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CLUSTER EXPANSION
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Colored particle Green functions in vacuum background random fields are written as path integrals. Averaging over random fields is done using the cluster (cumulant)expansion . The existence of a finite correlation length for vacuum background fields is shown to produce the linear confinement, in agreement with the results, obtained with the help of averaged Hamiltonians. A modified form of cluster expansion for nonabelian fields is introduced using the path-ordered cumulants.

Fig. - , ref. - 9

1. Introduction

A new confining mechanism has been suggested recently [1-4], which is due to strong random vacuum gluonic fields. As was discussed in [1 - 3], those vacuum background fields confine any colored object, a quark or a gluon, and moreover in the white system between colored constituents there occurs a linear potential even before perturbative gluon exchanges are taken into account [2, 3]. For a quark and an antiquark in nonabelian vacuum fields one has to introduce a local white $q\bar{q}$ state, referred e.g. to their center of mass; it is this state that propagates in the vacuum, while the colored (octet) state does not propagate [3].

In [1-3] a simple method of averaging the Hamiltonian over random vacuum field configuration was used, and in this way equations for averaged Green functions have been obtained, containing the averaged Hamiltonian. It has been implied in [1 - 3], but not explained, that the averaging of the Hamiltonian is a first step of a more systematic procedure. This procedure has been used in [4] to derive a linear confining potential between heavy quark and antiquark in a random Euclidean colorelectric field and is well known in the theory of random processes and the theory of solids as the cluster or the cumulant expansion [5].

The essential assumptions about the character of vacuum background fields in [1 - 3] and [4] are the same, as well as the conclusion about the existence of a linear confining potential for heavy quarks.

The euristic arguments in [1 - 3] concern the confinement of any colored objects, quarks or gluons, and use the properties of averaged Hamiltonians. The present paper treats the same subject, i.e. the behaviour of colored objects in an external random field, but we use here a different method - the Green functions are written in the form of a path integral and the cumulant expansion is used to average over random background field configurations. In this way we formulate a general systematic expansion to obtain effective Lagrangians of colored particles in a random background field. Using the cluster (cumulant) expansion we confirm in Section 2 the results of [2] on confinement of a nonrelativistic charge in a stochastic(in space)magnetic field. We also show, that taking into account higher cumulants does not change qualitatively the behaviour of Green functions. We discuss the parameter of the cumulant expansion and show that it converges for $Bd^2 \lesssim 1$, where B is an average value of the field strength, and d is a correlation length.

In Section 3 we discuss the quark and gluon Green functions. To this end we use the proper-time representation, which enables one to write the Green function as a path integral and then to average over random fields using the cluster expansion.

For some purposes, e.g. to calculate an average of the Wilson loop, one must use the cluster expansion of a path-ordered exponential of the nonabelian field,

$$P\exp\left(ig\int A_\mu dx_\mu\right)$$

The ordinary cumulants [5] are not convenient for this purpose, since they involve complicated expressions, and they should be replaced by the "path-ordered cumulants", introduced in Section 4 . The contribution of bilocal path-ordered cumulants is written here explicitly. Those cumulants have been exploited in a common paper by H.G.Dosch and the author to deduce the area law of the Wilson loop (to be published).

Section 5 is devoted to the behaviour of external currents in random background fiels. As an illustration we deduce here a linear potential between heavy quarks in analogy with the result of [4] .

In Conclusions main results and prospectives of the method are summarized.

2. Propagation of a nonrelativistic charged particle in a stochastic background field

We consider in this Section a charged nonrelativistic particle in the external magnetic field $A_\mu(x)$. The time-dependent Green function in the Euclidean space can be written as a functional integral [6]

$$G(X,0;T/A_\mu)=\langle X|e^{-\tau H}|0\rangle=\int[dx]e^{-\int_0^T L(\tau)d\tau} \qquad (1)$$

with

$$H=(p_\mu-eA_\mu)^2 \quad ,(M=1/2). \qquad (2)$$

$$L(\tau)=\frac{\dot{x}_\mu^2}{4}-ieA_\mu(x)\dot{x}_\mu \qquad (3)$$

and

$$[dx]=\lim_{\substack{N\to\infty\\ \varepsilon N=T}}\prod_{n=1}^{N}\frac{dx_\mu(n)}{\sqrt{2\pi\varepsilon}} \qquad (4)$$

The integral (1) is over all trajectories with $x(\tau=0)=0$; $x_\mu(\tau=T)=X_\mu$.

Note, that the electromagnetic field A_μ enters the functional integral (1) only through the phase integral along a trajectory (nonlinear, in general)

$$\mathcal{P}(X,0)=\exp\left(ie\int_0^X A_\mu(z)dz_\mu\right) \qquad (5)$$

Before doing any averaging over the field A , one must separate the gauge dependent factor, namely the phase factor $\mathcal{P}_0(X,0)$ over some fixed, e.g. linear, trajectory out of the Green function

$$G(X,0;T/A_\mu)=\mathcal{P}_0(X,0)f(X,0;T/A_\mu) \qquad (6)$$

The remaining part, f, is gauge invariant.

An equivalent way is to use the so-called coordinate (Fock-Schwinger) gauge $x_\mu A_\mu(x)=0$, in which case automatically the phase factor over the linear trajectory $\Phi_c(x,o)=1$. In this gauge one can write

$$A_\mu(x)=\int_0^1 \alpha\, d\alpha\; F_{\nu\mu}(\alpha x)\, x_\nu = \int_0^x \alpha(z)\, dz_\nu\, F_{\nu\mu}(z), \quad x\,\alpha(z)=z \qquad (7)$$

Assume now, that the external field $A_\mu(x)$ is a stochastic variable, i.e. $A_\mu(x)$ belongs to an ensemble of fields, with the probability measure $d\mu(A)$. This situation is generic in the field-theory case, where one should integrate eventually the Green function found in a given e.m. field over all fields $A_\mu(x)$ with a prescribed measure. Here we shall not specify the integration measure $d\mu(A)$ and consider the averaged Green function

$$\begin{aligned}\langle G(X,0;T)\rangle &\equiv \int d\mu(A)\, G(X,0;T/A_\mu)= \\ &= \int d\mu(A)\int[dx]\exp\Big[-\int_0^T\Big(\tfrac{1}{4}\dot{x}_\mu^2(\tau)-ieA_\mu(x(\tau))\dot{x}_\mu(\tau)\Big)d\tau\Big]= \\ &= \int d\mu(A)\int[dx]\exp\Big[-\int_0^T\tfrac{1}{4}\dot{x}_\mu^2\,d\tau+\tfrac{ie}{2}\int_\Delta d\sigma_{\mu\nu}(z)\,F_{\nu\mu}(z)\Big].\end{aligned} \qquad (8)$$

In (8) we have used the property that

$$\alpha(z)\, dz_\nu\, d\alpha_\mu = d\sigma_{\mu\nu} \qquad (9)$$

The surface $\Delta(x)$ in (8) is formed by the straight lines from the origin to all points of the given trajectory.

The averaging in (8) can be done using the cluster expansion [5] valid for any stochastic function $\xi(s)$:

$$\langle \exp\Big(i\int_0^t \xi(s)\,ds\Big)\rangle = \exp\sum_{n=1}^{\infty}\frac{i^n}{n!}\int_0^t\ldots\int_0^t ds_1\ldots ds_n \langle\langle \xi(s_1)\ldots\xi(s_n)\rangle\rangle \qquad (10)$$

where the usual average (with $d\mu(A)$) is denoted with single brackets, while the so-called cumulants $\langle\langle \xi(s_1)\ldots\xi(s_n)\rangle\rangle$ are defined as follows.

$$1 = 1 \tag{11}$$

$$12 = 1 \quad 2 + 12$$

$$123 = 1 \quad 2 \quad 3 + 1 \quad 23 + 12 \quad 3 + 13 \quad 2 + 123$$

Applying (10) to our Green function (8) we obtain

$$\langle G(X,0;T)\rangle = \int [dx] \exp\Big(-\int_0^T \tfrac{1}{4}\dot{x}_\mu^2 d\tau\Big) \cdot \exp \Xi \tag{12}$$

where

$$\Xi = \sum_{m=1}^{\infty} \frac{(ie)^m}{m!} \int_\Delta d\sigma_{\nu_1\mu_1}(y_1) \ldots \int_\Delta d\sigma_{\nu_m\mu_m}(y_m) \langle\langle F_{\nu_1\mu_1}(y_1)\ldots F_{\nu_m\mu_m}(y_m)\rangle\rangle \tag{13}$$

Assume now that the statistical ensemble of fields respects Euclidean $O(4)$ and translational invariance, then all cumulants depend on relative distances and*)

$$\langle\langle F_{\nu\mu}(y)\rangle\rangle = 0 \tag{14}$$

$$\langle\langle F_{\nu\mu}(y) F_{\rho\sigma}(y')\rangle\rangle = (\delta_{\nu\rho}\delta_{\mu\sigma} - \delta_{\nu\sigma}\delta_{\mu\rho})\langle\langle F^2(o)\rangle\rangle D(y-y') +$$
$$+ (h_\nu h_\rho \delta_{\mu\sigma} + h_\mu h_\sigma \delta_{\nu\rho} - h_\nu h_\sigma \delta_{\mu\rho} - h_\mu h_\rho \delta_{\nu\sigma}) D_r(y-y'),$$
$$h_\nu = y_\nu - y'_\nu . \tag{15}$$

It is natural to assume furthermore that $D(z)$ and all higher-order cumulants decrease fast enough, so that all integrals in (13) converge. If the characteristic distance (correlation length) for the cumulants is denoted by d (so that e.g. $D(z) \sim \exp(-|\frac{z}{d}|^\alpha)$) and the average field strength is denoted by B, $\langle F^2(o)\rangle \sim B^2$, then the expansion (13) is in powers of Bd^2.

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*) The second term in (15) has been suggested to the author by M.B.Voloshin. In what follows only the contribution of $D(z)$ is retained, since the effect of $D_r(z)$ is the same. The Bianchi identity connects $D(z)$ and $D_r(z)$ in such a way that confinement occurs only for nonabelian fields and abelian fields with monopole charges (for details see the aforementioned paper by H.G.Dosch and the author).

One may expect a good convergence of this expansion for $Bd^2 \lesssim 1$ due to fast decreasing numerical coefficients in (13), but the situation in general may depend on the details of the stochastic ensemble.

Let us take the first nonzero term of the expansion (13) and study the properties of the resulting Green function which has the form

$$\langle G(X,0,T)\rangle = \int [dx] \exp\Big[-\int_0^T \frac{\dot{x}^2}{4} d\tau - \frac{e^2}{8} \iint_{\Delta\Delta} d\sigma(1)\, d\sigma(2) \langle\langle F(1) F(2) \rangle\rangle\Big] \tag{16}$$

Our next aim is to calculate the functional integral (16) and to this end we rewrite the exponent in (16) using (15) in the form

$$-S = -\int_0^T \frac{\dot{x}^2}{4} d\tau - \frac{e^2}{2} \langle\langle F^2(0) \rangle\rangle \int_0^T\!\!\int_0^T [\, \dot{x}_\mu(\tau')\dot{x}_\mu(\tau'')\, x_\nu(\tau') x_\nu(\tau'') -$$
$$- \dot{x}_\mu(\tau') x_\mu(\tau'') \dot{x}_\nu(\tau'') x_\nu(\tau')] \mathcal{K}\big(x(\tau'), x(\tau'')\big)\, d\tau' d\tau'' \tag{17}$$

where we have defined

$$\mathcal{K}(x,y) \equiv \int_0^1 \alpha\, d\alpha \int_0^1 \beta\, d\beta\; D(\alpha x - \beta y) \tag{18}$$

We are interested in the large-distance behaviour of the Green function and for that purpose we need the properties of $\mathcal{K}(x,y)$ at large x, y. For $D(y)$ which falls off at large $|y|$ faster than $|y|^{-4}$, we find

$$\mathcal{K}(x,y) \sim \frac{\bar{c}\, d}{|x+y|}, \quad |x-y| < d,\ |x| \gg d, \quad \text{or } (\vec{x},\vec{y},0) \text{ on a straight line,} \tag{19}$$

$$\mathcal{K}(x,y) \sim \frac{\bar{c}'\, d^4}{|\vec{x}-\vec{y}|^4}, \quad |\vec{x}-\vec{y}| \gg d, \tag{20}$$

where d is the correlation length in $\mathcal{D}(y)$. From (19)-(20) we conclude that the main contribution in the integral (17) in the asymptotics $|X| \to \infty$, $T \to \infty$ is from the region where $x(t') \approx x(t'')$, that is from $\tau' \approx \tau''$.

To actually perform functional integral we split the time interval $[0, T]$ into N pieces of length ε each and use (4). We also introduce new variables

$$\xi_\mu(n) = x_\mu(n) - x_\mu(n-1), \quad n = 1, \dots N$$

$$x_\mu(N) = X_\mu, \quad x_\mu(0) = 0, \quad \sum_{n=1}^{N} \xi_\mu(n) = X_\mu \tag{21}$$

Inserting the unity in (16)

$$1 = \int d^k\xi(N)\, \delta\Big(\sum_{n=1}^{N} \xi(n) - X\Big) = \int \frac{d^k p}{(2\pi)^k}\, e^{ip\left(\sum_1^N \xi(n) - X\right)} \tag{22}$$

and having in mind that $\frac{\xi}{\varepsilon} = \dot{x}_\mu(n) = \frac{1}{\varepsilon}\left(x_\mu(n) - x_\mu(n-1)\right)$ we obtain the integral

$$\langle G(X,0;T)\rangle = \int [d\xi]\, \frac{d^k p}{(2\pi)^k}\, e^{-\sum^N \frac{\xi_\mu^2(n)}{4\varepsilon} + ip\left(\sum^N \xi(n) - X\right) - \frac{\varepsilon^2}{2} \langle\langle f^2(n) \rangle\rangle \theta} \tag{23}$$

with $[d\xi] = \int \prod_n \frac{d^k\xi(n)}{(2\pi\varepsilon)^{k/2}}$, and

$$\theta = \sum_{n,n'=1}^{N} \sum_{m=1}^{n} \sum_{m'=1}^{n'} [\xi_\mu(n)\xi_\mu(n')\xi_\nu(m)\xi_\nu(m') - \\ - \xi_\mu(n')\xi_\mu(m)\xi_\nu(n)\xi_\nu(m')] \mathcal{K}\left(\sum_{m=1}^{n}\xi(m), \sum_{m'=1}^{n'}\xi(m')\right); \tag{24}$$

when $\theta = 0$ one can easily perform integration over $[d\xi]\, d^k p$ in (23) to obtain the standard result for the free Green function

$$G(X, 0; T) = \left(\frac{M}{2\pi T}\right)^{k/2} \cdot \exp\left(-\frac{MX^2}{2T}\right) \tag{25}$$

In case of confining potentials of the type $V(x) \sim |x|^\beta$ one can also rather easily calculate the Green function by the method, described in Appendix, assuming $|X|^2 \gg MT$ We obtain

$$G(X, 0; T/V) \sim \exp\left(-\frac{MX^2}{2T} - 6X^\beta T\right)$$

Note, that in this way we get the dominant term in the asymptotics - the exponential, and not a preexponential factor. The method is applied in Appendix also to the case, when $\theta \neq 0$, as in (24).

In the general case (24) we obtain $(X, T \to \infty,\ X^2 \gg T),\ M = 1/2.$

$$\langle G(X,0,T)\rangle \cong \exp\left(-\frac{X^2}{4T} - e^2 \langle\langle F^2(0)\rangle\rangle |X| T d\right) \tag{26}$$

The last term in the exponent (26) can be associated with an equivalent potential contribution to the action, $S = \int_0^T V(z)\,d\tau$, which gives linear confining potential

$$V(x) = e^2 d\, |X| \langle\langle F^2(0)\rangle\rangle \tag{27}$$

Another way to visualize confinement is to compute from (26) the Green function in the energy representation, which can be done by the steepest descent method, yielding

$$G_E(X,0) \sim \exp\left(-\,const\, |X|^{3/2}\right) \tag{28}$$

which signals again the presence of a linear confining potential. This behaviour agrees with that obtained by averaging the Hamiltonian in Section 1 and in [2].

Consider now the contribution of higher-order correlators in Ξ, Eq.(13). Asuuming that all cumulants decrease fast enough in all relative distances, e.g.

$$\langle\langle F_{\nu_1\mu_1}(y_1) \ldots F_{\nu_n\mu_n}(y_n)\rangle\rangle \lesssim \prod_{i<k} |y_i - y_k|^{-b}, \tag{29}$$

with $b > 4$, one can easily see, that every additional integration over $d\sigma(y_n)$ in (13) is effectively damped by the fall-off of the cumulant (29) in the variable y_n, and the integral (13) reduces to the double integral $\iint d\sigma(y_1)\, d\sigma(y_2)\, C(y_1 - y_2)$, already considered above. Therefore all cumulants effectively contribute to the linear confining potential, if the series

(13) converges, and the parameter of expansion is, as discussed above, Bd^2.

3. Relativistic quarks and gluons in the vacuum background field

For a relativistic scalar particle in the external field the Green function satisfies an equation

$$\left[\left(-i\frac{\partial}{\partial x_\mu}-eA_\mu\right)^2+m^2\right]\mathcal{G}(x,y)=\delta(x-y) \tag{30}$$

Following the method of Schwinger [7] we consider the proper-time representation of $\mathcal{G}$:

$$\mathcal{G}=\left[(p-eA)^2+m^2\right]^{-1}=\int_0^\infty ds\cdot e^{-s[(p-eA)^2+m^2]} \tag{31}$$

and for the integrand we use the functional representation (4), where now x_μ is a 4-vector and the time τ is replaced by the proper time S. All formulas Eqs.(1-28) are valid in this relativistic case, but (26) is trivially changed into

$$\langle G(X,0;s)\rangle=\exp\left(-\frac{X^2}{2s}-c|X|s-m^2s\right) \tag{32}$$

Finally the integral (31) again computed by the steepest descent method yields

$$\langle G(X,0)\rangle\sim\exp\left(-\sqrt{2}\,|X|\sqrt{m^2+c|X|}\right) \tag{33}$$

The large-$|X|$ behaviour (33) displays the linear confinement of a relativistic particle in a stochastic external field.

The gluon Green function in the background vacuum field $D_{\mu\nu}(x,y)$ and the quark Green function $S(x,y)$ can be treated on the same footing if one introduces instead of S the quadratic (in D_μ) operator, namely [1, 7]

$$S = (-i\gamma_\nu D_\nu + im) G^q(x, y) \qquad (34)$$

then we can write

$$G^q(x,y) = \langle x | (m^2 - D^2 + g\sigma_{\mu\nu} F^a_{\mu\nu} t^a)^{-1} | y \rangle \qquad (35)$$

$$D_{\mu\nu}(x,y) = \langle x | (-\hat{D}^2 \delta_{\mu\nu} + 2g F^a_{\mu\nu} T^a)^{-1} | y \rangle \qquad (36)$$

where

$$t^a = \frac{\lambda^a}{2}, \; [T^a]_{bc} = i f_{bac}, \; \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu] \qquad (37)$$

$$D_\mu = \partial_\mu - igA_\mu, \; \hat{D}^2 = \hat{D}_\mu \hat{D}_\mu; \; \hat{D}_\mu f = \partial_\mu f - ig[A_\mu, f] \qquad (38)$$

We can now use the proper-time representation for (31) for the Green functions G^q and $D_{\mu\nu}$ and repeat all that was told above about relativistic Green functions with the final result (33). The only difference for nonabelian fields is that the phase integral (5) should be treated as a P-ordered exponential. The expression (7) in the Fock-Schwinger gauge is valid also for the nonabelian case. In the next Section we show, that with some modifications the cluster expansion of the type of (10) exists for the most general case, when $\xi(s)$ are noncommuting matrices. Hence we conclude , that the existence of a finite correlation length in the correlators (cumulants) (13) leads to the confinement of quarks and gluons. At this point it is important to stress that the gluon confinement actually requires the full gauge invariant correlator

$\langle F_{\mu\nu}(x) P\exp(ig\int_y^x A_\mu(z)dz_\mu) F_{\mu'\nu'}(y)\rangle \equiv G_2(x,y)$ to decrease faster than an exponent, i.e. [1] $G_2(x,y) \sim \exp(-|\frac{x-y}{d}|^\alpha), \alpha > 1$. Here $F_{\mu\nu}$ includes both vacuum background and gluonic fields.

Since the latter cannot be separated unambigiously, we require that the correlator $G_2(x,y)$ for vacuum background fields alone should fall off like $\exp\left(-\frac{|x-y|^{3/2}}{d^{3/2}}\right)$. As we have seen above in this Section, this property ensures that in this case both Green functions (35), (36) will have the same type of fall-off, hence the full $G_2(x,y)$ for both background fields and gluons decreases the same way and the resulting picture of confinement is internally consistent.

4. The cumulant expansion for nonabelian fields

In case of nonabelian fields the cumulant expansion (10) should be written for the P-ordered exponent, instead of the ordinary exponent on the l.h.s. of (10); the generic expression for the integral of a matrix-valued stochastic function $A(s)$ can be written in two different forms

$$P\exp\left(ig\int_0^t A(s)ds\right)=\sum_{n=0}^{\infty}(ig)^n\int_0^t ds_1\int_0^{s_1}ds_2\ldots\int_0^{s_{n-1}}ds_n\,A(s_1)A(s_2)\ldots A(s_n) \tag{39}$$

$$=\sum_{n=0}^{\infty}\frac{(ig)^n}{n!}\int_0^t ds_1\int_0^t ds_2\ldots\int_0^t ds_n\,P\{A(s_1)A(s_2)\ldots A(s_n)\} \tag{40}$$

where the symbol P places factors $A(s_i)$ in the order of decreasing argument $s_1 \ldots s_n$ from left to right.

Taking an average over stochastic distribution of $A(s)$ in both sides of (39-40) one could introduce the usual cumulants (11) also in our case of noncommuting variables, as is discussed in [5]. The answer suggested in ([5], Eqs.(12.2) and (12.3)) is essentially the same as before, Eq.(10) with an additional sign of the P-ordering. On this way one encounters a difficulty which casts some doubt on the direct use of the usual cumulant expression for non-commuting variables. Namely, in the cumulants (11) there appear averages of matrices, taken from different places of the matrix product and the final sum over the matrix indices in the product (40) is highly complicated. In some simple cases, like SU(2) matrices, one can easily check, that after averaging one does not recover simple exponential form, as in (10). In a similar situation this difficulty has appeared in the Wilson loop calculation in [8].

We show below that the very nature of the P-ordered exponent (36) suggests another definition of cumulants, which easily solves the difficulty and under additional assumptions $(d \to 0)$ brings back the same simple expression for the two-point cumulant contribution, $A(s_1)A(s_2)$, as in (10).

To define new, or path-ordered cumulants, we first remember, that the main purpose of introducing the cumulants is to obtain the objects which decrease fast enough whenever any relative distance inside a cumulant tends to infinity. The definition (11) satisfies this property for any splitting into clusters, including e.g. for triple cumulants $\langle 123 \rangle \to$ $\to \langle 13 \rangle \langle 2 \rangle + \langle 12 \rangle \langle 3 \rangle + \langle 1 \rangle \langle 23 \rangle$. However, the ordering in (39) prescribes that splitting is possible within a given order $s_1 > s_2 > \ldots > s_n$, so that e.g. $\langle 123 \rangle$ cannot be split into $\langle 13 \rangle$ and $\langle 2 \rangle$. Therefore we propose the following definition of cumulants in the case of the path-ordered exponents:

$$\langle 1 \rangle = \langle\langle 1 \rangle\rangle$$

$$\langle 12 \rangle = \langle\langle 12 \rangle\rangle + \langle\langle 1 \rangle\rangle \langle\langle 2 \rangle\rangle$$

$\langle 123\rangle = ((123)) + ((12))((3)) + ((1))((23)) + ((1))((2))((3))$ (41)

Higher terms $\langle 12\ldots n\rangle$ contain all splittings on the r.h.s. respecting the ordering of the symbol $12\ldots n$. Note, that the first two path-ordered cumulants $((1))$ and $((12))$ in (41) coincide with usual cumulants (10). Now assume, that $((1))=0$ and we are interested only in the contribution of the pair correlators,

$$((A(s_i)A(s_j))) = \Psi(s_i, s_j) \qquad (42)$$

then (39) has the form

$$\langle P \exp(ig\int_0^t A(s)ds)\rangle = \sum_{n=0}^{\infty}(ig)^n \int_0^t ds_1 \int_0^{s_2} ds_2 \, \Psi(s_1,s_2)\ldots \times$$

$$\times \int_0^{s_{n-2}} ds_{n-1} \int_0^{s_{n-1}} ds_n \, \Psi(s_{n-1}, s_n) \qquad (43)$$

In the most interesting cases $\Psi(s_1, s_2)$ is a commuting matrix, i.e. it is diagonal or proportional to a constant matrix. We shall also be interested in the case, when $\Psi(s_1,s_2)$ is a fast decreasing function of its argument, $\Psi(s_1-s) < |s|^{-1-\gamma}, \gamma>0$, for $|s| \gg d$. Then the effective region of integration consists of $\frac{n}{2}$ double integrals $\iint ds_{k-1}\, ds_k \, \Psi(s_{k-1}, s_k)$ with $|s_{k-1} - s_k| \lesssim d$ and one can rewrite the r.h.s. of (43) in the form

$$\langle P \exp(ig\int_0^t A(s)ds)\rangle = \sum_{k=0} \frac{(ig)^{2k}}{k!\,2^k}\left(\int_0^t ds_1 \int_0^t ds_2 \, \Psi(s_1,s_2)\right)^k + \ldots$$

$$= \exp\left(-\frac{g^2}{2!}\int_0^t ds_1 \int_0^t ds_2 \, \Psi(s_1, s_2) + \ldots\right) \qquad (44)$$

where dots stand for the omitted contribution of triple and higher order correlators.

Note that (44) coincides with the double correlator contribution in (10), but the form (44) is correct only for fast decreasing and commuting $\Psi(\xi,\xi)$ as discussed above.

5. Quark external currents in the vacuum background fields

In this Section we consider the grand partition function Z for the quarks, gluons and external currents in the vacuum fields $\bar{A}_\mu$, treating the latter as a stochastic ensemble of strong (of the order $O(1/g)$) fields.

First we disregard the influence of light quarks and gluons, since they contribute only to the order $O(g^0)$, and calculate Z for external currents (e.g. heavy quarks) in the vacuum fields $\bar{A}_\mu$. We absorb ghost and gauge-fixing terms into the stochastic measure $d\mu(\bar{A})$ and write Z as

$$Z = \int d\mu(\bar{A}) \exp\left(-\int J_\mu^a(x)\, A_\mu^a(x)\, d^4x\right) \qquad (45)$$

The expression in the exponent withput quark kinetic energy terms is not gauge invariant even for a covariant conserving current $J_\mu(x)$, but for our purpose of qualitative understanding we can keep it as it is.

J_μ should be understood as a sum of a heavy quark and antiquark currents which are nonzero along their trajectories. Since $Tr(J_\mu \bar{A}_\mu)$ is a commuting variable, we can use the usual cumulant expansion (10); keeping only two-point correlators we have

$$Z \sim \exp \frac{1}{2!} \int J_\mu^a(x) J_\nu^b(y) \langle\langle A_\mu^a(x) A_\nu^b(y) \rangle\rangle dx dy \tag{46}$$

It is convenient to use the modified coordinate gauge [9] for $A_\mu(x)$ with the center at the point x_0 to be specified later:

$$A_4(x,\tau) = \int_0^1 d\alpha\, F_{\nu\mu}[\alpha(x-x_0)+x_0,\tau](x-x_0)_\nu = \int_{x_0}^{x} F_{\nu 4}(z) dz_\nu$$

$$A_K(x,\tau) = \int_0^1 \alpha d\alpha\, (x-x_0)_\ell F_{\ell K}[\alpha(x-x_0)+x_0,\tau] \tag{47}$$

For the correlator of $A_\mu(x)$, $A_\nu(y)$ we obtain

$$\langle\langle A_\mu^a(x) A_\nu^b(y) \rangle\rangle = \int_{x_0}^{x} \alpha(z) \int_{x_0}^{y} \alpha(z') dz_\rho dz'_\sigma \langle\langle F_{\rho\mu}^a(z) F_{\sigma\nu}^b(z') \rangle\rangle \tag{48}$$

The expression (48) is not gauge invariant, unless x_0 lies on the straight line, connecting x and y, so that additional factors

$$\hat{\Phi}(z',z) = P \exp ig \int_z^{z'} A_\mu(s) ds_\mu \tag{49}$$

when inserted in (45) to make it gauge invariant, are equal to unity. This situation is realized for fixed sources (heavy quark and antiquark) at $\vec{x} = \vec{r}$ and $-\vec{r}$

$$x = (\vec{r},\tau)\ ,\ (-\vec{r},\tau). \qquad J_i = 0, \quad i = 1,2,3.$$

$$J_4^a = ig\,[q^a \delta^{(3)}(\vec{x}-\vec{r}) + \bar{q}^a \delta^{(3)}(\vec{x}+\vec{r})] \tag{50}$$

and x_0 can be chosen at the center of mass $x_0 = (0,\tau)$. Now one can introduce a gauge invariant correlator

$$\langle\langle \hat{\Phi}(z',z) F_{\rho\mu}(z) \hat{\Phi}(z,z') F_{\sigma\nu}(z') \rangle\rangle \equiv \mathcal{G}_{\rho\mu,\sigma\nu}(z,z') \frac{\hat{1}}{N_c} = \frac{1}{N_c}\left(\delta_{\rho\sigma}\delta_{\mu\nu} - \delta_{\rho\nu}\delta_{\mu\sigma}\right)\frac{1}{12}\, tr F^2(0) \cdot D(z-z') \tag{51}$$

Having in mind that $\hat{\Phi}(z,z') = \hat{\Phi}(z',z) = 1$ on the lines connecting $\vec{x}=+\vec{r}$, $-\vec{r}$ and $\vec{x}=0$, and with the halp of the relation

$$\sum_a t^a t^a = \frac{N_c^2-1}{2N_c} \hat{1} \tag{52}$$

we can express the correlator entering (45) as follows

$$\langle\langle F^a_{\rho\mu}(z) F^b_{\sigma\nu}(z') \rangle\rangle = \frac{2\delta_{ab}}{N_c^2-1} \mathcal{G}_{\rho\mu,\sigma\nu}(z,z') \tag{53}$$

Using (50) one obtains in the exponent of (46) a sum of combinations $q^a(i) q^a(j) \langle\langle A_4(x_i,x_4) A_4(x_j,x_4) \rangle\rangle$. This sum can be further simplified in the case of the total zero chrge, $Q^2 = (q^a+\bar{q}^a)(q^a+\bar{q}^a)=0$. Eq.(46) can be rewritten in the form

$$Z \approx \exp \frac{g^2}{2} \int_0^T d\tau \int_0^T d\tau' \frac{2q^a\bar{q}^a}{(N_c^2-1)} \int_{-\vec{r}}^{\vec{r}} dz_\nu \int_{-\vec{r}}^{\vec{r}} dz'_\nu \frac{1}{12} tr(F^2(0)) \cdot D(z-z') \tag{54}$$

Assuming that $D(z)$ falls off at large $|z|$ faster than $|z|^{-2}$, we can integrate $D(z-z')$ over relative time and distance

$$\int d(z'_\nu - z_\nu)\, d(\tau-\tau') D(z'-z) \equiv d^2 C \tag{55}$$

where d is the correlation length and C - some dimensionless constant. Now we expect to have in the exponent of (48) or (55) the effective action, or exp(-V(R)T), where V(R) is a potential between the external quark and antiquark separated by the distance R=2r . At large R we obtain from (54), (52) and (55)

$$V(R) = \frac{g^2 R}{2N_c} \frac{1}{12} \operatorname{tr} F^2(0) \cdot d^2 C \tag{56}$$

which coincides with the result of [4].

6. Conclusions

We have studied in the present paper the cluster expansion in the framework of the path integral method and have applied it to the problem of charged (colored)-particle propagation in background stochastic fields. To use the cluster expansion method in the general case of nonabelian fields, one must introduce the path-ordered cumulants, as it was done in Section 4.

Comparing the results for averaged Green functions, calculated here via the path integrals and earlier with the help of averaged Hamiltonians [2-3], one concludes that the latter method (for quadratic in A_μ Hamiltonians) reproduces the first nonzero cumulant contribution, containing bilocal cumulants. In this approximation results of the present paper and earlier results [2 - 3] agree. We also show here, that higher terms of the cumulant (cluster) expansion do not change the qualitative conclusion about the confinement of charges in the stochastic background fields with a finite correlation length. Due to this mechanism quarks and gluons are confined in the vacuum, if the correlators of vacuum gluonic fields fall off fast enough at large distances, and we argue that they should decrease faster than an experimental exponential of the distance.

It is likely that the confinement mechanism described above is only the first stage in the overall picture of confinement, since the nonpropagation of gluons gives rise (due to the Gauss theorem) to the formation of tubes of color fields between a quark and an antiquark and in this way amplifies the confining force due to background fields. We remind the reader, that up to now we have neglected all gluonic fields except for vacuum background, and we must take the quark fields into account in the next order in g(F).

In our approach the cumulants are inputs and we make an assumption about their properties. In principle, the cumulants should be calculated in a selfconsistent system of equations, connecting all cumulants, which must follow from the minimum of the free energy of the vacuum. This topic will be discussed in a subsequent publication.

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Appendix

An estimation of the path integral (23).

The integral (23) can be rewritten in the form, where the effective range of integration over $\xi_\mu(n)$ and p_μ is made clear:

$$\langle G(X,0;T)\rangle = \int [d\xi_\mu] \frac{d^k p}{(2\pi)^k} \exp\left[-\sum_{n=1}^{N} \frac{M}{2\varepsilon}\left(\xi_\mu(n) - \frac{ip\varepsilon}{M}\right)^2\right] \times$$

$$\times \exp\left[-\frac{N\varepsilon}{2M}\left(p_\mu + i\frac{X_\mu M}{N\varepsilon}\right)^2 - \frac{MX^2}{2N\varepsilon}\right] \cdot \exp\left[-\frac{e^2}{2}\langle\langle F^2(0)\rangle\rangle \cdot \theta\right] \quad (A.1)$$

Using (A.1) one can rewrite ξ_μ, p_μ as

$$\xi_\mu(n) = \frac{ip\varepsilon}{M} + \tilde{\xi}_\mu(n) = \frac{X\varepsilon}{T} + \sqrt{\frac{2\varepsilon}{M}}\,\tilde{\xi}_\mu(n) + i\varepsilon\beta_\mu \sqrt{\frac{2}{MT}} \quad (A.2)$$

$$p_\mu = -\frac{iX_\mu}{T} \cdot M + \beta_\mu \sqrt{\frac{2M}{T}} \quad (A.3)$$

where $\tilde{\xi}_\mu(n)$ and β_μ have finite limit as $\varepsilon \to 0$ and are of the order of unity. Let us estimate now $x_\mu^2(n)$ for $X^2 \gg \frac{T}{M}$

$$x_\mu(n) = \sum_{m=1}^{n} \xi_\mu(m) = \frac{n}{N} X_\mu + \sqrt{\frac{2\varepsilon}{M}} \sum_{m=1}^{n} \tilde{\xi}_\mu(m) + i\sqrt{\frac{2}{MT}}\,\varepsilon n \beta_\mu ;$$

$$x_\mu^2(n) = \left(\frac{n}{N}\right)^2 X^2 + O\left(\sqrt{\varepsilon}, \varepsilon n\right) \quad (A.4)$$

We first check our method on the exemple of the Green function for potential $V(x) = c\,|x|^\beta$. In this case one should replace in (A.1) the last exponent in (A.1) with $-\sum_{n=1}^{N} \varepsilon V(|x_\mu(n)|)$. Taking into account that the integration over $\tilde{\xi}_\mu(n)$ is carried out in (A.1) independently and hence $\langle (\sum_n^k \tilde{\xi}_\mu(n))^2 \rangle \sim \sum_{n=1}^k \langle \xi_\mu^2(n) \rangle$, keeping the dominant terms in (A.4) and in $V(|x_\mu(n)|)$ and integrating over $[d\tilde{\xi}_\mu]\, d^k p$, we obtain

$$G(X,0;T/V) \sim \exp\left(-\frac{MX^2}{2T} - \int_0^T d\tau\, V\left(\sqrt{\left(\frac{\tau}{T}\right)^2 X^2 + \tau c}\right)\right) \qquad (A.5)$$

where C is a constant of the order of unity. For $V(x) = c\, x^\beta$ we have

$$G(X,0;T/V) \sim \exp\left(-\frac{MX^2}{2T} - b\, X^\beta T\right) \qquad (A.6)$$

Using the steepest descent method one obtains the Green function in the energy representation for that potential

$$G_E(X,0/V) \sim \exp\left(-c\, X^{1+\frac{\beta}{2}}\right) \qquad (A.7)$$

Therefore one can see that the method, using estimates (A.2)-(A.4) produces correct results for the dominant (exponential) term in the asymptotics of the Green function for the class of confining potentials.

We consider now our case of (A.1) when θ given by (24). As is seen from (A.2) and (A.4) the leading contribution of the order of $O(\varepsilon)$ occurs when we substitute $\xi_\mu(n) \sim \sqrt{\frac{2\varepsilon}{M}}\, \tilde{\xi}_\mu(n)$ and $x_\mu(n) \approx \frac{n}{N} X_\mu$. In this way we obtain

$$\theta \approx \sum_{n,n'=1}^{N} \tilde{\xi}_i(n)\, \tilde{\xi}_i(n')\, \frac{2\varepsilon}{M} \frac{nn'}{N^2} X^2 \mathcal{K}\left(\frac{n}{N} X_\mu, \frac{n'}{N} X_\mu\right) \qquad (A.8)$$

where the subscript i of $\tilde{\xi}_i$ runs over k-1 transverse values, while the longitudinal component of $\tilde{\xi}_\mu$ is defined as $\tilde{\xi}_{\parallel} = \frac{\tilde{\xi}_\mu X_\mu}{|X|}$.

Integrating in (A.1) over dp we come to the following form of the Green function:

$$\langle G(X,0;T)\rangle \approx \int [d\tilde{\xi}_{\parallel}][d\tilde{\xi}_{\perp}] \exp\Big\{-\sum_{n=1}^{N}\tilde{\xi}_{\parallel}^{2}(n) - \\ -\sum_{n,n'=1}^{N}\tilde{\xi}_i(n)\tilde{\xi}_i(n')\, b_{nn'} - \frac{M}{2}\frac{X^2}{T}\Big\} \qquad \text{(A.9)}$$

where we have defined

$$b_{nn'} \equiv \delta_{nn'} + a_{nn'} = \delta_{nn'} + \frac{e^2}{2}\langle F^2(0)\rangle \frac{2\varepsilon}{M}\frac{nn'}{N^2} X^2 \mathcal{K}\Big(\frac{nX}{N},\frac{n'X}{N}\Big) \qquad \text{(A.10)}$$

Integration over $[d\tilde{\xi}]$ yields

$$e^{\frac{MX^2}{2T}}\langle G(X,0;T)\rangle \sim \exp\Big(-\frac{k-1}{2}\ln\det\hat{b}\Big) = \exp\Big(-\frac{k-1}{2}\operatorname{Tr}\ln\hat{b}\Big) \qquad \text{(A.11)}$$

Furthermore we have

$$\operatorname{Tr}\ln\hat{b} = \operatorname{Tr}\ln(1+\hat{a}) = \operatorname{Tr}\hat{a} - \frac{1}{2}\operatorname{Tr}\hat{a}^2 + \ldots \qquad \text{(A.12)}$$

The first term of (A.12) can be easily calculated, using (19-20) :

$$\operatorname{Tr}\hat{a} = \frac{e^2\langle F^2\rangle}{2}\sum_n \frac{2\varepsilon}{M}\frac{n^2}{N^2}X^2\mathcal{K}\Big(\frac{nX}{N},\frac{nX}{N}\Big) \approx \frac{e^2\langle F^2\rangle}{2M} X T d\bar{c}$$

$$\langle G(X,0;T)\rangle \sim \exp\Big(-\frac{MX^2}{2T} - \frac{k-1}{2}\frac{X}{d}\frac{\omega T}{2}\Big) \qquad \text{(A.13)}$$

$$\omega = \frac{e^2\langle F^2\rangle}{M} d^2\bar{c}$$

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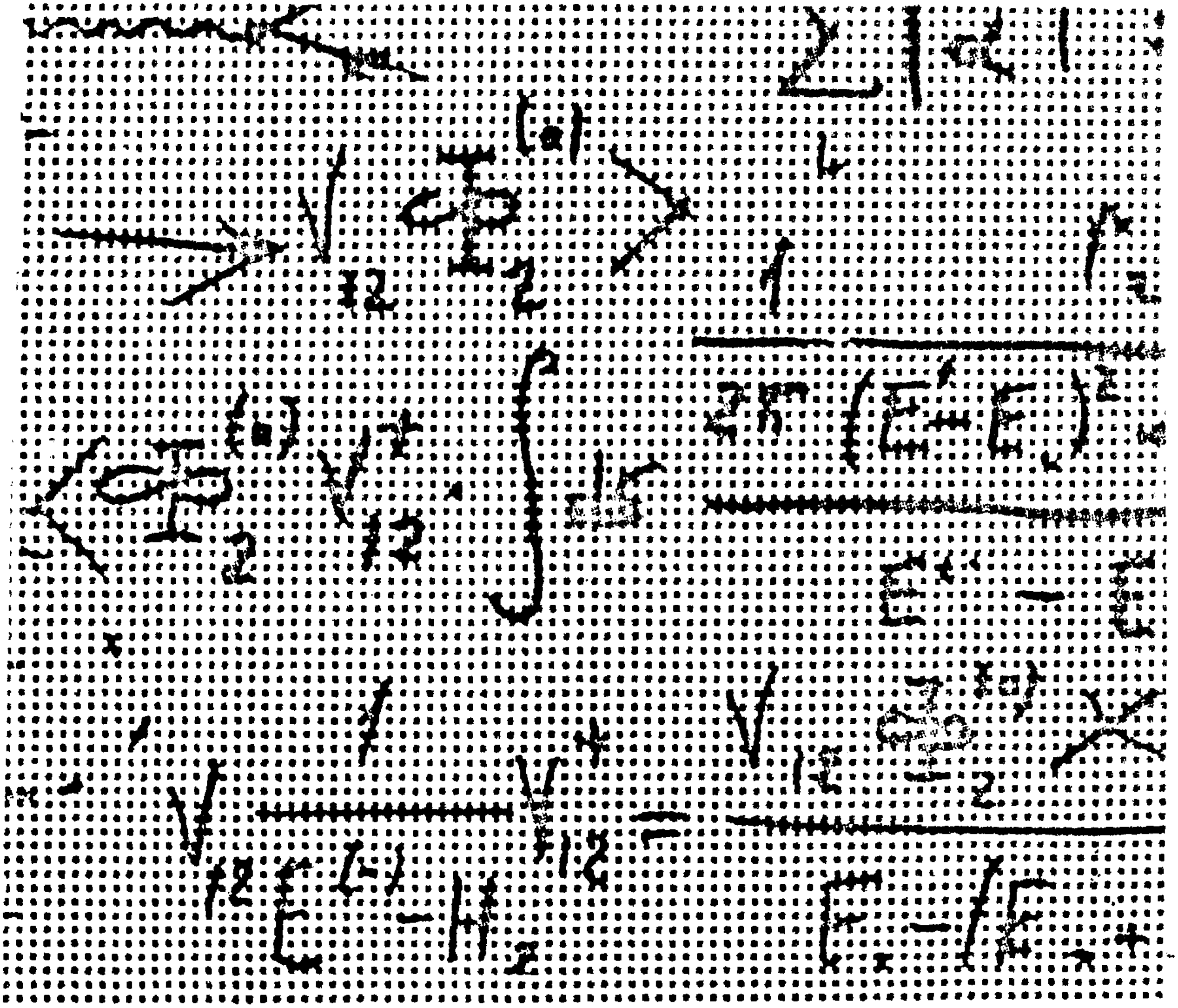

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