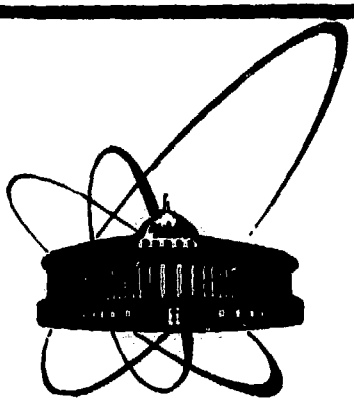

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A NEW PERTURBATIVE APPROACH TO QCD

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Basic problems of the strong interaction theory (the confinement, spontaneous breaking of chiral symmetry, hadron spectroscopy and the dual-resonance amplitudes) are now attempted to be solved by considering QCD in the framework of a strictly defined mathematical method. Main elements of the method are the Dirac quantization and the formula of asymptotical freedom. At the same time, one ignores that at long distances this formula becomes a phenomenological hypothesis, and the Dirac method cannot solve the problem of relativistic description of bound states even in QED [1].

If a hadron is a bound state of quarks and gluons, then to solve the above problems we have to consider the following questions:

I. What is the perturbation theory for bound states?

II. How can the relativistic-covariant perturbation theory be constructed?

III. How must one modify this perturbation theory to reproduce the basic features of hadronic physics?

I. PERTURBATION THEORY FOR BOUND STATES

In QED, bound states are described by means of the expansion not in the coupling constant but in the spatial components of gauge fields around an exact solution defined by their temporal component (i.e. by the Coulomb field). This perturbation theory will be called the "physical" one (PPT). PPT with an exact solution to the classical equation for the temporal component of the fields corresponds not to the Dirac quantization method (where all degrees of freedom are considered as quantum ones) but to the "minimal" method [2] (where only physical degrees of freedom are quantized).

On the classical level (after the exact solution of the Gauss equation) the gauge-invariant quantities like the Hamiltonian of Belinfante

$$H = \int d^3x \left[\frac{1}{2} (F^a_{0i})^2 + \frac{1}{4} (F^a_{ij})^2 + \bar{q} (i\gamma_i \partial_i + \gamma_i \hat{A}_i + m) q \right] \Big|_{A_0(A_i, q)} =$$

$$= \int d^3x \left[\frac{1}{2} (E^{Ta}_i)^2 + \frac{1}{2} (\partial_i A^{Ta}_j)^2 + \bar{q}^T (i\gamma_i \partial_i + m) q^T - \frac{e^2}{2} J^{Ta}_0 \frac{1}{\partial^2} J^{Ta}_0 + \ldots \right] \quad (1)$$

$$\left(J^{Ta}_0 = q^{+T} \frac{\lambda^a}{2} q^T + f^{abc} E^{Tb}_i A^{Tc}_i \, , \; E^{Ta}_i = \dot{A}^{Ta}_i \right)$$

depend on the classical variables (A_i^T and q^T), that are the functionals of the initial fields (A_i and q_i), in the lowest order of the perturbation theory these functionals have the form [2]

$$\hat{A}_i^T[A] = v[A](\hat{A}_i + \partial_i)\, v[A]^{-1}, \qquad v[A] = \exp\left\{\frac{1}{\partial^2}\partial_i \hat{A}_i\right\},$$

$$q^T[A,q] = v[A]\, q, \qquad \hat{A}_i = e\,\frac{A_i^a \lambda^a}{2i}. \tag{2}$$

The difference between the functionals (2) and the transversal variables ($\partial_i A_i = 0$) in the Dirac method consists in their transformation properties under Lorentz-transformations of the initial fields [2,3].

II. RELATIVIZATION

There exists the opinion that to restore the relativistic-covariance of bound states, one should set all field components (A_0, A_i) in the same conditions. There are two ways: either to use the relativistic gauge or to calculate all orders of the perturbation theory exactly. However, in practice, any relativistic gauge mixes up the nonperturbative bound effects with the effects induced by radiative corrections, and as a result, the theory becomes noncalculable [4]. The correct statement of the question is that how one gets the relativistic-covariant wave functions of the bound states in every order of the perturbation theory. As yet there is no solution of this problem even in QED [1].

Calculations show that the covariant description of bound states by means of PPT is possible if one chooses the quantization time vector (η_μ, $\eta_\mu^2 = 1$) so that the Coulomb field ($A_0 = (\eta \cdot A)$) moves together with particles, the bound states of which it forms.

For the lowest order of PPT $A_{||\mu} = \eta_\mu (A \cdot \eta) \neq 0$ and $A_{\perp\mu} \equiv A_\mu - A_{||\mu}$, we have the following system of the Dyson-Schwinger and Bethe-Salpeter equations, respectively,

$$\Sigma_{1,2}(p_\perp) = m_{1,2} + \int (dk)\, \underset{\sim}{V}(k_\perp - p_\perp)\, \gamma_{||\mu}\, G_\Sigma(k)\, \gamma_{||\mu}, \tag{3}$$

and

$$G_{\Sigma_1}^{-1}(k+\eta_1 \mathcal{P})\, \chi_\mathcal{P}(k)\, G_{\Sigma_2}^{-1}(k+\eta_2 \mathcal{P}) = \int (dq)\, \underset{\sim}{V}(k_\perp - q_\perp)\, \gamma_{||\mu}\, \chi_\mathcal{P}(q)\, \gamma_{||\mu}, \tag{4}$$

where

$$G_{\Sigma}(p) = \frac{1}{\not{p} - \Sigma(p_{\perp})}, \qquad (dk) \equiv i\,\frac{d^4k}{(2\pi)^4},$$

$$k_{||\mu} = \mathcal{P}_{\mu}(\mathcal{P}k)/\mathcal{P}^2, \qquad k_{\perp\mu} \equiv k_{\mu} - k_{||\mu},$$

$$\gamma_{||\mu} \equiv \mathcal{P}_{\mu}(\mathcal{P}k)/\mathcal{P}^2, \qquad \gamma_{\perp\mu} \equiv \gamma_{\mu} - \gamma_{||\mu},$$

$\mathcal{P}_{\mu}$ is the 4-momentum of the bound state $\chi_{\mathcal{P}}$, $\eta_{1,2}$ are the parameters depending on the particle masses m_1 and m_2 ($\eta_1 + \eta_2 = 1$, see [5]). The potential V is defined by the exact solution of the equation of motion for the temporal component of the field, $A_{||}$ ($A_{\perp} = 0$).

Relativistic-covariance (as a transformation property) of the Green functions $\langle\psi\psi\ldots\psi\psi\rangle$, in particular, solutions to eqs.(3) and (4), is achieved by including additional diagrams associated with the nonlocal Lorentz-transformations of the field into the "minimal" quantization scheme [2].

III. QCD AND PPT

The nonlocal functionals (2) may provide a new physical information which is absent when quantizing the theory by the Dirac method, for example, in QCD where stationary gauge transformations (as the mapping of a 3-dimensional space R_3 into SU(3)-group) have nontrivial topological properties, the functionals (2) are defined up to phase factors degenerated in the topological mapping numbers [2,6]. As a result of that degeneration, the physical fields (as factors of the field sources in the generating functional) differ from the "bare" fields used in the diagrams of the perturbation theory. Removal of the degeneration leads to vanishing of all the colour Green functions and the colour particle creation amplitudes, due to destructive interference of the phase factors of the degeneration. Consequently, we shall consider only the "bare" propagators of the perturbation theory and colourless bound states.

In the construction of such bound states a main problem of QCD is the introduction of a dimensional parameter. Usually, one associates it with the asymptotical freedom formula which is connected with "small" perturbative components (A_1). The constructive idea is to connect the dimensional transmutation, in the lowest order of PPT ($A_1 = 0$), with the boundary conditions for the equation for a "big" field component (A_0), i.e. to use such unnormalized solutions (A_0) to the classical equation which do not contradict the quantization principles for physical variables (2). For example, the expression $(\frac{1}{\partial^2} J_0^T)(x)$ in (1) can be redefined in the following form

$$e^2 \left(\frac{1}{\vec{\partial}^2} J_0^T\right)(x) = \int d^3 y \left[- \frac{\alpha_s}{|\vec{x} - \vec{y}|} - 2V_0 \ \vec{y} (\vec{x} - \vec{y}) \right] J_0^T (y) ,$$

where the second term vanishes when the Laplace operator acts on it, $\vec{\partial}^2 \left(\frac{1}{\vec{\partial}^2} J_0^T\right)(x) = J_0^T (x)$. Using the about expression we get the following interaction Hamiltonian

$$H_{int} (A_0 \neq 0, A_1 = 0) = \frac{e^2}{2} \int d^3 x \, J_0^T(x) \left(\frac{1}{\vec{\partial}^2} J_0^T\right)(x) = \tag{5}$$

$$= \int d^3 x d^3 y \, J_0^T(x) \, J_0^T(y) \, V(|x - y|) ,$$

where

$$V(|x - y|) = - \frac{\alpha_s}{|\vec{x} - \vec{y}|} + V_0 (\vec{x} - \vec{y})^2 , \qquad \alpha_s = \frac{e^2}{4\pi} ,$$

and the last expression was obtained owing to the symmetry under $\vec{x} \rightleftarrows \vec{y}$. The Hamiltonian (5) corresponds to redefinition of the Coulomb potential in an infinitesimal vicinity of the zero transfer momentum,

$$\underset{\sim}{V}(|\vec{p}|) = \left[\frac{4\pi\alpha_s}{|\vec{p}|^2} - V_0 (2\pi)^3 \left(\frac{\partial}{\partial \vec{p}}\right)^2 \delta(\vec{p}) \right]_{\vec{p} \neq 0} = \frac{4\pi\alpha_s}{|\vec{p}|^2} , \tag{6}$$

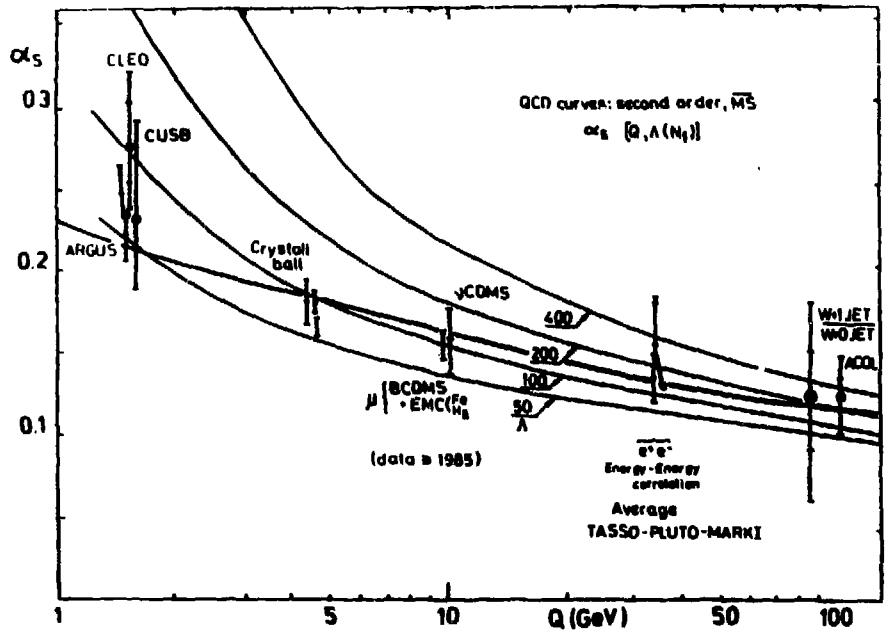
The phenomenological parameters (α_s, V_0) of this potential can be fixed from the spectroscopy of quarkonia [7] (which is an analogy of the Coulomb experiment in QED), which gives

$$V_0^{1/3} = 250 \text{ MeV} , \quad \alpha \simeq 0.2 . \tag{7}$$

The potential (6) explains the mass spectrum of light ($m_L \ll V_0^{1/3}$) and heavy ($m_H \gg V_0^{1/3}$) quarkonia (where $m_{L,H}$ are "bare" quark masses). Exactly calculated propagators of the quark and gluons describe the generation of their dynamical (or constituent) masses:

$$m_{q,A}(p) = \begin{cases} m_{q,A}(p = 0) \neq 0 , \\ m_{q,A}(p \to \infty) = 0 , \end{cases}$$

(see Appendix A). The dynamical mass of a gluon eliminates all infrared divergences, modifies the asymptotical freedom formula at small transfer momenta (Q^2) and leads to the finite coupling "constant" $\alpha(Q^2) \lesssim 0{,}2$,



The dependence of the strong coupling constant on Q. Given are experimental and theoretical values (thin lines) obtained by using the standard asymptotic freedom formula. The thick line corresponds to the modified formula values with $\alpha(Q^2 = 0) = 0.24$ *when* $N_f = 3$, $\Lambda = 110$ *MeV and* $\mu_0 = 700$ *MeV. (The authors would like to thank A.Roussarie (Saclay) for supplying with the data collected by him).*

for all Q^2 (see the Figure). The small coupling constant, in turn, allowed us to substantiate the nonrelativistic description of the hadron spectrum [7]. Thus, we may consider PPT as a self-consistent approach which has allowed us, in the framework of QCD, to describe any hadronic process with a given accuracy.

Equations (3) and (4) in their relativistic-covariant form, where the exchange-operator is

$$\underset{\sim}{K}(\mathcal{P}, q) = \frac{N_c^2 - 1}{2N_c} \underset{\sim}{V}(q_\perp)\, \gamma_{\parallel\mu} \otimes \gamma_{\parallel\mu}, \qquad (8)$$

describe not only the hadronic spectrum but their coherent interaction that in terms of the bilocal-meson fields [8] takes the following functional form

$$S = S_{free} + S_{int} ,$$

$$S_{free} \equiv Tr\left[-\frac{1}{2}(\mathfrak{M}, K^{-1}\mathfrak{M}) + \frac{i}{2}(G_\Sigma \mathfrak{M})^2 \right] , \qquad (9)$$

$$S_{int} \equiv i\, Tr\left[\sum_{n=3}^{\infty} \frac{(-1)^n}{n} (G_\Sigma \mathfrak{M})^n \right] ,$$

where Tr means summation over discrete indices and integration over continuous variables, for example, $Tr[\mathfrak{M}^2] \equiv \int d^4x\, d^4y\, tr[\mathfrak{M}(x,y), \mathfrak{M}(y,x)]$; $\mathfrak{M}(x,y)$ is the bilocal-meson field represented as the matrix

$$\mathfrak{M}_{\alpha\beta|ab} = \sum_{i=1}^{N_f^2-1} \mathfrak{M}^i_{\alpha\beta}\, \lambda^i_{ab/2}$$

with the Lorentz and flavour indices, (α, β) and (a, b) , respectively. Notice that the quark Green functions (G_Σ) and the bilocal field $(\mathfrak{M})$ satisfy the Dyson-Schwinger (3) and Bethe-Salpeter (4) equations, respectively,

$$\Sigma = m_0 + i\, KG_\Sigma ,$$

and

$$\mathfrak{M} = i\, KG_\Sigma\, \mathfrak{M} G_\Sigma .$$

The bilocal field can be decomposed over the solutions, $\chi^H_{\mathcal{P}}(q)$, to eq. (4) as

$$\mathfrak{M}(x,y) = \mathfrak{M}(z,X) = \int \frac{d^4\mathcal{P}}{(2\pi)^4} \sum_H \delta^+(\mathcal{P}^2 - \mu_H^2)\, \lambda_H K(z) \times$$
$$\times \left\{ e^{-i\mathcal{P}X} \chi^H_{\mathcal{P}}(z_\perp)\, a_H(\mathcal{P}) + e^{i\mathcal{P}X} \tilde{\chi}^H_{\mathcal{P}}(z_\perp)\, a^-_H(\mathcal{P}) \right\} , \qquad (10)$$

where

$$iK(z)\, \chi^H_{\mathcal{P}}(z_\perp) = \delta(z_{||})\, V(z_\perp)\, \gamma_{||} \int (dq)\, e^{iq_\perp z_\perp}\, \chi^H_{\mathcal{P}}(q)\, \gamma_{||} ,$$

$X = (x+y)/2$, $z = x - y$ are the absolute and relative coordinates, λ_H is the normalization factor, $a^+_H(a_H)$ is the creation (annihilation) operator for the hadronic state (other definitions have been introduced earlier).

As has been shown in refs.[8,9] , the perturbation theory in the bilocal fields, with the propagator $D(x_1,x_2|x_3,x_4) = \overline{\mathfrak{M}(x_1,x_2)}\, \mathfrak{M}(x_3,x_4)$ containing an infinite number of resonances describes self-dual amplitudes. The low-energy limit of these amplitudes for light quarks can be easily obtained from the expression (10).

In the limit of $V_0 \to \infty$ (i.e. at low energies) the bound-state wave functional (χ), due to the normalization condition, transforms to the δ-function, for example, by using the Gaussian successfully applied to the spectroscopy, we obtain

$$\lim_{V_0 \to \infty} \chi_\varphi(z) \sim \lim_{V_0 \to \infty} V_0^{1/2} \exp(-z^2 V_0^{2/3}) \sim \delta^{(3)}(z) .$$

As a result, in this limit the bilocal field ($\mathfrak{M}$) behaves as the δ-function, i.e. $\mathfrak{M}(x,y) \sim \delta^{(4)}(x-y)$. The same limit could be obtained by taking $K(z) \sim \delta^{(4)}(z)$. It is well known that just this "potential" for four-quark interactions has been used in the original formulation of the spontaneous breaking of chiral symmetry [10]. Today we know that the bilocal Lagrangian (9) with the potential of $\delta^{(4)}$-type is reducible to the chiral Lagrangians [11].

On the other hand, at high energies the Green functions of quarks and gluons turn into the "bare" propagators of the usual perturbation theory of QCD used to describe the quark-hadronic phenomenology of deep-inelastic processes.

To summarize, the physical perturbation theory allows us, from a common point of view i) to consider the light and heavy quarks and gluons; ii) to give the calculation method for radiative corrections, and to substantiate the zero-norm of the colour states and the relativistic interactions of hadrons in accordance with the conventional phenomenological models.

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Appendix A

The Green functions for a light quark and a gluon in the oscillator potential have calculated in refs.[3,7] . For the quark Green function we have [7] the expression

$$\Sigma(p) = E(p) \sin\phi(p) + \hat{p}[E(p)\sin\phi(p) - p], \quad \hat{p} = \frac{p_i \gamma_i}{|\vec{p}|}, \quad p = |\vec{p}| ,$$

where $\phi(p)$ satisfies the equation of the sine-Gordon type,

$$(\underline{p}^2 \phi')' = 2\underline{p}^3 \sin\phi(\underline{p}) - \sin 2\phi(\underline{p}) - 2\underline{m}\ \underline{p}^2 \cos\phi(\underline{p}) ,$$

$$\underline{E}(\underline{p}) = \underline{p}\sin\phi(\underline{p}) - \underline{p}^2\cos\phi(\underline{p}) - \frac{1}{2}(\phi')^2 + \underline{m}_L \sin\phi(\underline{p}) , \qquad (A1)$$

$$\phi' = \frac{d}{d\underline{p}}\phi(\underline{p}), \quad \underline{p} = (\frac{4}{3}V_0)^{1/3}p, \quad \underline{E} = (\frac{4}{3}V_0)^{1/3}E,$$

$$\underline{m}_L = (\frac{4}{3}V_0)^{1/3}m_L.$$

The gluon Green function is given by [3]

$$D_{ij}(q_0, \vec{q}) = \frac{1}{q_0^2 - \omega(|\vec{q}|)^2 - i\epsilon}(\delta_{ij} - q_i \frac{1}{\vec{q}^2} q_j), \qquad (A2)$$

Here $\omega(q)$ is the single-particle energy defined as a solution to the following equation

$$H|q> = \omega(q)|q>, \qquad (A3)$$

which after substitution of (A2) and (5) takes the form

$$\frac{\omega(p)^2 + p^2}{2\omega(p)} + N_c \int d^3q V_0 (-\frac{\partial}{\partial \vec{q}})^2 \delta^{(3)}(\vec{q}) \mathbb{C}(p+q|p)^2 = \omega(p), \qquad (A4)$$

where

$$\mathbb{C}(p|q) \equiv \frac{1}{2}[(\frac{\omega(p)}{\omega(q)})^{1/2} + (\frac{\omega(q)}{\omega(p)})^{1/2}]$$

or

$$\frac{\underline{\omega}^2 - \underline{p}^2}{\underline{\omega}} = (\frac{\underline{\omega}'}{\underline{\omega}})^2, \qquad (A5)$$

in the dimensionless variables $\underline{\omega} = (N_c V_0)^{-1/3}\omega$. To solve equation (A5), the boundary conditions $\omega(p=0) = \mu(p=0) = \mu_0 \neq 0$ are needed. In the limit $p \to \infty$ the equation turns into the algebraic one $\omega^2 - p^2 = \mu(p)^2 = \frac{1}{p} \to 0$. The value of μ_0 has been estimated from the massive loop that defines the modified asymptotical freedom formula:

$$\alpha^{Inf}(Q^2, \mu_0) \simeq \frac{1}{\beta_0 \int_0^1 dx \ln \frac{\mu_0^2 + x(1-x)Q^2}{x(1-x)\Lambda^2}} \Big|_{Q^2=0} = \frac{1}{\beta_0(\ln \frac{\mu_0^2}{\Lambda^2} + 2)} \simeq 0{,}2. \qquad (A6)$$

The Q^2-dependence of this parameter is shown in the Figure. In the limit of $Q^2 \gg \mu_0^2$ (A6) gives the usual asymptotical freedom formula, $a(Q^2) = 1/\beta_0 \ln \frac{Q^2}{\Lambda^2}$.

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Новый пертурбативный подход в КХД

Для описания связанных состояний в КЭД и КХД формулируется физическая теория возмущений по пространственным компонентам векторного поля вокруг точного решения, определяемого временной компонентой. Показано, что эта теория возмущений в КХД может быть доопределена так, что она воспроизводит все основные черты адронной физики: конфайнмент, спектроскопию легких и тяжелых кваркониев, дуально-резонансные амплитуды, киральные лагранжианы и партонную модель.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1988

Pervushin V.N. et al.
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A New Perturbative Approach to QCD

For the description of bound states in QED and QCD the physical perturbation theory on the spatial components of the gauge field over the exact solution, defined by the time one, is proposed. It is shown this perturbation theory in QCD can be redefined so that it reproduces the main elements of hadron physics: confinement, spectroscopy of light and heavy quarkonia, dual-resonance amplitudes, chiral Lagrangians and the parton model.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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