Renormalization Group Analysis of Quantum Chromodynamics at Finite Temperature

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*) Submitted in partial fulfillment of the requirements for the degree of Doctor of Science.
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Abstract

We re-investigate the behaviour of the effective coupling constant using the gauge invariant Wilson loop in Minkowsky space-time. The result tells us that the interaction of static q-¯q pair becomes weak at high temperatures and at large distances.

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Section 1. Introduction

As is well known, Quantum Chromodynamics (QCD) is a theory of quarks and gluons and is widely accepted as a fundamental theory of strong interactions in hadron world. QCD has several desirable properties as a fundamental theory. Main two of those are the asymptotic freedom at short distances[^1] and the confinement of quarks.[^2] In the last decade, along with the main interest of de-confinement transitions,[^3] the properties of QCD at finite temperature was vigorously studied.[^4] The understanding of the dynamics of QCD at finite temperature is also important as it may play the crucial roles in the development of early universe, heavy ion collisions and so on.

QCD at finite temperature (T=0) has already been investigated in a great deal, in particular, in lattice gauge theories.[^5] However it may be desirable to investigate QCD at T=0 from the point of view of perturbation theory. As predicted by lattice QCD suppose the de-confinement transition occurs at some critical temperature. Above the critical temperature, quarks and gluons are liberated and behave as free-like particles. Then, the system of quarks and gluons may be roughly considered as an ideal free gas. If this picture is close to the true situation any physical observable could be calculated by using a perturbative expansion in an effective coupling which decreases as the temperature is raised. However an important problem that whether the QCD interactions are in fact weak at high temperature remains open to be studied. Though there are affirmative conclusions[^6] they are unsatisfactory as the thermal effect is not treated
rigorously. Let us illustrate the points to be improved by considering a standard discussion using the renormalization group method. Consider a massless theory. At a finite temperature $T$, the renormalization group equation for the $n$-point $(1-P-I)$ Green's function is written as

$$
\left( p^2 \frac{\partial}{\partial \mu} + \mathcal{R}(g) \frac{\partial}{\partial g} \right) \Gamma^{(n)}(P_i, T, g(\mu); \mu) = n \mathcal{N}(g) \Gamma^{(n)}(P_i, T, g(\mu); \mu)$$ (1)

where $\mu$ and $\mathcal{N}(g)$ denote, respectively, the renormalization point and anomalous dimension and $g-g(\mu)$ is the coupling constant renormalized at zero temperature. From (1) we can relate $\Gamma^{(n)}$ at a high temperature $\sigma T$ ($\sigma \gg 1$), with the one at temperature $T$ as

$$
\Gamma^{(n)}(P_i, \sigma T, g(\mu); \mu) = \sigma^{4n} e^{-\int_{\mu}^{\mu_{\sigma T}} \frac{dz}{z} \mathcal{N}(z)} \Gamma^{(n)}(P_i, T, g_{\mu_{\sigma T}}; \mu)$$ (2)

Since in the case of QCD, $g(\sigma T)$ decreases logarithmically as $\sigma \to \infty$, it seems possible to calculate the right hand side with good accuracy by perturbation theory. However this scheme fails as we can see below. In 1-th order of perturbation theory temperature correction becomes large at low momentum as

$$
g^{2d}(\sigma T) \left( \frac{T}{P/\sigma} \right)^m = g^{2d}(\sigma T) \cdot \sigma^m \left( \frac{T}{P} \right)^m$$ (3)

where $P/\sigma$ is the typical momentum scale in $\Gamma^{(n)}$ in the right hand side of (2) and $m$ is some integer determined by $l$ and the mode. Therefore as $\sigma \to \infty$ higher order corrections dominate even though $g(\sigma T)$ decreases logarithmically. Then one should invent a
technique by which we can systematically improve the perturbation series at \( T \to 0 \). For this purpose a new method, namely the renormalization group at finite temperature was recently formulated (see Appendix C).\(^7\),\(^8\)

By the use of this new method we investigated the behaviour of the effective coupling constant with respect to the change of momentum and temperature in Ref. [9]. The result of the analysis is not satisfactory in several respects, however. One of the troubles is the gauge dependence of the given result. To resolve the issue we re-investigate the temperature dependence of the QCD interactions in a gauge invariant manner. This is the theme of this thesis. For this purpose we start with the Wilson loop and investigate the behaviour of the effective coupling constant which measures the strength of the interaction of a static quark and antiquark \((q\bar{q})\) pair in the thermal gluon medium. To calculate the effective coupling we use the finite temperature quantum field theory in the real time formalism.\(^10\) Furthermore we improve the result by the use of the renormalization group method at finite temperature.

At this stage we should mention the work of Gendenshtein (Ref. [7]). He already studied the same subject and obtained essentially the same result with ours. Furthermore Gale and Kapusta also dealt with the similar subject to ours in their recent work (Ref. [11]). What is new in our work is that our approach is manifestly gauge invariant as we extract the effective coupling constant from the Wilson loop. This justifies presenting this work.
The organization of the thesis is as follows: In the next section the formalism of the Wilson loop in Minkowsky space-time at finite temperature is presented. The Wilson loop enables us to investigate the interaction of a q-\bar{q} pair in a gaug invariant manner. In section 3 we calculate the temperature correction to the Wilson loop and extract the effective coupling constant according to the idea of Susskind\cite{12}. Then we improve the result by the use of the renormalization group method. In the last section we discuss the physical interpretation of the result given in section 3 and summarize the thesis.

In this thesis we use the real time formalism of quantum field theory at finite temperature. Its brief review in the path integral approach is given in Appendix A\cite{13}. The Feynman rules used in this thesis are collected in Appendices A and B. Appendix C is devoted to a review of the renormalization group method at finite temperature. As for the presentation of the idea we apply the approach of Susskind to a finite temperature case.
Section 2. Wilson loop in Minkowsky space-time at finite temperature

It is well known that the perturbative calculation in gauge theories is plagued with the gauge dependence. Hence there arises a problem of physical interpretation of the calculation. Such a problem would be absent if the result of the calculation is derived from a gauge invariant quantity. Therefore we use the Wilson loop, which is manifestly gauge invariant by construction, to investigate the behaviour of the effective coupling constant. To be more precise we consider a quark-antiquark (q-\bar{q}) pair separated by the distance R and calculate its static potential at finite temperature via the Wilson loop. Then we extract the effective coupling constant from the potential. The original idea of extracting the effective coupling has been proposed by Susskind.[12] It goes as follows:

Let us consider a q-\bar{q} pair, in the thermal medium of gluons, which is initially located at the same point in the remote past. Then q and \bar{q} are adiabatically separated and brought to the distance of R. This situation lasts for a long time (t_0). In the end the q-\bar{q} pair is adiabatically brought together and annihilates. As a whole the process represents the vacuum to vacuum transition and thus can be expressed by the elongated Wilson loop (Fig.1). By dropping the upper and lower ends of Fig.1 whose contributions are negligible when t_0 → ∞, we can extract the q-\bar{q} potential. Therefore it follows that the potential we look for is obtained by evaluating the thermal averaged transition amplitude for the process that q and \bar{q},

-7-
located respectively at $\hat{r}=(0,0,0)$ and $\hat{r}=(0,0,R)$ at time $-t_0/2$, be found at the same locations at time $t_0/2$.

In order to express the amplitude explicitly let us first introduce the creation, $\psi^\dagger_\alpha(\hat{r})$ ($\alpha=1-N$), and annihilation, $\psi_\alpha(\hat{r})$, operators for quark ($\alpha$ denotes the color of a quark). The corresponding operators for antiquark are denoted as $\psi^\dagger_\alpha$ and $\psi_\alpha$. They satisfy,

$$\{ \psi_\alpha(\hat{r}), \psi^\dagger_\beta(\hat{r}') \} = \delta_{\alpha\beta} \delta(\hat{r}-\hat{r}') .$$

All the other anti-commutation relations are vanishing.

To treat the gluon medium at finite temperature it is necessary to know the Hamiltonian. Therefore we turn to the problem of canonical quantization. Here it is carried out in the axial gauge. The Lagrangian to start with is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} F_{\mu\nu}^a (\partial^\alpha A_a^{\alpha\nu} - \partial^\alpha A_a^{\mu\nu} + g f^{abc} A_a^{\mu\nu} A_b^{\alpha\rho})$$

$$+ \psi^\dagger_\alpha i\left( \frac{\partial}{\partial \xi^a} - ig A_a^{\alpha} \right) \psi_\alpha + \psi^\dagger_\alpha i\left( \frac{\partial}{\partial \xi^a} - ig A_a^{\alpha} \right) \psi_\alpha ,$$

where $A_a^{\alpha}$ denotes the matrix in the fundamental representation of the gauge group $SU(N)$. In the axial gauge ($A_3^a=0$) the dynamical variables are $A_a^a$, $F^a_{01}$ ($1=1,2$), $\psi_\alpha$ ($\psi^\dagger$) and $\psi_\alpha$ ($\psi^\dagger$). $A_a^a$, $F^a_{01}$ and $F^a_{1j}$ ($1,j=1,2,3$) are determined by them. The Hamiltonian is found to be,

$$\mathcal{H} = \int d^3x \left( \frac{1}{2} \left( \vec{E}_a^2 + \vec{B}_a^2 \right) \right) ,$$

$$\vec{E}_a^i = F_{a}^{\mu i} , \quad \epsilon_{ijk} B^{jk} = \partial_i A_j - \partial_j A_i + g f^{abc} A_i^a A_j^b A_k^c .$$
$A_3^a$ and $E_3^a$ are given by

$$A_3^a = 0,$$

$$E_3^a(x, y, z) = \int d\vec{z} \left\{ \sum_{i=1}^{2} \partial_i E_i^a - g f^{abc} \sum_{i=1}^{2} E_i^a A_i^c - g (\psi^T \gamma^a \psi - \bar{\psi}^c \gamma^a T^a \psi) \right\}$$ (7)

Then we introduce a gauge invariant operator which creates the $q\bar{q}$ pair at a time $-t_0/2$. It is given by

$$\psi^a(\vec{r}, -\frac{t_0}{2}) \left( P_c^a \exp i \int d\vec{r}' \vec{A}(\vec{r}', -\frac{t_0}{2}) \right) \psi_c^a(\vec{r}, -\frac{t_0}{2})$$ (8)

where $\psi^a$, $\psi_c^a$ and $A^a$ are the Heisenberg fields. In the above $C'$ denotes the path connecting $q$ and $\bar{q}$ and $P_c^a$ denotes the ordering along the path $C'$. Since the choice of path is irrelevant in the large $t_0$ limit we adopt a straight line. Then in $A_3^a = 0$ gauge (8) becomes

$$\psi^a(\vec{r}, -\frac{t_0}{2}) \psi_c^a(\vec{r}, -\frac{t_0}{2})$$ (9)

The amplitude is expressed as

$$\text{amp} = \text{tr} \left( e^{-\beta H} \psi_c^a(\vec{r}, \frac{t_0}{2}) \psi^a(\vec{r}, \frac{t_0}{2}) \psi_c^a(\vec{r}, -\frac{t_0}{2}) \psi^a(\vec{r}, -\frac{t_0}{2}) \right)$$

$$= \int DA \, DA' \, \langle A | e^{-\beta H} | A' \rangle \langle A' | \psi_c^a(\vec{r}) \psi^a(\vec{r}) \psi_c^a(\vec{r}) \psi^a(\vec{r}) | A \rangle, \quad \beta = \frac{1}{\hbar}$$ (10)

In (10),

$$|A\rangle = |A_1, A_2; \phi \rangle, \quad DA = \prod_{a, \vec{z}, c} dA_c^a(\vec{z}) (c = 1, 2),$$ (11)
and \(|A_1, A_2; 0\rangle\) denotes the eigen state of the gauge fields \(A_1^a\) and \(A_2^a\) without static quarks,

\[
\Psi_\xi(\vec{r}, t) |A\rangle = \Psi_\xi(\vec{r}, t) |A\rangle = 0 .
\]  
(12)

Using the completeness condition

\[
\int DA D(\xi', \xi) D(\eta', \eta) e^{-\frac{i}{\hbar} \int d^4x (\tilde{\xi} \cdot A + \tilde{\eta} \cdot A')} |A, \xi, \eta\rangle \langle A, \xi', \eta'| = 1 ,
\]

\[
\xi = \xi' \eta , \quad \Psi_\xi(\vec{r}) |A, \xi, \eta\rangle = \tilde{\xi} \eta \Psi_\xi(\vec{r}) |A, \xi, \eta\rangle ,
\]  
(13)

\[
\eta' = \eta \xi' , \quad \Psi_{\eta'}(\vec{r}) |A, \xi, \eta\rangle = \tilde{\eta} \xi' \Psi_{\eta'}(\vec{r}) |A, \xi, \eta\rangle ,
\]

and carrying out the integration over \((\xi, \xi')\) and \((\eta, \eta')\) the second factor in (10) becomes,

\[
\langle A' | \Psi_{\eta'}(\vec{r}) \Psi_{\xi}(\vec{r}) e^{-i \gamma L} \Psi_{\xi'}(\vec{r}) \Psi_{\eta}^{*}(\vec{r}) |A\rangle
\]

\[
= (\delta'(\eta'))^2 \int_A D A_r \delta(\Lambda_r^a) e^{i \int_{\gamma_r} dt d^4 r \mathcal{L}} .
\]

\[
\left( \Theta_t \exp \left[ i \int_{\gamma_r} dt A_r(\vec{\sigma}, t) \right] \right)_{\eta \rho} \left( \Theta_t \exp \left[ i \int_{\gamma_r} dt A_r(\vec{\sigma}, t) \right] \right)_{\rho \eta} ,
\]  
(14)

\[
\delta A_r = \prod_{\mu = 0, 1, 2, 3} d A_r^a(\vec{r}, t) \quad (\mu = 0, 1, 2, 3) ,
\]

where the Lagrangian \(\mathcal{L}\) is given by,

\[
\mathcal{L} = -\frac{1}{4} \left( \partial \mu A_r^a - \partial_r A_r^a + g f^{abc} A_r^a A_r^b A_r^c \right)^2 .
\]  
(15)
The first factor can be rewritten as

$$A_l e^{-i\mathcal{H}(-\frac{t}{2},-\frac{t}{2})}A_l$$

by choosing the path of continuation to the complex time plane as shown in Fig. 2. From (14) and (16) it follows that

$$\text{amp} = (\mathcal{D}_2(\tau)) \int \mathcal{D}A_\mu \delta(A_\mu^*) e^{i\int_t^{t+\tau} dt d^2 \mathcal{L}} \times \left( P \exp i\int_{t^x}^{t+y} dt A_\nu(\vec{r},t) \right)_{\mu} \times \left( P \exp i\int_{t+y}^{t^x} dt A_\nu(\vec{r},t) \right)_{\mu},$$

where P. B. denotes the periodic boundary condition which is the so-called KMS condition.

$$A_\mu^*\left(-\frac{t}{2},\vec{z}\right) = A_\mu^*\left(-\frac{t}{2},-i\vec{p},\vec{z}\right).$$

The last step is to rewrite (17) into the manifestly gauge invariant form. For the purpose we perform a gauge transformation.

$$A_\mu \rightarrow UA_\mu U^{-1} + \frac{i}{\hbar} \partial_\mu U U^{-1}, \quad A_\mu = A_\mu^* T^\mu, \quad U = U^*(\vec{r},t) T^\mu.$$
By integrating (20) over $U(x,t)$ one finds

$$\text{amp} = \left( \delta'_{\lambda_0} \right)^2 \int_{P_R} \delta A_r \ e^{-i \int dt \int d^4 \mathcal{L} \ tr \left( P \exp \ i \frac{1}{\hbar} \int dx \ A_r \right)}$$

$$= \left( \delta'_{\lambda_0} \right)^2 \ e^{-it \mathcal{V}(\tau, \mathcal{T})}, \ A_r = A_r^a T^a,$$

where large $\tau_0$ limit is implicit. Thus the rectangular Wilson loop (Fig. 3) comes out which ensures the gauge invariance of the expression (21).

To evaluate perturbatively the thermal average of the Wilson loop we must eliminate the degeneracy of the gauge freedom. This can be done as in the zero temperature case. The Feynman rules at finite temperature in the real time formalism are briefly reviewed in Appendix A and the rules for the Wilson loop are collected in Appendix B.

Before closing this section we like to mention that one could have chosen Polyakov loops\textsuperscript{[3]} or Wilson loop (Fig. 4) in imaginary time formalism as a gauge invariant quantity to start with. In working with Polyakov loops one faces the problem that one gluon exchange process is absent. As regards Wilson loop the trouble is that there exists an ambiguity of how we connect the $q$-$\bar{q}$ pair, as stated by Nadkarni.\textsuperscript{[17]} On the other hand the real time formalism requires no change in the shape of the Wilson loop in going from $T=0$ to $T \neq 0$. This explains our choice of the Wilson loop in Minkowsky space-time.
Section 3. Finite temperature correction for the Wilson loop and effective coupling constant

In this section we evaluate the quantum correction to the \( q \cdot \bar{q} \) potential using the Wilson loop and then derive the renormalization group equations (RGE's). Here we would like to give a comment. In (21) \( g \) and \( A_\mu \) denote the bare coupling constant and bare fields, respectively. Now, in this section, we perform a perturbative expansion in powers of a coupling constant renormalized at zero temperature. Therefore there exist the contributions from counter terms which cancel the ultra-violet divergences. However we do not explicitly write them for the sake of simplicity. At the stage where the divergences are subtracted their existence are to be considered implicitly. The precise definition of a renormalized coupling constant is given by a renormalization prescription later in Eq.(36). In the following we represent a renormalized coupling constant as \( g \) for notational simplicity.

From (21) the potential, \( V(R,T) \), is given by

\[
V(R,T) = \lim_{t_0 \to +\infty} \frac{e^{i} \log \sum_{\text{loop}} \langle i g \rangle \int \prod_{\mu} dx_\mu \cdots dx_\mu \int DA_r e^{i \int dt \cdots 2\pi \frac{\delta^4 L}}
\]

\[
= \text{tr} \left( P A_\mu (x) \cdots A_\mu (x) \right).
\]

We calculate the potential at the \( g^4 \) order by choosing the Feynman gauge. Following the steps stated in Ref. [12], we find that the diagrams to be evaluated are those shown in Fig. 5. Then one arrives at the expression,
\[ V(R,T) = \lim_{t_0 \to 0^+} \frac{\gamma}{t_0} \int \frac{d^4 k}{(2\pi)^4} e^{-ikR} \left\{ I_a - \frac{1}{2} C (I_b + I_c + I_d) + C I e \right\}, \tag{23} \]

\[ C_+ = \frac{N^2 - 1}{2N}, \quad C = N \text{ for SU}(N) \]

In the above, $I_a - I_e$ denote the "Abelian part" of the corresponding diagrams. To be definite, for instance, the quantity $I_a$ is given by

\[ I_a = (i \delta)(-i \delta) \int_{-t_0}^{t_0} dt_1 \int_{-t_0}^{t_0} dt_2 \Delta(z_i - z_j), \tag{24} \]

\[ z_i = (t_i, \vec{r}_i), \quad z_j = (t_j, \vec{r}_j), \]

where

\[ \Delta_{\mu \nu}(x) = \delta_{\mu \nu} \Delta(x) = \delta_{\mu \nu} \int \frac{d^4 k}{(2\pi)^4} e^{-i k x} \left( \frac{-i}{k^2 + i\epsilon} - \frac{2\pi \delta(k^2)}{e^{ikx} - 1} \right), \tag{25} \]

\[ \delta_{\mu \nu} = (+1, -1, -1, -1), \]

is the real time gluon propagator. Among the integrals $I_a - I_e$, $I_e$ has already been computed in Ref. [9]. $I_a$ is easily calculated to be

\[ I_a = i t_0 g^2 \int \frac{d^4 k}{(2\pi)^4} \frac{e^{ik\vec{r}}}{k^2} \tag{26} \]

Now we come to the evaluation of $I_b$, $I_c$, and $I_d$. After some examination we can extract the finite temperature part which has $R$ dependence when $t_0 \to 0^+$ and which thus contributes to the q-\overline{q} potential. For $I_c$ it is given by
\[ I_c = (i g)^i \left( \frac{d^4 k_1}{2 \pi^4} \cdots \frac{d^4 k_n}{2 \pi^4} \right) e^{-i(k_1 \cdot x_1 + \cdots + k_n \cdot x_n)} \left( 2 \pi \right)^n \delta^0 (k_1 + \cdots + k_n) \]

\[
\int \frac{d^4 p}{(2 \pi)^4} \Delta_0 (p) \Delta_0 (k_1 + k_2) \frac{i}{\omega_1 + \gamma + i \epsilon - \omega_3 + \gamma + i \epsilon} \frac{i}{\omega_2 + \gamma + i \epsilon - \omega_4 + \gamma + i \epsilon}
\]

where \( \Delta_0 \) and \( \Delta_\beta \) denote, respectively, the \( \mathcal{T} \)-\( \mathcal{Q} \) and \( \mathcal{T} \)-\( \mathcal{Q} \) parts of the thermo propagator \( \Delta(k) \) and \( \omega \) denotes the time component of momentum \( k \). Integrating over \( \vec{k}_1 \) and \( \vec{k}_2 \) makes the remaining integrand be the function of \( \vec{k}_1 + \vec{k}_2 \). Therefore, a shift of the integration variable \( \vec{k}_1 \) as \( \vec{k}_1 \rightarrow \vec{k}_2 - \vec{k} \) makes the \( \vec{k}_2 \) integration trivial. In the course of this, the factor \( (s^3(0))^2 \) appears. This is because the positions of the quark and antiquark are fixed. Since this constant is irrelevant for our discussion, we drop it (see (21)). Then, integrating over \( \omega_3 \) and \( \omega_4 \) and using the formula

\[ \lim_{t_0 \to \infty} \frac{\sin (\omega_1 + \omega_2) t_0}{\omega_1 + \omega_2} = \pi \delta (\omega_1 + \omega_2) \]

we obtain

\[ I_c = -g^4 \left( \frac{d^4 k}{(2 \pi)^4} \right) e^{i \vec{k} \cdot \vec{R}} \left( -i t_0 \right) \int_0^\infty \frac{dp}{2 \pi^2} \frac{1}{p^4 (e^{Ap} - 1)} \left( p + P \cos \theta P l - \frac{2}{t_0} \sin \theta P l \right) \]

\[ \equiv \frac{i t_0 g^4 \int d^4 k}{2 \pi^4} \frac{e^{i \vec{k} \cdot \vec{R}}}{\vec{k}^2} \left( I_1 + I_3 + I_3 \right), \]

\[ P \equiv | \vec{P} |. \]
Each integral is divergent at $P=0$ although the sum is finite. Therefore, we introduce a convergence factor $P^\varepsilon$ and also use a formula,

$$\frac{1}{e^{\beta P} - 1} = \frac{1}{\beta P} + \frac{2P}{\beta} \sum_{n=1}^{\infty} \frac{1}{n^s + \left(\frac{2\pi n}{\beta}\right)^2} - \frac{1}{2}, \quad (30)$$

to find

$$I_1 = \frac{1}{2} \left( \frac{1}{\xi} + \gamma + \log \frac{\beta}{2\pi} \right),$$

$$I_2 = -\frac{\pi \xi}{2\beta} - \frac{1}{2} \log \left( 1 - e^{-\frac{2\pi \xi}{\beta}} \right) + \frac{1}{2} \left( \frac{1}{\xi} + \gamma + \log \xi \right), \quad (31)$$

$$I_3 = \frac{\pi \xi}{2\beta} - \frac{\beta}{2\pi \xi} \left( \xi(2) - \sum_{n=1}^{\infty} \frac{1}{n!} e^{\frac{2\pi \xi n}{\beta}} \right) - \left( \frac{1}{\xi} - 1 + \gamma + \log \xi \right).$$

Then, summing up $I_1$, $I_2$ and $I_3$, we obtain the finite answer,

$$I_1 + I_2 + I_3 = \frac{1}{2} \left( \log \frac{\beta}{2\pi \xi} + 2 \right) - \frac{1}{2} \log \left( 1 - e^{-\frac{2\pi \xi}{\beta}} \right)$$

$$- \frac{\beta}{2\pi \xi} \left( \xi(2) - \sum_{n=1}^{\infty} \frac{1}{n!} e^{\frac{2\pi \xi n}{\beta}} \right). \quad (32)$$

As $t_0 \to 0$ only the first term survives. Therefore we find

$$I_c = \frac{d^2 P}{(2\pi)^2} \frac{e^{i\hat{P}\hat{R}}}{i\hat{R}^2} \cdot \frac{1}{4\pi^2} \left( \log \frac{\beta}{2\pi \xi} - 2 \right). \quad (33)$$

The contribution of $I_d$ is the same as that of $I_c$. The evaluation of $I_b$ is much lengthier but can be done by following the similar procedures as for $I_c$. In the end the whole contribution including $T=0$ part turns out to be
where
\[ \frac{\lambda}{\xi} = \frac{2}{\xi} - \gamma - \log \frac{K^4}{4\pi\mu^2} , \]
\[ \xi = 4 - D \quad (D: \text{space-time dimension}), \]
\[ \Omega(\beta K) = \frac{\beta K}{\pi} \left( \frac{\eta}{2} \xi(2) - \sum_{n=1}^{\infty} \frac{1}{n} \tan\left(\frac{\beta K}{4\pi n}\right) \right) - 4 \left( \log \frac{\beta K}{4\pi} + \gamma + 2 \right) + \frac{16\xi(2)}{(\beta K)^4} - 4F_0 + 2F_1 , \]
\[ F_n(\beta K) = \int_0^\infty \frac{x^n}{\exp(\beta K x) - 1} \cdot \log \left| \frac{1 + x}{1 - x} \right| . \]

Now that we have found the corrected potential, the next step is to do the renormalization and then to derive the RGE. By subtracting a divergent part with the renormalization prescription
\[ V(K, \beta = \infty)_{K = \mu} = \frac{-g^3 C_f}{\mu^3} = -\frac{g^3 C_f}{\mu^3} , \]
we arrive at the renormalized potential
\[ V(R, T) = \left( \frac{d^3 k}{(2\pi)^3} \right) e^{\mathbf{p} \cdot \mathbf{R}} \left( -\frac{g^3 C_f}{\mu^3} \left( \frac{1}{6} \log \frac{K}{\mu} + \omega(\frac{K}{\mu}) \right) \right) \]
We define the temperature dependent effective coupling constant, 
\( g^2(K, T) \), by

\[
V(K, T) = V(K, \beta) = - \frac{\alpha(K, T)}{4\pi} \equiv - \frac{4\pi \alpha(K, T)}{K} ,
\]

(38)

From (37) and (38), within one-loop approximation, \( \alpha(K, T) \) is given by

\[
\alpha(K, T) = \alpha \left\{ 1 - \frac{\alpha C}{4\pi} \left( \frac{22}{3} \log \frac{K}{\mu} + \Omega(K/T) \right) \right\} , \quad \alpha = \frac{g^2}{4\pi} .
\]

(39)

Differentiating the \( \alpha(K, T) \) by \( \frac{3}{\partial K} \) and \( \frac{3}{\partial T} \), we obtain the RGE's

\[
K \frac{\partial \alpha(K, T)}{\partial K} = - \frac{C}{4\pi} \alpha^2(K, T) \left( \frac{22}{3} + K \frac{\partial^2 \Omega(K/T)}{\partial K} \right) + O(\alpha^3(K, T)) ,
\]

(40-1)

\[
T \frac{\partial \alpha(K, T)}{\partial T} = - \frac{C}{4\pi} \alpha^2(K, T) T \frac{\partial^2 \Omega(K/T)}{\partial T} + O(\alpha^3(K, T)) .
\]

(40-2)

Since we are interested in the region of \( (K, T) \) where \( \alpha(K, T) \) is small \((<<1)\) we approximate the right hand sides of (40) by the first terms. Then, solving the above equations, we find the effective coupling constant

\[
\alpha(K, T) = \frac{\alpha}{1 + \frac{\alpha C}{4\pi} \left( \frac{22}{3} \log \frac{K}{\mu} + \Omega(K/T) \right)} , \quad \alpha = \frac{g^2}{4\pi} ,
\]

(41)

where \( g \) is the renormalized coupling constant defined by (36).

Introducing the RG invariant scale parameter \( \Lambda \) by

\[
\Lambda = 1 \exp \left( - \frac{3}{22} \cdot \frac{16\pi^2}{C} \cdot \frac{1}{g^2} \right) ,
\]

(42)
we rewrite (41) as

\[ \alpha(K, t) \equiv \omega \left( \frac{k}{\Lambda}, \frac{t}{\Lambda} \right) = \frac{4\pi}{C \left( \frac{22}{3} \log \frac{K}{\Lambda} + \mathcal{O} \left( \frac{K}{\Lambda} \right) \right)}. \] (43)

This is the RG improved effective coupling constant which has the gauge invariant physical meaning. In the next section we examine its behaviour.
Section 4. Discussion and Summary

Since $\Omega(T/K)$ is positive definite, $\alpha(K,T)$ is free from singularity at any temperature so long as $K>A$. Furthermore, as high temperature expansion gives

$$\Omega \sim 32 \pi^2 \left( \frac{T}{K} \right)^2 - 4 \pi^2 \left( \frac{T}{K} \right) + \frac{22}{3} \log \left( \frac{T}{K} \right) + \cdots, \quad \frac{T}{K} \gg 1, \quad (44)$$

we find that with fixed $K$, $\alpha(K,T)$ decreases like $(K/T)^2$ as temperature increases. This result is qualitatively the same as in the behaviour of the effective coupling constant defined by the tri-gluons and fermion-gluon coupling in Ref. [9]. We note also that the log($K$) in $\Omega(K/T)$ exactly cancels the one in $T=0$ part, as expected.[6]

Now let us see the behaviour of $\alpha(K,T)$ with the change of $K$ with $T$ fixed. As $K$ becomes large $\alpha(K,T)$ goes to the zero temperature result, as expected. This is because $\Omega(K/T)$, which represents temperature correction, goes to zero when $T/K\to 0$. A more interesting feature arises for small momentum ($K<<T$) region. For small $K$ with fixed $T$, $\alpha(K,T)$ becomes

$$\alpha(K,T) \sim \alpha(T) \cdot \frac{K^4}{K^4 + \frac{4 \pi C}{3} \alpha(T) \cdot T^4 - \pi C \alpha(T) \cdot TK}, \quad (45)$$

where

$$\alpha(T) = \frac{1}{4\pi} \frac{22}{3} \log \left( \frac{T}{\Lambda} \right) \quad \alpha, \quad (46)$$
Here we like to stress that in order to use the expression (13) for the investigation of the behaviour of \( \alpha(K,T) \) at low momentum it should be small for all range of \( K \). This is because we approximated the right hand sides of (40) by the first terms. If \( \alpha(K,T) \) becomes large at some momentum scale, say, at \( K_0 \), the approximation breaks down at \( K_0 \) and we cannot study the behaviour of \( \alpha(K,T) \) in low momentum region \( (K<K_0) \) from (43). This is the case if the temperature is sufficiently low. However this situation is expected to change at high temperature. We note that the relative sign of \( \log(K/A) \) and \( \alpha(K/T) \) is opposite when \( K<A \). Since \( \alpha \) becomes large as \( (T/K)^2 \) for small \( K \), we may infer that if temperature is sufficiently high \( (T>>A) \), \( \alpha(K,T) \) remains small as we let \( K \) go to zero from the large momentum region. A numerical check indeed confirms this assertion (Fig.6). Thus (45) can be used at low momentum region as long as temperature is sufficiently high. This high temperature condition can be found in (45) if we note the sign of the squared Debye mass, \( m^2_{el} \),

\[
M^1_{el} = \frac{4}{3} \pi \alpha(t) T^1
\]  

Only when \( T>A \), \( m^2_{el} \) is positive.

Thus the result tells us that much above the temperature \( A \), \( \alpha \) remains small for all range of \( K \). Since it follows that at high temperature \( (T>>A) \) the interaction of a static \( q-\bar{q} \) pair is weak at any distances this result may support the de-confinement transition near the temperature \( A \).

In the following we summarize the thesis. By the use of the Wilson loop and the renormalization group method at finite
coupling which measures the strength of the interaction of a static q-\bar{q} pair. Then it is found that at high temperature the interaction is weak at any distances. This result indicates that at high temperature pure QCD is in the de-confined phase. We like to stress that our prediction is gauge invariant since our RGE's are derived from the Wilson loop which is manifestly gauge invariant.
Acknowledgements

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Footnotes

F1) I thank Mr. S. Misono for numerical check of the behaviour of $a(K,T)$.

F2) This result agrees with that of Gendenshtein in Ref. [7].
Appendix A. Real time Feynman rules for gauge theories at finite temperature

Real time Feynman rules for gauge theories at finite temperature is obtained by considering the path-integral, \[ \mathcal{Z} = \int D\phi \exp \left\{ i \int_C \mathcal{L} \right\} \quad \mathcal{L} = -\frac{1}{4} (\partial_\mu A_\nu^a - 2 i \frac{\alpha}{2} F_{\mu\nu} A_\rho^a + \lambda f^{abc} A_\rho^a A_\sigma^b A_\tau^c) \] (A-1)

where the time integration is carried out along the contour C as shown in Fig. 2 (see (21)). In the above P. B. denotes the KMS boundary condition. \[ A_\rho^a \left( -\frac{\tau}{2}, z \right) = A_\rho^a \left( -\frac{\tau}{2} - i\rho, z \right). \] (A-2)

By the use of the standard Faddeev-Popov trick we can eliminate the gauge degeneracy. If we impose a covariant gauge condition such as

\[ \partial^\mu A_\mu^a (x) = U^a(x), \quad a = 1, 2, \ldots, N = \frac{3}{2} \] (A-3)

and averaging over all functions \( U^a(x) \) with Gaussian weight, we arrive at the expression

\[ \mathcal{Z} = \int D\phi \mathcal{D}(\xi C) \exp \left\{ i \int_C \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{2N} (\partial_\mu A_\nu^a)^2 \right. \\
\left. - \partial^\mu \partial^\nu (\partial_\mu A_\nu^a - g f^{abc} A_\rho^a A_\sigma^b) \right\} \right], \] (A-4)

\[ F_{\mu\nu} \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\rho^a A_\tau^b A_\sigma^c, \]

\( \alpha \): gauge parameter

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In the above we used an abbreviated notation

\[ \int_c = \int_c dt \int d^3x', \int_c dt' \int d^3x d^3x', \ldots. \]  \hspace{1cm} (A-5)

The ghost fields, \( a^a \) and \( \bar{a}^a \), should obey the same boundary conditions as that of the gauge fields as the functional determinant is evaluated in the space of gauge fields. To evaluate (A-5) perturbatively we first add the source functions and replace the interaction Lagrangian by the functional derivative with respect to sources. At this point we note that the source function should be defined on the complex time plane. This is because the time coordinates of the fields are continued to the complex plane in (A-1). Then it is enough for us to consider the free part with sources of the full integral (A-8).

The free part factorizes into the ones which involve only the gauge fields or ghost fields. Therefore, choosing the Feynman gauge, we consider the path-integral

\[ Z_s(J) = \int_{\mathcal{P.B.}} D\phi \exp i \int_c \left( -\frac{i}{2} \partial_a \partial^a \phi + J\phi \right), \]  \hspace{1cm} (A-6)

where we have collectively denoted the \( A^a_\mu (\mu=0,1,2,3) \) and \( a^a (\bar{a}^a) \) as \( \phi \). Since the path integral (A-6) is a Gaussian type one can easily perform the integration. The result is

\[ Z_s(J) = \exp \left( -\frac{i}{2} \int_c J(x') D(x' - x)J(x) \right), \]  \hspace{1cm} (A-7)

where \( D(x' - x) \) is a Green's function on the contour \( c \),

\[ \square_c D(x' - x) = \delta_c(x' - x), \]  \hspace{1cm} (A-8)

\[ \delta_c(x' - x) = \delta_c(t' - t) \delta^3(x' - x). \]
In (A-8) $\delta$-function on the contour $C$, $\delta(t-t')$, is defined by

$$\int_C dt \delta(t-t') f(t) = f(t'). \quad (A-9)$$

The boundary condition of $D(x'-x)$ is the same as that of $\phi$. To obtain the $D(x'-x)$ explicitly we consider the Ansatz,

$$D(x'-x) = D_0(x'-x) \theta(t'-t) + D_1(x'-x) \theta(t-t'), \quad (A-10)$$

where $\theta(t'-t)$ is a step-function on the contour $C$. Then, from (A-2), (A-8) and (A-10), we obtain

$$D(x'-x) = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot(x'-x)} \frac{-i}{2\omega_k} \frac{1}{1 - e^{-\beta\omega_k}} \times \left\{ (e^{-i\omega_k(t'-t)} + e^{-\beta\omega_k + i\omega_k(t'-t)}) \theta(t'-t) \right\}$$

$$+ \left\{ (e^{i\omega_k(t'-t)} + e^{-\beta\omega_k - i\omega_k(t'-t)}) \theta(t-t') \right\}, \quad (A-11)$$

$$\omega_k = \sqrt{\omega^2_k}.$$  

We shall now take the limit $t_0 \to 0^+$ which is implicit in (21) and (A-1). In this limit contributions to (A-6) from the vertical segments vanish (see Fig. 4). Therefore only the contributions from the horizontal segments, $C_1$ and $C_2$, remain in this limit. Thus, dropping the irrelevant constant we arrive at the expression,

$$Z_0(J) = \exp \left\{-\frac{i}{2} \int_{C_1, C_2} J(x') L(x'-x) J(x) \right\}. \quad (A-12)$$
In (A-12) we define

\[ \int_{c, c^*} J(x') D(z'-x') J(x) = \int_{0}^{\infty} \int_{0}^{\infty} d^{3} x' d^{3} x'' J_{a}(t, x') D_{a}(t^{'-t}; z'-z') J_{a}(t, x') \]  \hspace{1cm} (A-13)

where \( a, t=1, 2 \) and

\[ J_{1}(x) = J(t, x'), \quad J_{1}(x) = J(t - \frac{i\rho}{2}, x') \]  \hspace{1cm} (A-14)

and

\[ D_{a}(x'-x) = D(x'-x), \]
\[ D_{2a}(x'-x) = D(x-x'), \]
\[ D_{1b}(x'-x) = D^{c}(t'-t + \frac{i\rho}{2}; z'-z'), \]
\[ D_{2b}(x'-x) = D^{c}(t'-t - \frac{i\rho}{2}; z'-z'). \]  \hspace{1cm} (A-15)

In momentum space the propagator \( iD_{ab} \) is written as

\[
\begin{pmatrix}
-\frac{-i}{\mathcal{K}^2 + i\varepsilon} & 2i\pi \delta(k^0) e^{\frac{\mathcal{E}_{2}}{\mathcal{E}^{2} + i\varepsilon}} & \frac{2i\pi \delta(k^0)}{e^{\mathcal{E}_{1} l_{1l}}} - 1

\frac{2i\pi \delta(k^0)}{e^{\mathcal{E}_{1} l_{1l}}} & \frac{i}{\mathcal{K}^2 + i\varepsilon} & 2i\pi \delta(k^0) e^{\frac{\mathcal{E}_{2}}{\mathcal{E}^{2} + i\varepsilon}} \end{pmatrix} \]  \hspace{1cm} (A-16)

\( \equiv i \hat{D}(k) \).
Thus in the Feynman gauge the propagators for the gauge ($\hat{a}_{\mu \nu}^{\text{ag}}$) and ghost ($\hat{a}_{\mu \nu}^{\text{gh}}$) fields are given in the matrix notation by

$$i\hat{\Delta}_{\mu \nu}^{\text{ag}} = \delta^{\mu \beta} \delta_{\nu \lambda} i\hat{D}, \quad \omega, \beta = 1, \ldots, N$$

$$i\hat{\Delta}_{\mu \nu}^{\text{gh}} = \delta^{\mu \beta} i\hat{D}, \quad a, b = 1, \ldots, N^2 - 1$$

In the real time formulation of finite temperature quantum field theory there arise the fictitious fields which live on the lower horizontal segment $C_2$ (see Fig. 2). Due to this the propagator has a matrix structure as in (A-16). (1-1) element of the propagator, $i\hat{D}$, represents the propagation of the physical free quantum. As is easily seen from (21), external legs of the Green's functions are always physical fields. Therefore fictitious fields appear in the Feynman diagrams only as internal building blocks.

As a final note we would like to comment on the rules for the vertices. Since the vertices arise through the functional derivative by the external sources,

$$\exp\left\{-i \int_C \mathcal{L}_{\text{int}} \left( \frac{i}{\gamma} \frac{\delta}{\delta J} \right) \right\},$$

two kinds of vertices are found to exist. One is the usual vertex which involves only physical fields. The other is the one which involves only the fictitious fields. The rules for the latter vertex is given by that of usual one with extra factor ($-$
1). The reason for the need of the factor $(-1)$ is that, in (A-18), the direction of time integration on $C_2$ is opposite to that on $C_1$. 
Appendix B. Feynman rules for the Wilson loop

In this Appendix we summarize the rules for the Wilson loop. The Wilson loop represents the propagation of quark and antiquark. For a quark the color flows toward the positive time direction and for an antiquark toward the negative direction. The rules for a static $q-ar{q}$ pair are easily read off from (22) and are given by [12]

\[
\delta^{\mu_1 \mu_2} \delta^{\nu_1 \nu_2} \epsilon(t' - t) = \delta^{\nu_1 \nu_2} \int \frac{d^4k}{(2\pi)^4} e^{-ik(t' - t)} \frac{i}{k_\mu - i\epsilon} = \delta^{\mu_1 \mu_2} \delta^{\nu_1 \nu_2} \epsilon(t' - t)
\]

\[
\epsilon_{\alpha} = +i g (T^a)_{\alpha \beta}, \quad \epsilon_{\alpha} = -i g (T^a)_{\alpha \beta}.
\]
Appendix C. Renormalization group at finite temperature

The method of renormalization group (RG) was first found and formulated by Stuckelberg and Peterman and Gell-Mann and Low. RG method can be generalized to finite temperature cases to investigate the asymptotic behaviour of the physical systems. In this Appendix we give a brief review of the renormalization group method at finite temperature. As for the presentation of the idea we follow the approach of Susskind.

Let us consider the static potential in Quantum Electrodynamics (QED) at finite temperature. Suppose the electromagnetic medium is composed of photons and charged massless fermions. For the sake of simplicity we set the chemical potential equal to zero. Then, within the one-loop approximation, potential energy of the static test charges of electron and positron pair at temperature $T$ is given by

$$V(R, T) = -\int_{(2\pi)^4} d^4 \vec{p} \sum_{\beta} \frac{e^2}{(2\pi)^3} \log \left[ 1 - \frac{e^2}{4\pi^2} \left( -\frac{1}{3} \log \frac{\vec{p}^2}{\mu^2} + \mu^2 \frac{T^2}{\mu^2} \right) \right], \quad \beta = \vec{P} \mu$$

where $\vec{R}$ denotes the difference of the position vectors of electron and positron and $e$ is the renormalized coupling constant at momentum scale $\mu$ at zero temperature. In the above, $\mu(K/T)$ denotes the temperature correction due to the fermion loop,

$$\mu(K/T) = \delta(2) \left( \frac{T}{K} \right)^{\frac{1}{2}} - \left( 1 - \frac{3}{2} \frac{T}{K} \right) W_0 - 2 \left( 1 + \frac{3}{2} \frac{T}{K} \right) W_1,$$

$$W_n = \int_0^\infty dx \frac{x^n}{\exp \left( \frac{K x}{2T} \right) + 1} \left( x + \frac{x^2 - 1}{2} \log \left| \frac{x + 1}{x - 1} \right| \right).$$
Since at low momentum or at high temperature \( (K/T<<1) \), \( \Omega \) behaves as

\[
\Omega(K/T) \sim 8 \gamma_2 \left( \frac{T}{K} \right)^2 + \frac{2}{3} \left( 7 + \log \frac{K}{\pi T} - \frac{4}{3} \right) + \cdots , \quad \frac{K}{T} \ll 1. \tag{C-3}
\]

the temperature correction becomes large in the region of \( K/T<<1 \). Therefore, when \( e^2 (T/K)^2 >> 1 \), naive perturbation theory breaks down. In order to improve the perturbation theory one may sum up the series of chain diagrams (Fig. 7). The result of this improvement yields the potential

\[
\langle \mathcal{V}(R, T) \rangle = -\int \frac{d^3 K}{(2\pi)^3} \frac{1}{R^3} \frac{e^2}{K^3} \frac{1}{1 + \frac{e^2}{4\pi^2} \left( -\frac{2}{3} \log \frac{K}{\pi T} + \Omega(K/T) \right)} . \tag{C-4}
\]

At low temperature (C-4) gives the screened Debye potential at large distances.

In analogy with zero temperature cases the above improvement can be done systematically by the use of the renormalization group method at finite temperature. It goes as follows: According to Susskind, we define the effective coupling constant \( \alpha(K, T) \) by

\[
\langle \mathcal{V}(R, T) \rangle = \int \frac{d^3 K}{(2\pi)^3} e^{i k \cdot \mathbf{R}} \frac{-4\pi \alpha(K, T)}{K^2} . \tag{C-5}
\]

Then we set up the differential equations, namely renormalization group equations for \( \alpha(K, T) \). In perturbation theory \( \alpha(K, T) \) is expressed as a power series in \( \alpha (= e^2 / 4\pi) \),

\[
\alpha(K, T) = \alpha \left( 1 + \alpha C_1 + \alpha^2 C_2 + \cdots \right) , \quad C_n = C_n(K, T), \tag{C-6}
\]

\[
-33-
\]
where $\alpha$ is a renormalized charge at zero temperature and $C_n(K,T)$ denotes the $n$-loop correction to the potential with $\alpha^n/(K)^2$ factored out. Differentiation of the $\alpha(K,T)$ with respect to $K$ and $T$ gives

$$K \frac{\partial \alpha(K,T)}{\partial K} = \alpha^2 \left( K \frac{\partial C_i}{\partial K} + \alpha K \frac{\partial C_n}{\partial K} + \ldots \right),$$

$$T \frac{\partial \alpha(K,T)}{\partial T} = \alpha^2 \left( T \frac{\partial C_i}{\partial T} + \alpha T \frac{\partial C_n}{\partial T} + \ldots \right).$$

(C-7)

We can re-express the right hand side of (C-7) by $\alpha(K,T)$ instead of $\alpha$. For instance, at the $\alpha^3(K,T)$ order, we obtain

$$K \frac{\partial \alpha'(K,T)}{\partial K} = \alpha^3(K,T) K \frac{\partial C_i}{\partial K} + \alpha^2(K,T) K \frac{\partial C_n}{\partial K} \left( C_i - C_n^2 \right) + O(\alpha^4(K,T)),

(C-8)

$$T \frac{\partial \alpha'(K,T)}{\partial T} = \alpha^3(K,T) T \frac{\partial C_i}{\partial T} + \alpha^2(K,T) T \frac{\partial C_n}{\partial T} \left( C_i - C_n^2 \right) + O(\alpha^4(K,T)).$$

Since we are interested in the region where $\alpha(K,T)$ is small we approximate (C-8) by

$$K \frac{\partial \alpha'(K,T)}{\partial K} = \alpha^3(K,T) K \frac{\partial C_i}{\partial K},$$

$$T \frac{\partial \alpha'(K,T)}{\partial T} = \alpha^3(K,T) T \frac{\partial C_i}{\partial T}.$$

(C-9)

Solving (C-9) we obtain the improved effective coupling constant,

$$\alpha'(K,T) = \frac{\alpha}{1 - \alpha C_i} = \frac{\alpha}{1 + \alpha \left[ -\frac{2}{3} \log \frac{K}{\mu} + \Omega(K,T) \right]},$$

(C-10)

By the insertion of (C-10) into (C-5) the expression (C-4) reappears. Thus, also at finite temperature, we can improve the
one-loop result by the use of the renormalization group method as is explicitly stated in the above.
References

    43.
[5] See, for example,
    See also
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See also
Figure Captions

Fig. 1 The elongated Wilson loop which represents the vacuum to vacuum transition through the pair creation and annihilation of q-\bar{q} pair.

Fig. 2 The continuation path in the complex time plane.

Fig. 3 Wilson loop which represents the amplitude that a static q-\bar{q} pair propagates during the time period T.

Fig. 4 Wilson loop in the world of imaginary time.

Fig. 5 The Feynman diagrams which contribute to the static q-\bar{q} potential.

Fig. 6 Results of the numerical calculation of the behaviour of 1/a(K,T) with respect to K at T=0.1A (graph (1)) and at T=10A (graph (2)). In the graph (1) only the behaviour of 1/a(K,T) in the region K>K_0 is reliable.

Fig. 7 Series of chain diagrams which contributes to the static potential in QED at finite temperature.
FIG. 3

FIG. 4
FIG. 5

a

b

c

d

e
(1) $T = 0.1 \Lambda$
(2) $T = 10 \Lambda$

FIG. 6
FIG. 7