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THE DISLOCATION DISTRIBUTION FUNCTION NEAR A CRACK TIP GENERATED BY EXTERNAL SOURCES *

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ABSTRACT

The dislocation distribution function near a crack tip generated by external sources is calculated. It is similar to the shape of curves calculated for the crack tip emission case but the quantative difference is quite large. The image forces enlarges the negative dislocation zone but does not change the form of the curve.

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1. INTRODUCTION

Since one of the present authors (Lung and Xiong, 1983) pointed out the possibility of the existence of negative dislocations at a crack tip, Narita and Takamura (1985) showed once in NaCl that some of the dislocations observed near the crack tip have negative Burgers vectors. Recently, it is worth mentioning that Ohr (1987) has indeed observed negative dislocations at the immediate crack tip in stainless steel. Negative dislocations are attracted to the crack tip. As they approach the crack tip, these negative dislocations antishield the crack tip, i.e., the local K becomes greater than the applied K. Thus, the presence of the antishielding dislocations close to a crack tip can play a significant role in promoting brittle crack extension (Thomson, 1986; Li, 1986).

It is reasonable to think that these negative dislocations are generated from sources other than the crack tip, such as Frank-Read sources or grain boundaries.

In this paper we calculate the dislocation distribution function near a crack tip generated by external sources. It should be remembered that the direction of the frictional stress for the negative dislocations moving toward the crack tip is opposite to their moving direction and then it is parallel to the direction of the crack extension stress. In our present calculation we consider the effect of image forces due to the crack tip on the dislocations which was neglected in the previous work (Lung and Xiong, 1983) and which was shown necessary to be considered (Lung and Wang, 1984; Zhou and Lung, 1988).

2. DISLOCATIONS MOVING TOWARD THE CRACK TIP

A dislocation source which exists in front of a sharp crack can generate dislocation pairs. The positive ones would move away from the crack tip due to the repellent force between them. The negative ones would move towards the crack tip due to the attractive force between them. The frictional stress would change sign at the location of the dislocation source. This problem is not easy to handle.

At first, we assume that all the dislocations move toward the crack tip. This is easy to be understood for negative dislocations though it would not be true for positive dislocations. Under this assumption we can calculate the dislocation distribution function as the previous work (Lung and Xiong, 1983) simply.

In our previous work, unlike the original BCS-type crack model, we treated this problem within a homogeneous and continuous system. We only consider the plastic zone which does not include both the crack and plastic zone in a whole system to avoid discontinuity and inhomogeneity, because the crack plane is a free surface and the resistance to motion of the dislocations in the crack region (σ_0) is different from that in the plastic zone (σ_1).

We use the image force expansion for a semi-infinite length crack as the upper limit of the effect though we have derived a more precise one for a finite length crack (Zhou and Lung, 1988a).

Suppose one of the dislocations with a Burgers vector of magnitude $-b$ is located at a distance x from a crack of length $2c$ (Fig.1). In this case the resultant shear stress on it is zero when the system is in equilibrium. The condition leads to the following integral equation for the density function $D(x)$ of the dislocation:

$$A'\int_0^{a} \frac{D^{I}(x')\,dx'}{x - x'} - A'\int_0^{a} \frac{D^{I}(x')\,dx'}{x + x'} + \sigma_1 + \sigma^{\infty} + \sigma^{c} = 0 \qquad (1)$$

where $A' = Gb/2\pi(1-\nu) = -A$, G is the shear modulus, $-b$ is the Burgers vector. The first term is the interaction between this dislocation and the others. The second term is the image force acting on this dislocation. $+\sigma_1$ is the frictional stress for the dislocation motion (which is taken as the yield stress), σ^{∞} is the applied stress taken as constant and σ^{c} is the elastic stress field due to a crack. For small scale yielding, one may write

$$\sigma^{c} = \frac{K}{(2\pi x)^{1/2}}, \qquad K = \sigma^{\infty}(\pi c)^{\frac{1}{2}} \qquad (2)$$

Let $u = x'^2$, $w = x^2$, where a is the plastic zone size, u and w are dimensionless parameters, then from (1)

$$A\int_0^{1} \frac{D^{I}(u)\,du}{w - u} = \sigma_1 + \sigma^{\infty} + \sigma^{c}$$

This singular integral equation can be solved by a finite Hilbert transformation (e.g. Chambers, 1976). We obtain

$$D^{I}(x) = \frac{1}{\pi^2 A}\left(\frac{a^2-x^2}{x^2}\right)^{\frac{1}{2}}\Big\{(\sigma_1+\sigma^{\infty})\pi + \sigma^{\infty}\sqrt{\frac{c}{a}}\Big[2F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right) -$$

$$\sqrt{\frac{2ax}{a^2-x^2}}\Big[KZ\left(\sin^{-1}\sqrt{1-\frac{x}{a}}, \frac{1}{\sqrt{2}}\right) + \frac{\pi}{2}\Lambda_0\left(\sin^{-1}\sqrt{1-\frac{x}{a}}, \frac{1}{\sqrt{2}}\right)\Big]\Big]\Big\} \qquad (3)$$

The condition for it to exist leads to the relation

$$\int_0^{a^2}\left(\frac{x^2}{a^2-x^2}\right)^{\frac{1}{2}}(\sigma_1 + \sigma^{\infty} + \sigma^{c})\,dx^2 = 0 \qquad (4)$$

From (4) we obtain

$$\alpha = \frac{-\pi}{\pi + \frac{4}{3}\sqrt{\frac{c}{a}}F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right)} \qquad (5)$$

where $\alpha = \sigma^{\infty}/\sigma_1$.

Substitute (5) into (3) and let $\eta = \frac{x}{a}$, we obtain

$$D^{I}(\eta) = \frac{\sigma_1}{\pi^2 A}\left(\frac{1-\eta^2}{\eta^2}\right)^{\frac{1}{2}}(1+\alpha)\Big\{\pi - \frac{3\pi}{4F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right)}\Big[2F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right) -$$

$$\sqrt{\frac{2\eta}{1-\eta^2}}\left(KZ\left(\sin^{-1}\sqrt{1-\eta}, \frac{1}{\sqrt{2}}\right) + \frac{\pi}{2}\Lambda_0\left(\sin^{-1}\sqrt{1-\eta}, \frac{1}{\sqrt{2}}\right)\right)\Big]\Big\} \qquad (6)$$

where $F\left(\frac{\pi}{2}, \frac{1}{\sqrt{2}}\right)$ is the complete elliptic integral of first kind. The definition of $KZ(\beta,k)$ and $\Lambda_0(\beta,k)$ may be found in the "Handbook of Elliptic Integrals for Engineers and Physicists" (Byrd and Friedman, 1954).

3. DISLOCATIONS MOVING AWAY FROM THE CRACK TIP

Secondly, we assume that all the dislocations move away from the crack tip. This is the case in the first approximation that only dislocation sources very near the crack tip can be operated or it is equivalent to the case that dislocations are emitted from a crack tip. They have positive Burgers vectors and hence are repelled by the crack tip stress field.

Eq.(1) changes to be

$$A\int_0^{a} \frac{D^{II}(x')\,dx'}{x - x'} - A\int_0^{a} \frac{D^{II}(x')\,dx'}{x + x'} - \sigma_1 + \sigma^{\infty} + \sigma^{c} = 0 \qquad (7)$$

Calculating as above, we obtain

$$D^{II}(\eta) = \frac{\sigma_1}{\pi^2 A}\left(\frac{1-\eta^2}{\eta^2}\right)^{\frac{1}{2}}(1-\alpha)\left\{\pi - \frac{3\pi}{4F(\frac{\pi}{2},\frac{1}{\sqrt{2}})}\left[2F\left(\frac{\pi}{2},\frac{1}{\sqrt{2}}\right) - \sqrt{\frac{2\eta}{1-\eta^2}}\left(KZ\left(\sin^{-1}\sqrt{1-\eta},\frac{1}{\sqrt{2}}\right) + \frac{\pi}{2}\Lambda_0\left(\sin^{-1}\sqrt{1-\eta},\frac{1}{\sqrt{2}}\right)\right)\right]\right\} \tag{8}$$

The relationship betwen $D^{I}(\eta)$ and $D^{II}(\eta)$ is

$$D^{II}(\eta) = \frac{1-\alpha}{1+\alpha} D^{I}(\eta) \tag{9}$$

Usually, we use x/c as a unit. It is easy to show that

$$D(\eta) = D\left(\frac{c}{a}\xi\right)$$

where $\xi = \frac{x}{c}$, therefore

$$D(\xi) = D\left(\frac{a}{c}\eta\right) \tag{10}$$

4. NUMERICAL RESULTS AND DISCUSSIONS

Fig.2 shows the dislocation distribution curve calculated by Eq.(6). Fig.3 shows the dislocation distribution curve calculated by Eq.(8) or Eq.(9).

From Fig.2 and Fig.3 the following results appear:

(i) In Fig.3 the curves are qualitatively similar but the dislocation density $D^{I}(x)$ is higher than $D^{II}(x)$.

(ii) Comparing the curves calculated by Lung and Xiong (1983) and the present authors, the effect of image force is shown. It seems that the negative dislocation zone is enlarged by the image forces. It might be the reasonable results due to the large repellent forces of a large number of negative image dislocations of the original positive dislocations in the plastic zone. As we have used the expression of image forces for a semi-infinite length crack, the effect is overestimated. It would be reasonable to suggest that the real distribution might be between the two curves.

(iii) Dislocation generation processes due to the external sources are more easier than that due to the crack tip (Zhou and Lung, 1988b). Dislocation Free Zone or a low density dislocation zone would be left after annihilation of the crack generated positive dislocations and external source generated negative

dislocations. Anyhow, this process is quite complicated and not clear up till now. Further investigations and calculations are needed.

(iv) Eq.(8) shows some dislocation pairs might be generated even in crack emission processes.

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FIGURE CAPTIONS

Fig.1 Schematic figure of dislocation distribution in the plastic zone at a crack tip.

Fig.2 Dislocation distribution near a crack tip generated by external sources.

Fig.3 Dislocation distribution curves calculated by various methods ($\alpha = 0.5$).

a) $D^{I}(x/c)$,

b) Lung and Xiong (1983), $D(x/c)$

c) $D^{II}(x/c)$.

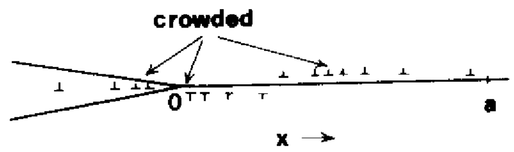

Fig.1

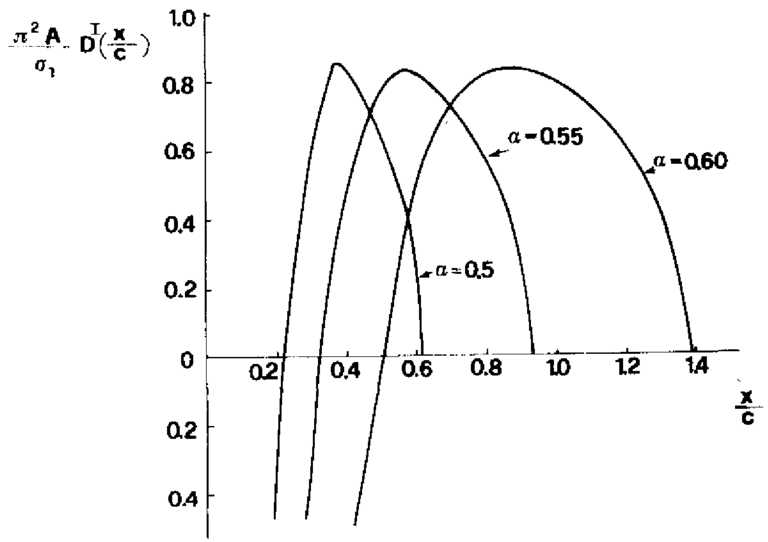
$\frac{\pi^2 A}{\sigma_1} D^I(\frac{x}{c})$
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$\frac{x}{c}$
$\alpha = 0.55$
$\alpha = 0.60$
$\alpha = 0.5$

Fig.2

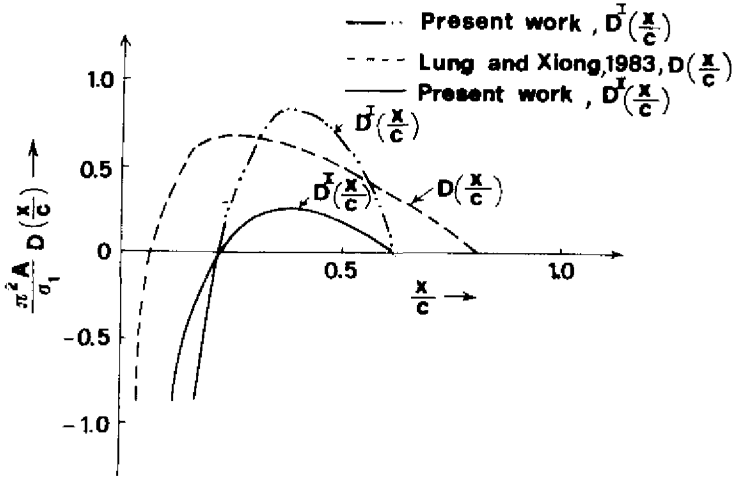
Present work , $D^I(\frac{x}{c})$
Lung and Xiong,1983, $D(\frac{x}{c})$
Present work , $D^{II}(\frac{x}{c})$
1.0
0.5
0
-0.5
-1.0
0.5
1.0
$\frac{\pi^2 A}{\sigma_1} D(\frac{x}{c})$
$\frac{x}{c}$
$D^I(\frac{x}{c})$
$D^{II}(\frac{x}{c})$
$D(\frac{x}{c})$

Fig.3