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INTERNATIONAL
ATOMIC ENERGY
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ORGANIZATION

IC/88/154

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

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OSCILLATORY MAGNETOCONDUCTANCE OF QUANTUM DOUBLE-WELL CHANNELS *

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ABSTRACT

The recently observed flux-periodic interference effect between parallel quantum double-well channels is theoretically studied in a discrete model that takes into account tunneling between channels. We obtain oscillatory magnetoconductance with small modulations which is attributable to the tunneling. Our treatment includes the effect of evanescent modes.

MIRAMARE - TRIESTE
July 1988

* To be submitted for publication.

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Flux periodic effects in the magnetoconductance of multiply connected metallic samples have been the subject of several recent theoretical and experimental investigations. Different types of oscillatory behaviour have been reported in metal rings[1], or cylinders[2], and more recently in a molecular-beam-epitaxy (MBE) grown quantum well structure[3]. After a number of theoretical papers the origin of h/e and $h/2e$ oscillations is well understood.[4-6] The theoretical work, however, deals with *diffusive* transport through an array of weak elastic scatterers in which the inelastic scattering is neglected -the weak localization regime. On the other hand, advances in semiconductor microtechnology have made it possible to fabricate extremely high-mobility conductive channels, thus motivating the interest in the *ballistic* regime.[7] In this limit elastic as well as inelastic scattering is negligible. In the present Letter we will be concerned with this regime in conection with the experiments of Ref.3 in which magnetoconductance oscillations superimposed on a relatively large monotonic decrease in the conductance is observed as a function of applied magnetic field. We propose a discrete exactly solvable model that contains the essential physics and conclude that the observed behaviour is dominated by tunneling of electrons between the interfering channels that conduct in parallel.

We consider the restricted structure depicted in Fig. 1(a) with two end regions ($x < 0$ and $x > L$) and a middle region ($0 < x < L$) consisting of two channels in parellel. In each region there is perfect translational invariance in the x direction. The transverse motion in the y direction is, however, confined by the potential double-well $V(y)$ giving the two channels, with tunneling between them. The spectrum consists of transverse mode subbands in the y direction with k^2 dispersion in the x direction. The potential barrier in the y direction $V(y)$ is, however, different in the end and middle regions and ,therefore , the corresponding subbands are different. We describe this situation with the discrete model shown squematically in Fig. 1(b). The dynamics of electrons is governed by a tight binding Hamiltonian with sites placed in a two-branch "ladder" geometry. The effect of the magnetic field B is included as the London factor multiplying the hopping matrix elements connecting sites i and j, *i.e.*,

$$t_{ij}(B) = t_{ij}(0) e^{i\frac{2\pi}{\phi_0}\int_i^j \mathbf{A}\cdot d\ell}, \qquad (1)$$

where $\mathbf{A} = (By, 0, 0)$ is the vector potential.

The Hamiltonian thus reads

$$H = \sum_{n=-\infty}^{\infty} D_n^\dagger E_n D_n + D_n^\dagger V D_{n+1} + D_{n+1}^\dagger V^\dagger D_n \qquad (2)$$

where D_n, E_n and V are the following matrices in the channel index with

$$E_n = \begin{pmatrix} \epsilon_n^{(a)} & t_n \\ t_n & \epsilon_n^{(b)} \end{pmatrix}, \qquad (3)$$

$$V = \begin{pmatrix} te^{i\theta} & 0 \\ 0 & te^{-i\theta} \end{pmatrix}, \qquad (4)$$

and

$$D_n = \begin{pmatrix} a_n \\ b_n \end{pmatrix}. \qquad (5)$$

Here $\epsilon_n^{(i)}$ is the orbital energy of an electron at site n in the $i-th$ branch ($i = a, b$, being respectively the upper and the lower branch) and t_n is the hopping matrix element between orbitals in different branches. We will present results for $t_n = t$, $\epsilon_n^{(a)} = \epsilon_n^{(b)} = 0$

in the end regions ($n < 0$ and $n > N$) and $t_n = t'$, $\epsilon_n^{(a)} = \epsilon_n^{(b)} = \epsilon$ in the middle region ($0 \le n \le N$). Also $\theta = \pi\phi/\phi_0$, with ϕ the magnetic flux transversing a single plaquet [see Fig. 1 (*b*)]. Finally $a_n^\dagger$ and $b_n^\dagger$ are fermion creation operators asociated with the two branches. The spin indices are omitted for being irrelevant to our treatment as there is no spin-flip scattering. We note in passing that for the experimental situation of large de Broglie wave length of the incident electrons the discrete model can be approximated to its continuum limit. This leads to a two component field with pseudo-spin 1/2 in a transverse pseudo-field. In each region the spectrum consists of two subbands and the eigenstates are, depending on the energy E, propagating or evanescent modes of the form $|\psi_{n,E}>= \Gamma^\dagger_{n,E}|O>$, with

$$\Gamma_{n,E} = e^{ink}\begin{pmatrix} Aa_n \\ Bb_n \end{pmatrix}. \qquad (6)$$

Here the constants A and B are determined by the conditions

$$\frac{B}{A} = \frac{E - 2tcos(\theta + k)}{t_n}, \qquad (7)$$

$$|A|^2 + |B|^2 = 1, \qquad (8)$$

and the energy of the two subbands in each region is given by

$$E_\pm(k,\theta) = \left(\frac{\epsilon_n^{(a)} + \epsilon_n^{(b)} + 4tcos\theta cosk}{2}\right) \pm \sqrt{\left(\frac{\epsilon_n^{(a)} - \epsilon_n^{(b)} - 4tsin\theta sink}{2}\right)^2 + t_n^2}. \qquad (9)$$

For the experimental situation of low carrier concentration and low temperatures only the lower subband at the end channels will be occupied, and so we consider the scattering problem of a single propagating mode incident from the left . Our complete treatment includes one evanescent mode in each end region, two evanescent and two propagating modes in the middle region, and, of course, the propagating mode in the two end regions corresponding to the incident , reflected and transmitted amplitudes.

Matching the two-component wave function and its derivative at the two boundaries separating end regions from the middle regions, we solve for the transmission coefficient for a given energy (the Fermi energy). The computed transmission coefficient (see Fig.2) exhibits resonances characteristic of finite size systems together with an overall monotonic decrease. It should be noted that while the evanescent modes do not contribute to the transmitted current *per se*, they do affect indirectly the trasmission coefficient through the matching conditions. The local piling up of charges at the boundaries due to the evanescent modes does not affect the difference in the chemical potentials due to strong screening effects. In contrast with the conclusion of Ref.7 we also find that even in the perfectly symmetrical case ($\epsilon^{(a)} = \epsilon^{(b)}$) small modulations of the transmission coefficients can be obtained. This is so because of two reasons. First, we have included tunneling between the channels and second, our solution contains the effect of the magnetic field not only in the middle region but also in the end regions. For samples of characteristic lengths $L \sim \mu m$ the magnetic field cannot be confined to the middle region, thus affecting the incident and scattered amplitudes. In our solution this implies that the propagating wave vector of the incident wave is affected by the magnetic field, giving rise to an oscillating behaviour which is not strictly periodic in the flux transversing the middle region between channels. The immediate consequence of tunneling between channels is a bonding anti-bonding type spliting of the eigenstates (subbands). For zero magnetic field, when only the lower subbands are occupied, transport is strictly single moded in the sense of the

unoccupied bands playing no role in transport because of parity: lower (upper) bands correspond to states with even (odd) symetry in the y direction. When the magnetic field is turned on this symmetry is broken, the states are no longer of even or odd parity, and the evanescent modes corresponding to the upper bands come into action. This is imposed by the matching conditions at the boundaries between regions. Although transport occurs through a single propagating mode, the presence of the evanescent modes converts the problem into a multimoded one (an exponentially decreasing function can be expressed as a Fourier superposition of travelling waves). In general in the multimode case a semiclassical path integral calculation can be done to evaluate the transmission coefficient T. In this approximation T is the square modulus of the sum of paths tansversing the middle region, each path contributing with a phase given by the integral of the potential vector (see equation 1). This gives

$$T \sim sin^2(\pi S B/\phi_0)/B^2 \tag{10}$$

where S is the average area enclosed by a typical pair of paths; in

Fig. 1(a) such area is $S = L(W+b)/2$. In Ref.3 a systematic monotonic decrease is reported, the magnitude of the oscillations being small ($\Delta\sigma/\sigma \sim 0.08\%$). In addition to the mentioned reasons the origin of this behaviour can be the asymmetry between channels which inhibits large oscillations . Note that in the extreme asymmetric case transport takes place only through the lower energy branch (the paths through the upper energy branch contribute in a negligible amount) and the interference is destroyed.

In conclusion we have proposed and solved a discrete model for ballistic transport through two coupled channels in a transverse magnetic field taking into account the inter-channel tunneling and the evanescent modes. The calculated transmission coefficient shows *small* modulations which is atributable to the tunneling between channels.

ACKNOWLEDGMENTS

The authors would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for financial support and hospitality at the International Centre for Theoretical Physics, Trieste. One of the authors (A.G.R.) is also supported by the Consejo Nacional del Investigaciones Cientificas y Tecnicas (CONICET, Argentina).

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FIGURE CAPTIONS

FIG. 1. (a) Restricted structure suitable for observation of oscillatory magnetoconductance. In each region there is translational invariance in the x direction. The potential V' in the "hole" in the middle region is not necesarily infinite thus allowing for tunneling of electrons between channel a and channel b. (b) Scheme of the discrete model proposed. The dots represent localized orbitals and the links between dots represent hopping matrix elements: vertical links correspond to t_n and horizontal links correspond to complex matrix elements given by Eq. (1). The magnetic flux traversing each plaquet is ϕ.

Fig. 2. Calculated transmission coefficient as a function of the total flux traversing the middle region, *i.e.*, $\Phi = N\phi/\phi_0$, where N is the number of plaquets in the middle region. The parameters were chosen so as to reproduce qualitatively the experimental situation of a single *propagating* mode in each region (for the definition of parameters see Eqs. 2-4): (a) $E/t = 2.6$, $\epsilon/t = 0.55$, $t'/t = 0.3$, $N = 50$. (b) $E/t = 2.8$, $\epsilon/t = 0.8$, $t'/t = 0.1$, $N = 100$. (c) $E/t = 2.7$, $\epsilon/t = 0.7$, $t'/t = 0.1$, $N = 100$. In each case t was taken negative.

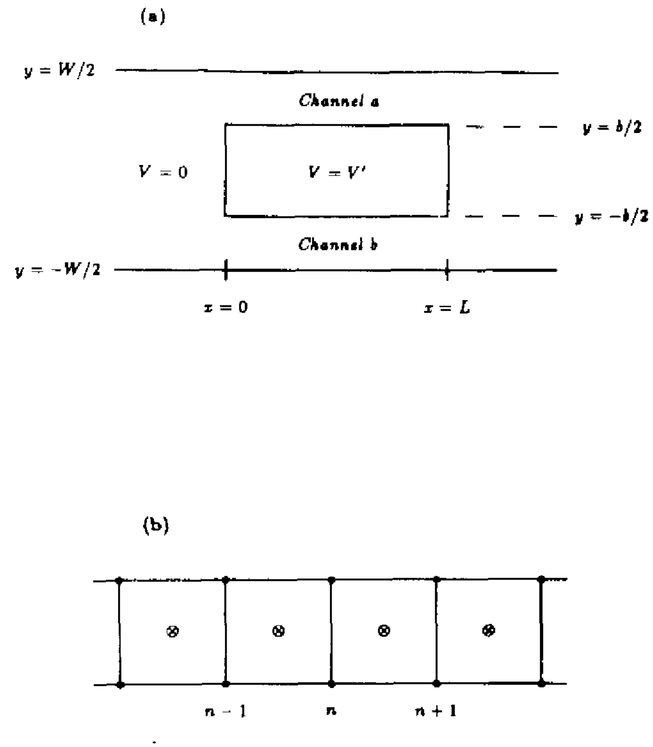

Fig. 1

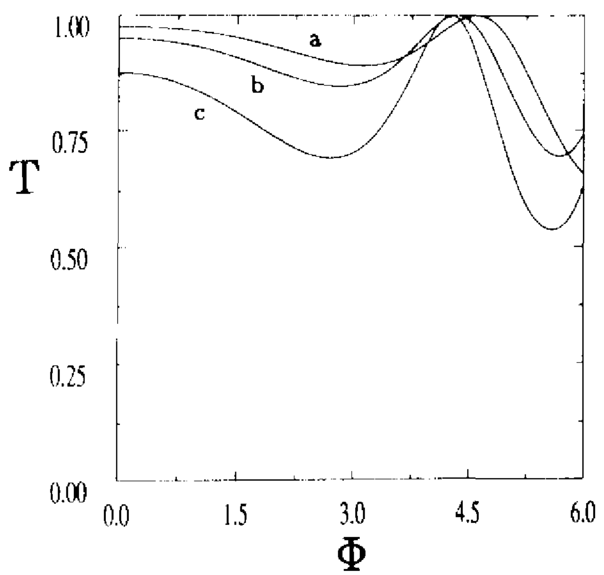

Fig.2