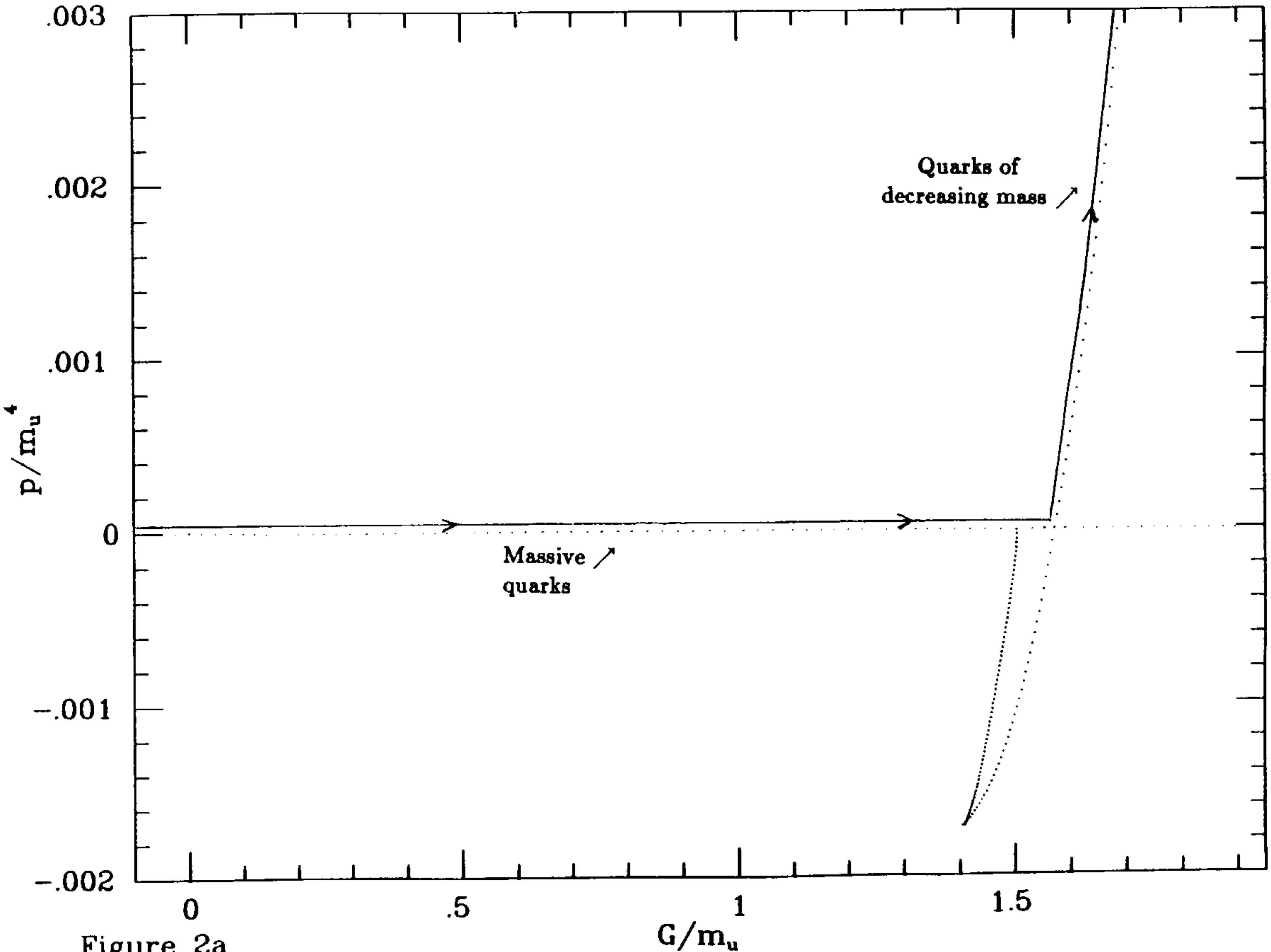
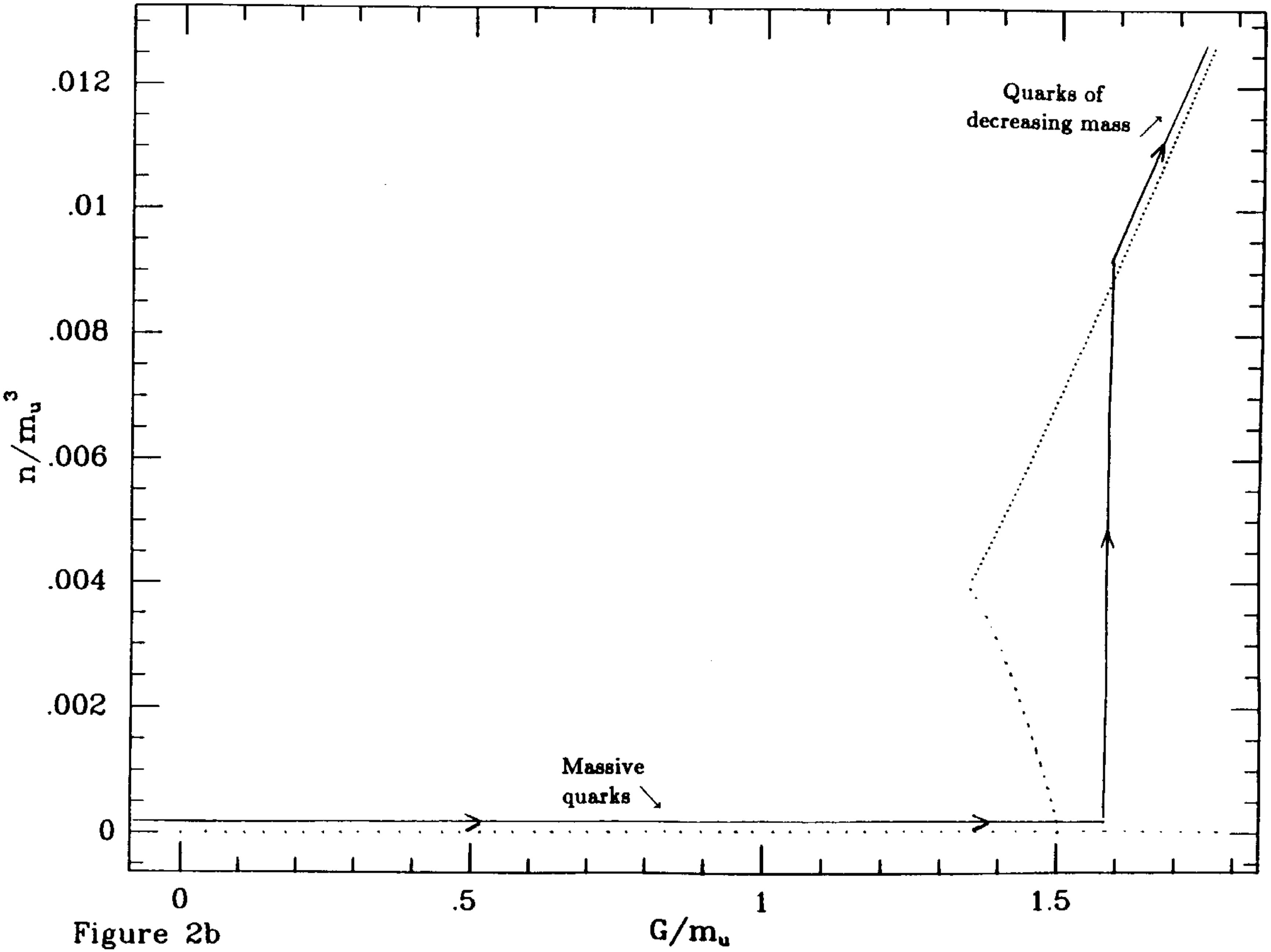

1
.8
.6
.4
.2
0
0
.5
1
1.5
Massive quarks
Quarks of decreasing mass
m_{eff}^u/m_u
G/m_u

Figure 1



.003
.002
.001
0
-.001
-.002
p/m_u^4
Quarks of
decreasing mass
Massive
quarks
0
.5
1
1.5
G/m_u

Figure 2a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0
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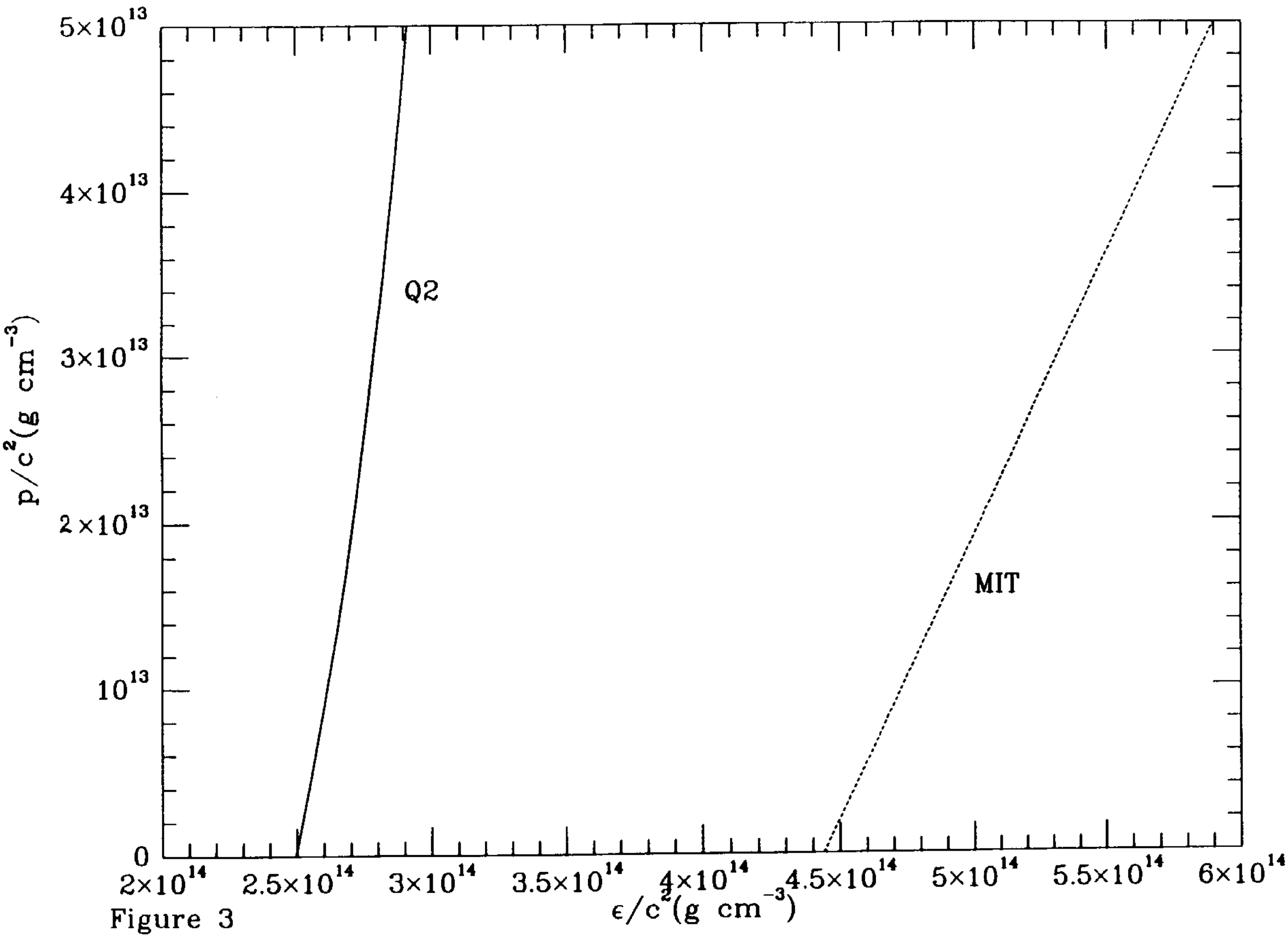
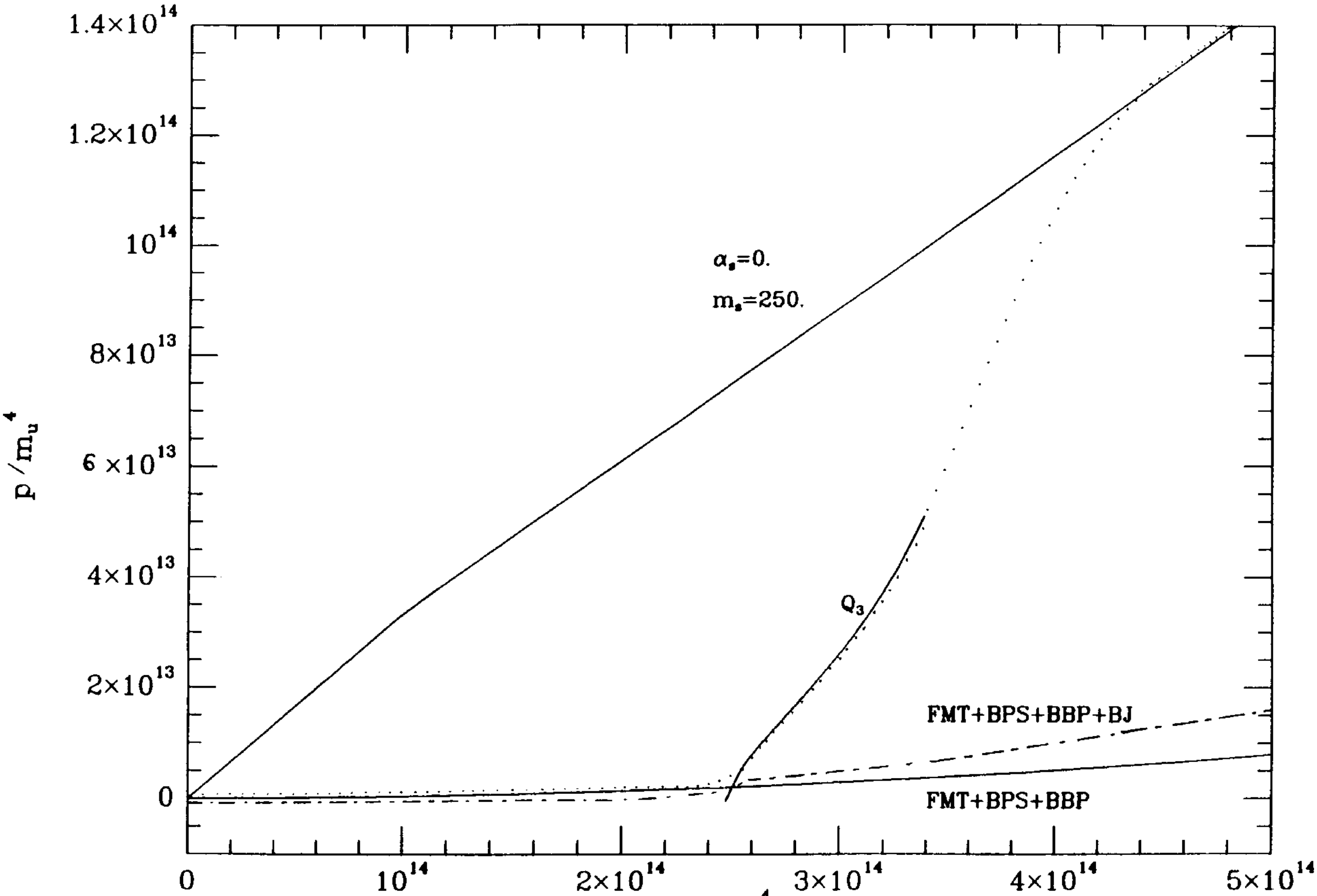
1
1.5
G/m_u
Quarks of
decreasing mass
Massive
quarks

Figure 2b


5×10^13
4×10^13
3×10^13
2×10^13
10^13
0
p/c^2(g cm^-3)
Q2
MIT
2×10^14
2.5×10^14
3×10^14
3.5×10^14
4×10^14
4.5×10^14
5×10^14
5.5×10^14
6×10^14
ε/c^2(g cm^-3)
Figure 3



1.4×10^14
1.2×10^14
10^14
8×10^13
6×10^13
4×10^13
2×10^13
0
p/m_u^4
α_s=0.
m_s=250.
Q_3
FMT+BPS+BBP+BJ
FMT+BPS+BBP
0
10^14
2×10^14
3×10^14
4×10^14
5×10^14
ε/m_u^4

Figure 4

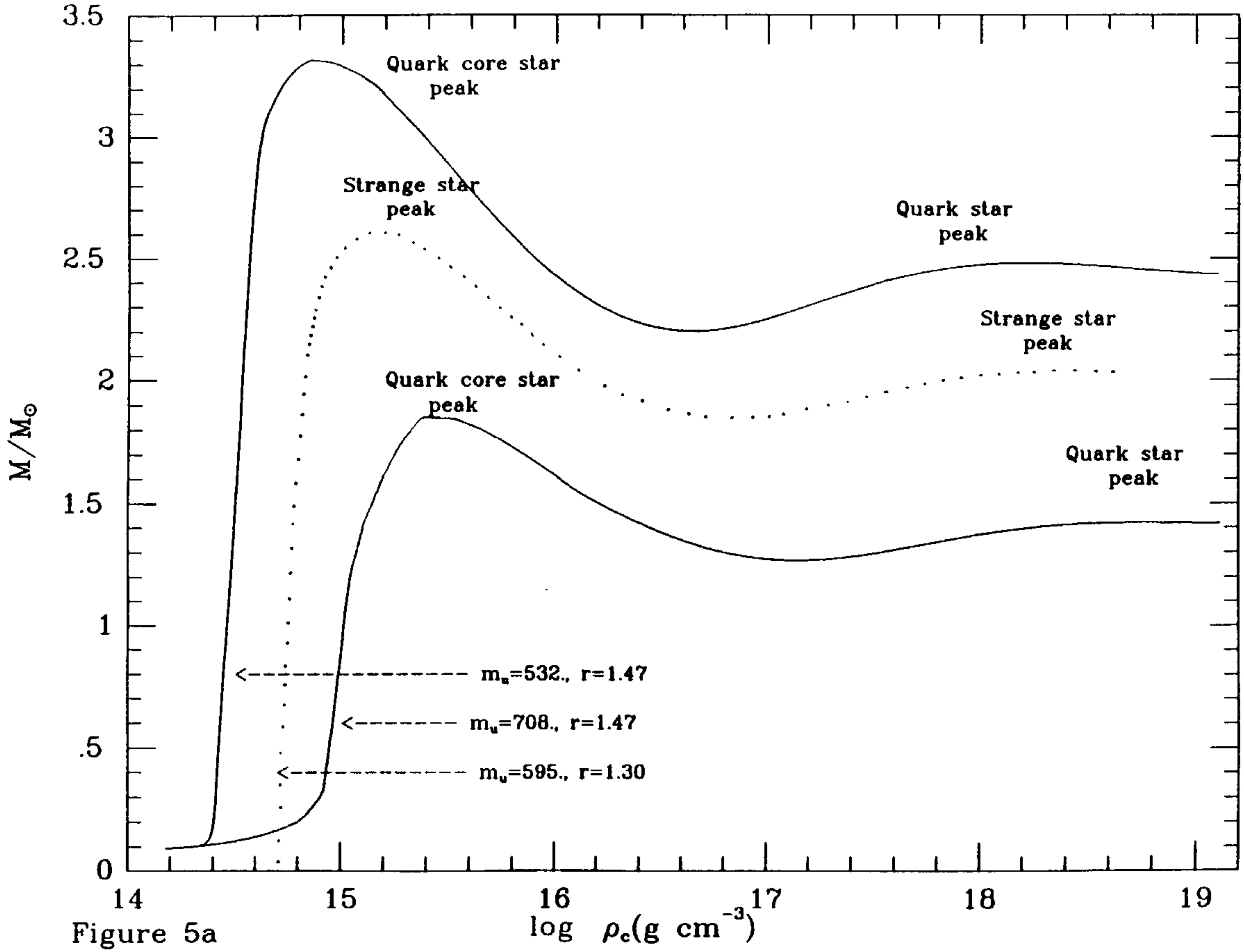
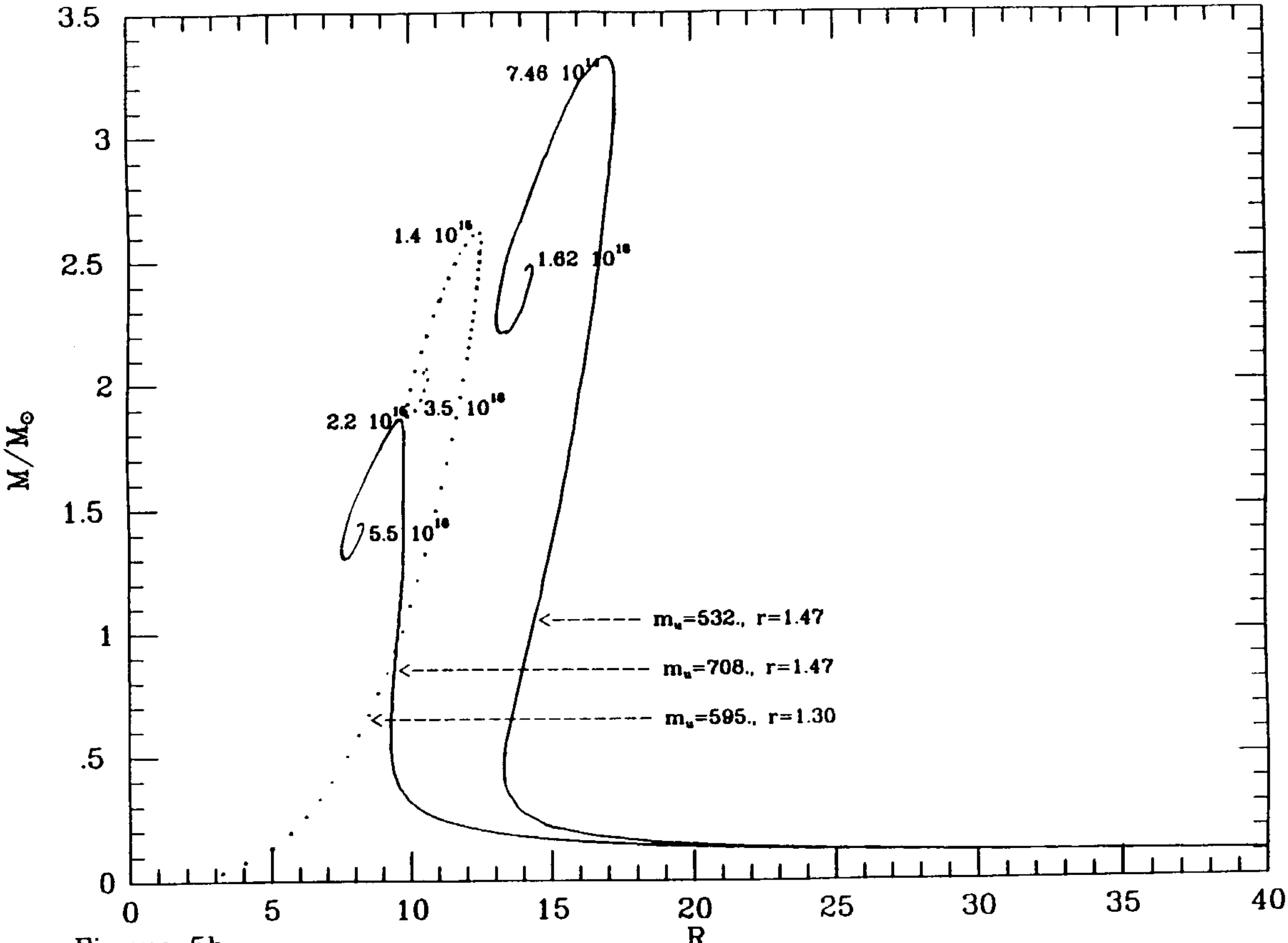

Quark core star
peak
Strange star
peak
Quark core star
peak
Quark star
peak
Strange star
peak
Quark star
peak
m_u=532., r=1.47
m_u=708., r=1.47
m_u=595., r=1.30
M/M⊙
log ρc(g cm^-3)
3.5
3
2.5
2
1.5
1
.5
0
14
15
16
17
18
19

Figure 5a



3.5
3
2.5
2
1.5
1
.5
0
M/M$_\odot$
0
5
10
15
20
25
30
35
40
R
7.46 10^{14}
1.62 10^{16}
1.4 10^{15}
2.2 10^{16}
3.5 10^{16}
5.5 10^{16}
m$_u$=532., r=1.47
m$_u$=708., r=1.47
m$_u$=595., r=1.30

Figure 5b

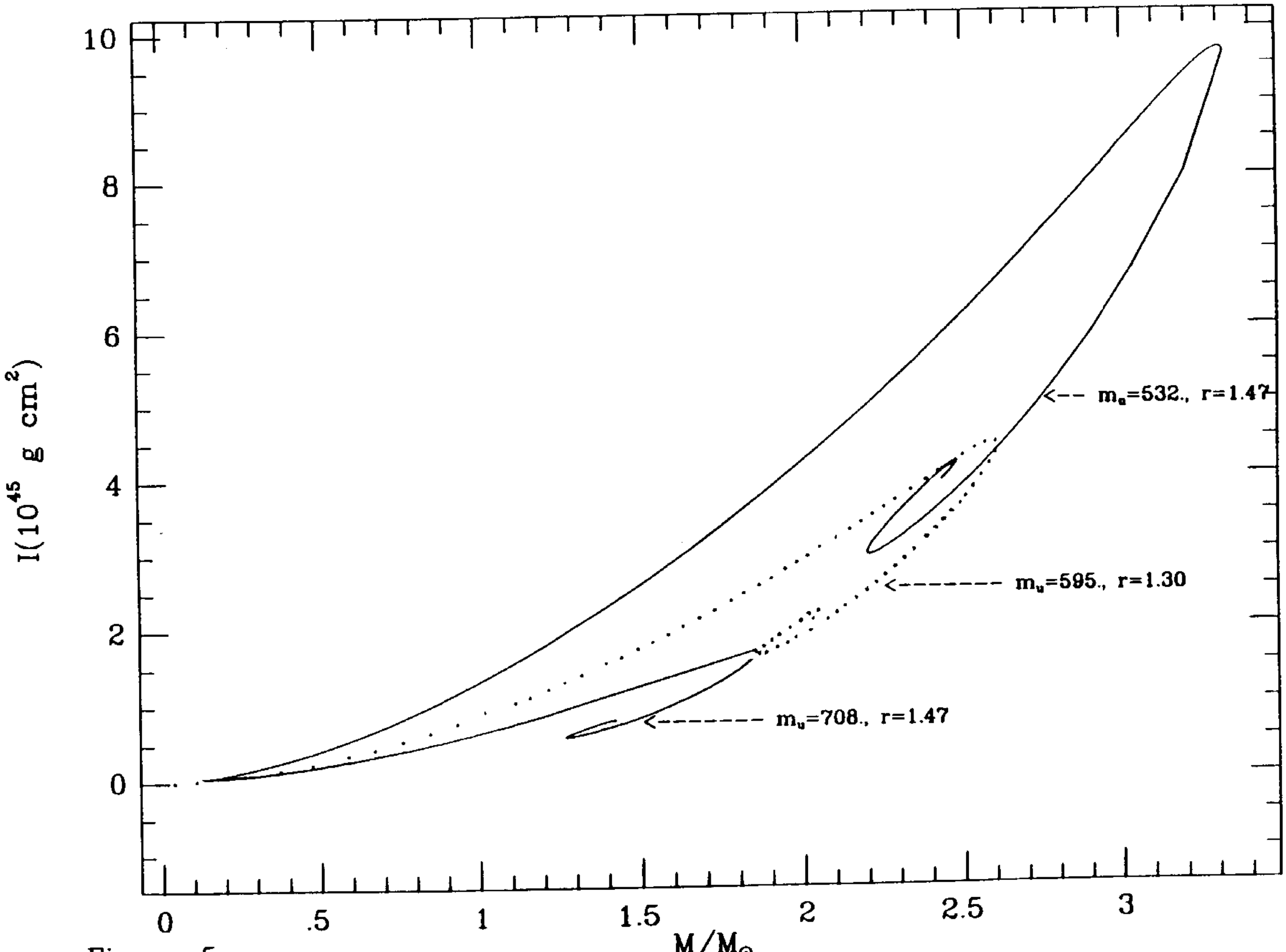
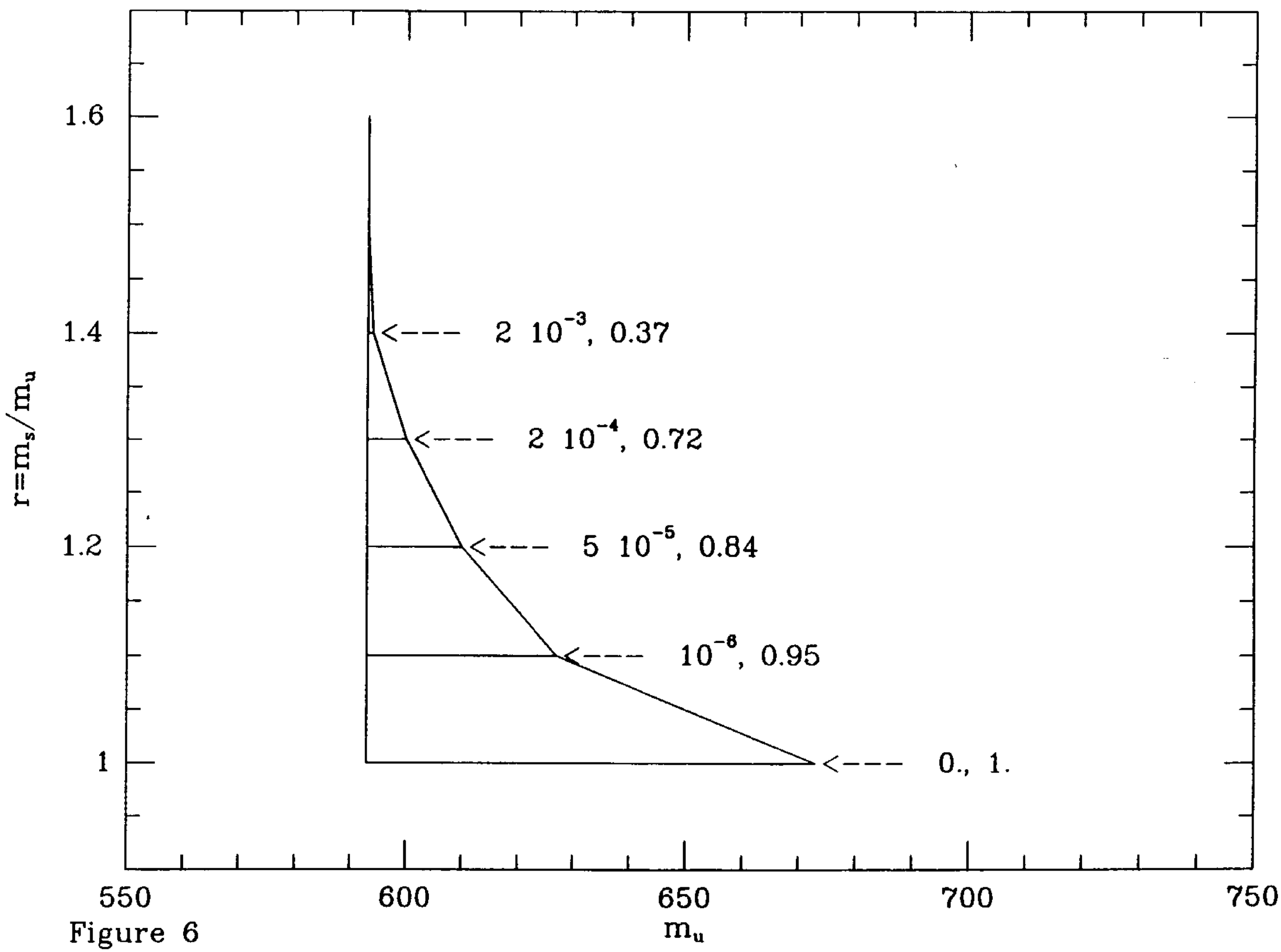

$I(10^{45}$ g cm$^2)$
10
8
6
4
2
0
0
.5
1
1.5
2
2.5
3
$M/M_\odot$
m_u=532., r=1.47
m_u=595., r=1.30
m_u=708., r=1.47

Figure 5c



1.6
1.4
1.2
1
r=m_s/m_u
2 10^-3, 0.37
2 10^-4, 0.72
5 10^-5, 0.84
10^-6, 0.95
0., 1.
550
600
650
700
750
m_u

Figure 6