TRITA-PFU-87-09

FINITE LARMOR RADIUS EFFECTS ON Z-PINCH STABILITY

J. Scheffel and M. Faghihi

TRITA-PFU-87-09

FINITE LARMOR RADIUS EFFECTS ON Z-PINCH STABILITY

J. Scheffel and M. Faghihi

Stockholm, October 1987

Department of Plasma Physics and Fusion Research
Royal Institute of Technology
100 44 Stockholm, Sweden

FINITE LARMOR RADIUS EFFECTS ON Z-PINCH STABILITY

Jan Scheffel and Mostafa Faghihi

The Royal Institute of Technology, S-100 44 Stockholm, Sweden

ABSTRACT

The effect of finite Larmor radius (FLR) on the stability of m=1 small axial wavelength kinks in a z-pinch with purely poloidal magnetic field is investigated. We use the Incompressible FLR MHD model; a collisionless fluid model which consistently includes the relevant FLR terms due to ion gyroviscosity, Hall effect and electron diamagnetism.

With FLR terms absent, the Kadomtsev criterion of ideal MHD $2r dp/dr + m^2B^2/\mu_0 \geq 0$ predicts instability for internal modes unless the current density becomes singular at the centre of the pinch. The same result is obtained in the present model, with FLR terms absent. When the FLR terms are included, a normal mode analysis of the linearized equations yields the following results. Marginally unstable (ideal) modes are stabilized by gyroviscosity. The Hall terms have a damping, however not stabilizing, effect, in agreement with earlier work. Specifying a constant current and particle density equilibrium, the effect of electron diamagnetism vanishes. For a z-pinch with parameters relevant to the EXTRAP experiment, the m=1 modes are then fully stabilized over the cross-section for wavelengths $\lambda/a \leq 1$, where a denotes pinch radius.

As a general z-pinch result a critical line density limit $N_{max} = 3\text{-}5\cdot10^{18}$ m^{-1} is found, above which gyroviscous stabilization near the plasma boundary becomes insufficient. The result holds for wavelengths close to, or smaller than, the pinch radius and for realistic equilibrium profiles. This limit is far below the required limit for a reactor with contained alpha particles, which is in excess of 10^{20} m^{-1}.

1. BACKGROUND

A variety of early experiments in the field of fusion research were performed with z-pinches. The stability analysis of the cylindrically symmetric configuration, using ideal MHD theory, resulted in a classification of the unstable modes with respect to the poloidal mode number m, of which m=0 (sausage instability) and m=1 (kink) have received special attention. The m=1 internal mode is present even if the plasma column is surrounded by a conducting wall. Ideal MHD predicts instability for this mode unless the current density is singular at the axis r=0. The reason for abandoning the z-pinch as potential fusion concept in the early sixties was, however, its globally unstable character. Surface kinks and interchanges plagued the experiments.

The EXTRAP concept is a certain class of z-pinches, being suggested by B. Lehnert[5] in 1974. Experiments in linear and sector geometry have later established that once the external and sausage modes are stabilized, instability against the internal m=1 modes do not occur during the discharge time, which generally is of the order 100 Alfven times. In EXTRAP, stabilization of external and sausage modes seems to be produced by the strongly inhomogeneous external magnetic field; a set of external currents being anti-parallel to the plasma current generating an octupole field. The magnetic separatrix also smoothens the pressure profile at the boundary so as to fulfill the MHD m=0 interchange condition, which is weaker than the m=1 criterion for high beta. In a pursuit to explain the stability of the m=1 modes in the internal region of the pinch, where the influence of the external conductors is negligible, the present investigation of finite Larmor radius (FLR) effects on the stabilization has been carried out.

Our results clearly indicates that the FLR terms of the ion pressure tensor, the so-called magnetic viscosity or gyroviscosity, do have a strong stabilizing effect on m=1 internal modes. Since our approach has been to consider modes which ideally have the largest growth rates, i.e. those with large axial wavenumbers and few radial nodes, and since these modes are global to their nature, our results are general to their nature. A classification into internal or external modes is, in fact, not relevant here, especially considering that the present modes are unsensitive to the form of the boundary conditions.

Early studies resulted in the proposal that the effects of FLR terms actually can be stabilizing[2,3,4]. These investigations were carried out either in particle orbit- or Vlasov- models in simplified zero-dimensional or slab geometries. Roberts and Taylor[5] were first to demonstrate that

FLR effects can be accounted for in macroscopic plasma models. When the Larmor radius is moderate in comparison with the plasma radius this approach is preferable due to its inherent simplicity. Tayler[6] demonstrated in an early paper, employing a surface current model, that collisional viscosity has a damping, however not stabilizing, effect.

Since the EXTRAP plasma is not collision-dominated, we approach the FLR stability problem with a collisionless fluid model; see Ref[7] for an extensive discussion and heuristic derivation. This Incompressible FLR MHD model incorporates the full ion gyroviscous pressure tensor in the moment equation and the Hall and electron diamagnetism terms in Ohm's law. Here, however, we will assume constant particle density so that electron diamagnetism effects are neglected. The influence of the Hall term alone has been treated elsewhere[8] and found not to be stabilizing, however reducing the growth rate. Due to the magnetic field null region near the axis r=0 the model has its limitations. The results obtained in the part of parameter space where the model really is applicable should, however, be indicative of the results expected from a more realistic, fully kinetic model.

The strength of the FLR analysis of the z-pinch is that, although it is applicable only as long as $r_L \ll \lambda \ll 2\pi r$, this is exactly where the FLR (or kinetic, LLR) effects are expected to be most weak, namely in the low-beta region close to the boundary. In this sense the FLR criteria we obtain are sufficient for stability, since longer wavelengths are not as fatal within ideal MHD and shorter wavelengths are strongly affected by kinetic effects.

The classical, ideal MHD stability criterion for short wavelength kinks of a cylindrical plasma has been derived by Kadomtsev[9] ; $2r dp/dr + m^2B^2/\mu_0 \geq 0$. Here p and B denote equilibrium plasma pressure and magnetic field, respectively. The poloidal mode number $m \neq 0$. Obviously the m=1 modes are the most difficult ones to stabilize. It is the purpose of this paper to settle whether or not the FLR contributions from the ion pressure tensor can stabilize the m=1 modes.

A few words about the effect of pressure anisotropy on the Kadomtsev criterion may be mentioned. In a previous study we investigated the effect of pressure anisotropy on the m=1 internal modes within the double adiabatic model[10]. In this model we actually found a stabilizing effect for strong anisotropy ($p_\perp \gg p_{//}$) and fairly high perpendicular plasma beta values. Whether these conditions are met in experiments is yet not known. In isotropic models, however, internal kinks are essentially unsensitive to the form of the energy equations, since they are associated with the Alfven wave.

The Sections of this paper are organized as follows. In Section 2 we write down the basic equations governing the problem. The equations are linearized and specific stability criteria are derived in Section 3. A discussion of the results concludes the paper in Section 4. In the Appendix a list of the approximations used is given.

2. BASIC EQUATIONS

The incompressible FLR-MHD equations for a collisionless plasma with density ρ and flow velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ are

$$\frac{d\rho}{dt} = 0 \tag{1}$$

$$\rho\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p - \nabla\cdot\Pi \tag{2}$$

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = \frac{m_i}{e\rho}\left(\mathbf{j}\times\mathbf{B} - \nabla p_e\right) \tag{3}$$

$$\nabla\cdot\mathbf{v} = 0 \tag{4}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{5}$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{j} \tag{6}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{7}$$

Here j is the current density, mi the ion mass, p the scalar plasma pressure, p_e the electron pressure and Π the pressure tensor due to ion gyroviscosity. For simplicity we henceforth let $\mu_0=1$.

There is a somewhat spread misconception that the gyroviscous part of the pressure tensor is not given correctly in Braginskii[11], due to the assumption of high collisionality. However, Oraevskii et. al.[12] and others (Thompson[13], MacMahon[14], Chhajlani and Bhand[15]) have all, in a Vlasov plasma, arrived at exactly the same pressure tensor components as those of Braginskii as well as of Mikhailovskii and Tsypin[16]. With the magnetic field lines in the azimuthal direction, we have

$$\Pi = \mu\begin{pmatrix} \frac{1}{2}\left(\frac{\partial v_r}{\partial z}+\frac{\partial v_z}{\partial r}\right) & \frac{\partial v_\varphi}{\partial z}+\frac{1}{r}\frac{\partial v_z}{\partial v_\varphi} & \frac{1}{2}\left(\frac{\partial v_z}{\partial z}-\frac{\partial v_r}{\partial r}\right) \\ \frac{\partial v_\varphi}{\partial z}+\frac{1}{r}\frac{\partial v_z}{\partial \varphi} & 0 & -\left(\frac{1}{r}\frac{\partial v_r}{\partial \varphi}+\frac{\partial v_\varphi}{\partial r}\right) \\ \frac{1}{2}\left(\frac{\partial v_z}{\partial z}-\frac{\partial v_r}{\partial r}\right) & -\left(\frac{1}{r}\frac{\partial v_r}{\partial \varphi}+\frac{\partial v_\varphi}{\partial r}\right) & -\frac{1}{2}\left(\frac{\partial v_r}{\partial z}+\frac{\partial v_z}{\partial r}\right) \end{pmatrix} \tag{8}$$

The parameter $\mu = p_i/\Omega_i$ is the gyroviscosity coefficient; p_i is the perpendicular ion pressure and Ω_i is the ion gyrofrequency. This form of the pressure tensor holds true as long as $r_L/L_C \ll 1$, where r_L represents the Larmor radius and L_C a characteristic scale length perpendicular to the magnetic field.

One might ask whether the full model, consisting of the set of Eqs.(1)-(8), conserves energy. The answer is yes; local conservation of energy with gyroviscosity, Hall effect and electron pressure gradient terms included is easily proven, validating the form of the pressure tensor (8).

An exact dispersion relation for the Incompressible FLR-MHD model with all FLR terms included was derived for a homogeneous plasma in Ref[7]. Only stable oscillations were found, as expected when no driving force for instability is present. The incompressibility condition results in that exclusively the propagation of the modified shear Alfvén wave is described by the model.

3. EQUILIBRIUM

We will consider equilibria in a cylindrical, pure z-pinch with a circular cross-section. Cylindrical coordinates (r,φ,z) are used. Equilibrium quantities are functions of r only. The magnetic field is taken to be purely azimuthal; $\mathbf{B}=B(r)\mathbf{e}_\varphi$. Consequently the current density is directed along the z-axis; $\mathbf{j}=j(r)\mathbf{e}_z$. The plasma is bounded radially at r=a by a fixed conducting wall; we are considering internal modes.

Due to the presence of the FLR terms, the equilibrium cannot generally be considered static. And if there is an equilibrium flow, additional terms will appear in the linearized stability equations; the convective velocity derivative of the moment equation scales similarly in Larmor radius as the gyroviscous terms of the pressure tensor.

We consider for simplicity constant particle density equilibria, in consistence with the incompressibility condition. Incompressibility also requires that the radial equilibrium flow velocity $u_r = 0$, to avoid infinite flow near the axis. The equilibrium equations then become

$$p' + BB' + \frac{B^2}{r} + \frac{(r\mu u_z')'}{2r} = 0 \tag{9}$$

$$E_r - Bu_z = -\frac{m_i}{e\rho}\left(BB' + \frac{B^2}{r} + p_e'\right) \tag{10}$$

where primes denote differentiation with respect to r. Whereas earlier the radial momentum balance only involved pressure p and magnetic field B, the appearance of u_z couples the equation to Ohm's law. Obviously we can, for simplicity, choose $u_z = 0$, resulting in a non-zero radial electric field. Physically we then have a situation where the $\mathbf{E}\times\mathbf{B}$ drift exactly cancels the ion diamagnetic drift. We can then, as for the ideal case, use the simple equilibrium equation $p'+BB'+B^2/r = 0$ in the stability analysis.

3. THE LINEARIZED EQUATIONS

3.1 Model and method

A small-amplitude perturbation of the equilibrium is now performed. We Fourier decompose the perturbed quantities by assuming them to be proportional to exp[i(ωt+mφ+kz)]. The linearized equations become, for arbitrary particle density profile

$$i\omega\rho v_r + (Bb_\varphi)' + \frac{2B}{r}b_\varphi - \frac{imB}{r}b_r + p_1' +$$
$$+ \frac{ik}{2r}(r\mu)'v_r - \frac{mk}{r}\mu v_\varphi - \mu\left(\frac{m^2}{r^2}+\frac{k^2}{2}\right)v_z + \frac{1}{2r}(r\mu)'v_z' + \frac{\mu}{2}v_z'' = 0 \qquad (11)$$

$$i\omega\rho v_\varphi - \frac{(rB)'}{r}b_r + \frac{im}{r}p_1 +$$
$$+ \frac{mk}{r}\mu v_r + \frac{ik}{r^2}(r^2\mu)'v_\varphi + \frac{im}{r^2}(r\mu)'v_z + \frac{im}{r}\mu v_z' = 0 \qquad (12)$$

$$i\omega\rho v_z + ikBb_\varphi - \frac{imB}{r}b_z + ikp_1 +$$
$$\left(\frac{m^2}{r^2}+\frac{k^2}{2}\right)\mu v_r + \frac{ik}{2r}(r\mu)'v_z - \frac{1}{2r}(r\mu)'v_r' - \frac{im}{r}\mu v_\varphi' - \frac{\mu}{2}v_r'' = 0 \qquad (13)$$

$$-i\omega b_r + \frac{imB}{r}v_r + ik\alpha B'b_r + \alpha\left[\frac{m^2B}{r^2}b_z + \frac{ikB}{r}(imb_\varphi + b_r)\right] = 0 \qquad (14)$$

$$-i\omega b_\varphi - ikBv_z - (Bv_r)' + \frac{ik\alpha B}{r}b_\varphi + \alpha\left[im\left(\frac{Bb_z}{r}\right)' - \frac{ikB}{r}(imb_r - b_\varphi)\right] +$$
$$+ \frac{ik\alpha B\rho'}{\rho}b_\varphi - \frac{im\alpha B\rho'}{\rho r}b_z + \frac{ik\alpha\rho'}{\rho}\gamma p_1 - \frac{ik\alpha\rho_1}{\rho}\left[\frac{B^2}{r} + BB' + \gamma p'\right] = 0 \qquad (15)$$

$$-i\omega b_z + \frac{imB}{r}v_z - \alpha\left[\frac{1}{r}(rB'b_r)' + \frac{imB}{r^2}b_\varphi\right] - \frac{\alpha}{r}\left[(imBb_\varphi + Bb_r)' - \frac{imB}{r^2}(imb_r - b_\varphi)\right] +$$
$$+ \frac{\alpha B'\rho'}{\rho}b_r + \frac{\alpha B\rho'}{\rho r}b_r - \frac{im\alpha\rho_1'\gamma p}{\rho r} + \frac{im\alpha\rho_1}{\rho r}\left[\frac{B^2}{r} + BB' + \gamma p'\right] = 0 \qquad (16)$$

$$i\omega\rho_1 + \rho' v_r = 0 \qquad (17)$$

$$(rv_r)' + imv_\varphi + ikrv_z = 0 \qquad (18)$$

where capital and small letters denote equilibrium and perturbed quantities, respectively. For avoiding confusion we use index "$_1$" for the perturbed pressure and density. There are eight equations for eight unknowns v, b, ρ_1 and p_1. We have introduced the parameters $\alpha \equiv m_i/(e\rho)$ and $\gamma \equiv p_e/p$, assuming the electron and total pressure to have similar radial dependence. In the following we will consider constant particle density equilibria only, so that α becomes a constant. The terms due to electron diamagnetism (∇p_e-terms) will then all vanish, leaving only gyroviscous and Hall FLR terms.

In incompressible (collision-dominated) MHD (which equations become the same as ours for absent FLR terms), the sufficient and necessary criterion for stability of an incompressible and collisionfree pure z-pinch is, for small axial wavelengths

$$2rp' + m^2B^2 \geq 0. \qquad (19)$$

This result also exactly coincides with the compressible ideal MHD (Kadomtsev) criterion[9]; it requires the current density profile to be unphysically singular at the axis. It is our aim to find the modification due to the FLR terms. Since our model formally (not physically, however; the model is derived assuming collisionfree plasma) can be regarded as a perturbation of ideal incompressible MHD, we will only consider the *ideally most unstable modes.* These are the m=1 short axial wavelength modes with no radial nodes[17]. It has been shown that for the Hall model, which also yields a fourth order differential dispersion equation, the most unstable modes may have somewhat larger radial node numbers. This has so far not been shown for the present model.

Obviously the internal short axial wavelength modes are unsensitive to the form of the energy equation (i.e. to the assumption of incompressibility), since the criterion (19) is the same both for ideal MHD and the present model. It is indeed also seen in the case of a homogeneous plasma that the Alfvén wave couples only weakly to the energy equation, yielding essentially incompressible perturbations.

For deriving the dispersion relation, we will use an approximate method. When combined, the system of Eqs. (11-18) will be of fourth order in v_r (the ideal case is only of second order);

$$A_4 v_r'''' + A_3 v_r''' + k^2 A_2 v_r'' + k^2 A_1 v_r' + k^4 A_0 v_r = 0 . \qquad (20)$$

Denoting the characteristic scale length for radial perturbations by L_c one finds the following scalings. The undifferentiated term scales to terms with even derivatives as $(kL_c)^n A_0/A_n$, and to terms with odd derivatives as $k(kL_c)^n A_0/A_n$. Since we assume few radial nodes, we have $L_c \approx a$.

We also have $kr \gg 1$, yielding $kL_c \gg 1$. From the above scalings we then find $A_0/A_n \ll 1$. In conclusion we have found that in the limit of small axial wavelength and few radial nodes, the dispersion relation can approximately be given by $A_0 = 0$. This result can be obtained in a more rigorous, but less obvious, way by solving Eq.(20) with the WKBJ method. Assuming $v_r \sim \exp(\int\phi dr)$, and after expanding ϕ in powers of k, it is found that $\phi \approx k\phi_0$ for large k. The lowest order solution to Eq.(20) is then $\phi_0{}^2 = -(A_2/(2A_4)) \pm ((A_2/(2A_4))^2 - A_0/A_2)^{1/2}$. To limit the number of radial nodes, we must require $\int k\phi_0 dr$ to be itself limited, yielding that ϕ_0 must tend to zero as k becomes large. This is equivalent to $A_0 \rightarrow 0$ for large k.

Since the dispersion relation is obtained without actually solving the full differential equation with associated boundary conditions, it becomes clear why the present modes are unsensitive to the boundary conditions. This independence of boundary conditions has also been discussed elsewhere[8].

In order that the FLR terms should give finite contributions for large k we will use the orderings $\varepsilon \equiv \mu k = O(k^0)$ and $\gamma \equiv \alpha k = O(k^0)$. Notable is that the FLR terms only appear in the coefficients of ω, not elsewhere. This feature, which is general for FLR terms, independently of the scaling, can be seen immediately, e.g. by multiplying Eqs.(11)-(13) with ω using the induction Eqs.(14)-(16). Since $v_z = O(k^{-1})$, as can be seen from the incompressibility condition, we also have $b_r = O(k^0)$, $b_\varphi = O(k^0)$ and $b_z = O(k^{-1})$.

3.2 Stability conditions

Using these scalings, the Eqs.(11-18) can finally be combined to yield

$$A_0(r,\omega,\varepsilon,\gamma) \equiv a_4\omega^4 + a_3\omega^3 + a_2\omega^2 + a_1\omega + a_0 = 0 \qquad (21)$$

where

$$a_4 = -4\rho^2 r^4 B \qquad (22)$$

$$a_3 = 4\rho r^3[\gamma\rho(2B^2 - rp') - 2(r\varepsilon)'B]$$

$$a_2 = r^2[8\rho B(rp' + 3B^2) + \varepsilon^2 B - 4\varepsilon' B(r^2\varepsilon)' + 8\gamma\rho(r\varepsilon)'(2B^2 - rp') + 4\gamma^2\rho^2 B(2rp' + B^2)]$$

$$a_1 = r[16\varepsilon B(rp' + 2B^2) + 8\varepsilon' rB(rp' + 3B^2) + 4\gamma\rho(2r^2p'^2 + 5rB^2p' + 2B^4)$$
$$+ \gamma(\varepsilon^2 - 4\varepsilon'^2 r^2 - 8\varepsilon\varepsilon' r)(rp' - 2B^2) + 8\gamma^2(r\varepsilon)'\rho B(2rp' + B^2)]$$

$$a_0 = -4B^3(2rp' + B^2) + 8\gamma rp'^2(r^2\varepsilon)' + 4\gamma rB^2p'(5r\varepsilon' + 8\varepsilon) + 4\gamma B^4(2\varepsilon' r + 3\varepsilon)$$
$$+ (-\varepsilon^2\gamma^2 B + 8\varepsilon\varepsilon'\gamma^2 rB + 4\varepsilon'^2\gamma^2 r^2 B)(2rp' + B^2)$$

In the above we have included all terms of all orders in ε and γ.

From (22) some interesting orderings can be obtained. Comparing ideal, gyroviscous and Hall terms we find the orderings gyroviscous / ideal = $O[(kr_L\sqrt{\beta})^n]$, Hall / ideal = $O[(kr_L/\sqrt{\beta})^n]$ and consequently gyroviscous / Hall = $O(\beta^n)$, where $\beta = 2p/B^2$, and where n is the order in Larmor radius. For constant density, however, we can write $\beta=2.9\cdot 10^{-17} na^2(r_L/a)^2$, so that the orderings become instead gyroviscous / ideal = $O[(kr_L{}^2/a)^n]$, Hall / ideal = $O[(ka)^n]$ and gyroviscous / Hall = $O[(r_L/a)^{2n}]$. So the gyroviscous terms are the dominating FLR terms in the near-axis region, whereas the Hall term becomes important towards the boundary.

Our analysis of the dispersion relation given by Eqs.(21)-(22) will take the following shape. First we will consider the modified Kadomtsev criterion for marginally unstable (ideal) modes. For these cases we will assume the FLR terms to be small, so that terms of higher than second order in Larmor radius can be neglected. We will then perform a perturbation analysis for a constant current density equilibrium, in order to obtain the growth rate. This will give us some physical insight into FLR stabilization of more realistic equilibria. Finally complete numerical solutions for the constant current density equilibrium will be given.

Gyroviscous terms consequently dominate in regions with high β, i.e. towards the axis. For small ω the condition for stability becomes $a_1{}^2 - 4a_0a_2 \geq 0$. Explicitly this can be written

$$2rp' + B^2 + \frac{[2\varepsilon(rp'+2B^2)+\varepsilon' r(rp'+3B^2)]^2}{2\rho B^2(rp'+3B^2)} \geq 0 \qquad (23)$$

This is the m=1 Kadomtsev criterion (19) modified to include gyroviscosity, valid for marginally unstable modes and large k. It is seen that gyroviscosity has a stabilizing influence on ideally unstable modes, since the terms in the numerator remain finite as $2rp'+B^2$ (i.e. a_0) tends to zero.

As β becomes small, the Hall term will dominate and the modified Kadomtsev criterion becomes instead

$$2rp' + B^2 + \gamma^2 \frac{\rho[2r^2p'^2+5rB^2p'+2B^4]^2}{8B^4(rp'+3B^2)} \geq 0 \qquad (24)$$

The form of Eq.(24) is deceptive, because it seems as if the Hall terms indeed stabilized marginally unstable modes. The numerator, however, becomes identically zero if we perform the substitution $2rp'=-B^2$. The threshhold consequently becomes the same as for the ideal case. No stabilizing effect can therefore be concluded in this case, in agreement with Spies and Faghihi[8].

To see the joint contribution of gyroviscosity and Hall effect, we will now consider the more realistic constant current density equilibrium. With $j = j_0$, the magnetic field and pressure profiles become $B(r) = B_0 r/a$ and $p(r) = B_0^2(1-r^2/a^2)$, respectively. Here a represents the pinch radius. Upon solving Eqs.(21-22) perturbatively up to second order in Larmor radius, the eigenfrequency becomes, for small FLR parameters

$$\omega(r,\varepsilon,\gamma) = i\left[-\frac{B_0}{\sqrt{\rho}\,a} + a\frac{4r^2\varepsilon'^2+4r\varepsilon\varepsilon'+5\varepsilon^2}{32\rho\sqrt{\rho}\,B_0 r^2} + \frac{\sqrt{\rho}\,B_0}{8a}\gamma^2 + \frac{2r\varepsilon'+\varepsilon}{8\sqrt{\rho}\,r\varepsilon}\varepsilon\gamma\right]$$

$$+\left[-\frac{2r\varepsilon'+\varepsilon}{4\rho r} + \frac{B_0}{2a}\gamma\right] \qquad (25)$$

Not surprisingly, this eigenfrequency corresponds in the ideal case to the unstable oscillation of the shear Alfvén wave. Notable here is that the stabilizing FLR terms are of second, not first, order. The first order FLR terms appear only in the real part of ω. The cross term, proportional to $\varepsilon\gamma$, is however destabilizing due to ε' being negative.

To settle whether the FLR terms can absolutely stabilize a constant current density equilibrium we have to numerically solve the full dispersion relation (21), including terms of all orders in Larmor radius. The FLR terms are now allowed to be large as compared to the ideal terms. This is justified as long as the fluid requirement $r_L << a$ is fulfilled. We will investigate a case relevant to the EXTRAP z-pinch experiments presently being carried out in Stockholm.

Experiments carried out in the linear EXTRAP z-pinch L1 are typically characterized by a pinch radius a of order 0.01m, hydrogen densities n of order $5\cdot10^{21}$ m^{-3} and plasma currents I_p of order 25kA. We shall employ these parameters, and variations of them, in the numerical computations together with the Bennett equilibrium condition $\mu_0 I_p^2=16\pi NkT$. Since we assume constant particle number density, the line density becomes simply $N=\pi a^2 n$. Our free parameters will primarily be the axial wavelength of the perturbation λ, and the local radius r. The temperature T, as well as the Larmor radius r_L, then follows selfconsistently from Bennett's relation. We find the following:

* The constant current density profile can be fully stabilized wherever the model is valid. The Hall FLR terms are found to be unimportant for stability, but they slightly reduce the growth rate. Their impact is a maximum near the edge, where at most a reduction of the ideal growth rate with about a factor two was found.

* Shorter wavelengths are more easily stabilized, due to the (first order) FLR terms being proportional to k, as discussed above. The stabilizing effect of gyroviscosity is small near the boundary, where $\beta < 1$. Wavelengths $\lambda/a=0.5$ kan be stabilized as far as to a local Larmor radius distance from the boundary for a=0.01-0.02 m, but not quite for a=0.03 m. Thus, an increase of plasma radius, other parameters held constant, is destabilizing. The physical reason is that T decreases for constant Ip, and with it r_L.

* Variations of I_p only does not alter stability, since r_L remains unchanged; $\sqrt{T}$ increases as B.

* The line density has an upper critical limit. An increase of line density, with I_p held constant, decreases the temperature and thereby decreases r_L. Numerically this limit is a function of λ; with $\lambda/a=0.5$ we find $N_{max} = 4.8\cdot10^{18}$ m^{-1} (with kr=10.7, kr_L=2.0, β=0.77 and T=23 eV) and with $\lambda/a = 1$ we find $N_{max} = 2.4\cdot10^{18}$ m^{-1} (with kr=4.5, kr_L=2.3, β=1.9, T=82 eV) for absolute stabilization over the cross-section, extending to a Larmor radius distance from the boundary. These limits are slightly higher than the so far obtained experimental results, where the line densities are of the order of $2\cdot10^{18}$ m^{-1}.

The latter result is of vital importance for a reactor, where line densities in excess of 10^{20} m^{-1} may be necessary. We should remember here that a peaked density profile would of course yield higher critical line densities, since the density is allowed to be large near the centre, where the gyroviscous stabilization is more forceful. A peaking of density is unlikely, however, to give the nearly two orders of magnitude required. There remains the possibilities that either electron diamagnetism provides additional FLR stabilization, or that kinetic large Larmor radius (LLR) effects are needed. We cannot expect drastically different results using other current density, or pressure, profiles since the plasma beta cannot be made sufficiently large within a Larmor radius distance from the boundary.

To study these profile effects, we have computed stability threshholds using the form $j=j_0(1-(r/a)^q)$. Our constant current density case then corresponds to $q\rightarrow\infty$. Bearing in mind that $\sqrt{\beta} \sim \sqrt{n}r_L$ directly measures Larmor radius for constant density, we find that more peaked current density profiles have the effect of decreasing β and the Larmor radius towards the boundary, so that lower critical line densities are obtained in these cases.

3.3 The number of Larmor radii θ_i

A simple measure of FLR stabilization is the total number of ion Larmor radii θ_i that can be contained within a pinch radius. We here define $\theta_i = \int dr/r_L$, where the integration should extend from the axis to the boundary. Using $r_L=m_i w_\perp/(eB)=(2/3)^{1/2} m_i w/(eB)$, with w as the thermal velocity, a general form for θ_i becomes

$$\theta_i = \sqrt{\frac{3e^2\mu_0}{2m_i}} \int_0^a \sqrt{\frac{n}{\beta}}\, dr \tag{26}$$

Taking the particle and current density profiles to be constant, we obtain exactly

$$\theta_i = \sqrt{\frac{3e^2\mu_0}{4\pi m_i}} \sqrt{N} \,. \tag{27}$$

The correspondence between line density and FLR stabilization is clearly revealed in Eq.(27). Numerically we find for a hydrogen plasma $\theta_i=2.15\cdot10^{-9}\sqrt{N}$ and for a deuterium plasma $\theta_i=1.52\cdot10^{-9}\sqrt{N}$. For the EXTRAP experiments, which are performed with hydrogen, we find $\theta_i=4.8$ for $N=5\cdot10^{18}\ m^{-1}$ and $\theta_i=22$ for $N=10^{20}\ m^{-1}$. In deuterium $\theta_i=15$ for $N=10^{20}\ m^{-1}$. Obviously a fluid model with FLR corrections becomes nearly ideal as reactor line densities are approached. To see that Eq.(27) has a wider significance when physically more realistic profiles are considered, we note the following (Bennett equilibrium is assumed).

A peaked density profile has the effect of decreasing the integrand of Eq.(26) where it is significant, that is near the boundary. Consequently θ_i becomes smaller for a peaked density profile and FLR stabilization is strengthened, integrated line density held constant. A peaked current density profile will, on the other hand, cause θ_i to increase since β is smaller away from the axis in this case. Roughly Eq.(27) then gives the number of Larmor radii along a radius for a large class of experimentally realistic profiles. Future experiments will reveal whether our stability limit of $\theta_i \approx 4$ can be overcome with the aid of stabilizing mechanisms not included here.

4. SUMMARY AND DISCUSSION

A main result of this paper is that FLR stabilization of constant density z-pinch equilibria to m=1 modes has been found to be sufficient only up to line densities of order $3\text{-}5\cdot 10^{18}\ m^{-1}$, for wavelengths close to or less than the pinch radius. Since reactor line densities in excess of 10^{20} m^{-1} may be required, this result tells us that we must hope for kinetic effects for additional stabilization of these modes. A peaked density profile would allow a higher line density limit, but cannot yield the two orders of magnitude desired. Peaking of the current density profile, on the other hand, tends to lower the line density limit. Consequently density and current density profile effects tend to cancel each other. It is uncertain whether electron diamagnetism, which we have neglected due to our assumption of constant density, would yield additional stabilization.

In terms of number of ion Larmor radii, it is found that whereas about 20 Larmor radii are required for reactor conditions, the FLR stabilization due to gyroviscosity becomes insufficient already for more than four to five Larmor radii.

We have also found that stabilization increases with decreasing plasma radius, but is insensitive to variations of total plasma current, other parameters held constant. Shorter perturbation wavelengths are more easily stabilized, as expected, since the FLR terms are proportional to the axial wavenumber k. With present parameters of the linear EXTRAP z-pinch experiment ($a=0.01m$, $n=5\cdot 10^{21}m^{-3}$, $I_p=25kA$) we find that wavelengths up to $\lambda/a=1$ can be fully stabilized as far as to a Larmor radius distance from the boundary.

Here we have concentrated on studying the influence of gyroviscosity and Hall effect on the stability of short axial wavelength kink modes in a cylindrical z-pinch, when the poloidal mode number is non-zero. The approach has been to apply a normal mode analysis to a collisionless and incompressible fluid model. In general our model accounts for all finite Larmor radius effects in the moment equation and Ohm's law, i.e. gyroviscosity, Hall effect and electron diamagnetism. With electron diamagnetism dropped, we find analytically that the Kadomtsev m=1 criterion becomes modified due to the stabilizing effect of gyroviscosity. No absolute stabilization can be attributed to the Hall term; in our numerical calculations it is also found that the Hall term only has a weakening effect on the growth rate.

The present results indicate that the classical z-pinch should be more stable than what ideal MHD theory predicts, i.e. even in the absence of stabilizing external fields such as the EXTRAP octupole field. One may think that this result is in conflict with early z-pinch experiments, which went unstable within a few Alfvén times. Our answer to this is that it is essential how the pinch is initially formed; large surface currents are highly destabilizing. A classical z-pinch does feature surface currents, whereas EXTRAP e.g. is characterized by a current build-up from a central channel, so that equilibria with centre-peaked current density profiles are obtained.

ACKNOWLEDGEMENTS

Thanks are due to Prof. B. Lehnert and Dr. G. Spies for valuable discussions on this investigation.

5. APPENDIX : APPROXIMATIONS

A short summary of the approximations made for the present analysis will be given here. We perform three types of approximations; to obtain the general *one-fluid equations* from the combined Boltzmann's and Maxwell's equations, to obtain the *collisionless* Incompressible FLR MHD equations and to obtain the *dispersion relation* for internal kinks from the linearized equations. For a more detailed discussion; see Ref[7].

First we give a few definitions. We denote characteristic equilibrium and perturbation scale lengths by L_c and $\lambda=2\pi/k$, respectively. Also let v_c and ω/k denote characteristic equilibrium and perturbation velocities, respectively.

To obtain the *one-fluid equations* we must assume

* ω/k and $v_c << c$, the velocity of light. This in order to neglect the displacement current.
* $\omega <<$ electron plasma frequency and $\lambda >>$ Debye length, so that Poisson's equation can be dropped and charge neutrality assumed.
* $\omega <<$ electron cyclotron frequency. This makes the Nernst term of the generalized Ohm's law negligible.
* Electron mass << ion mass; in simplifications of the moment equation and Ohm's law.

It is necessary to make the following approximations when assuming an *isotropic collisionless* model:

* Isotropy must be established by kinetic mechanisms, not by collisions.
* $r_L << a$, i.e. the Larmor radius must be sufficiently small compared to the macroscopic plasma dimensions in order to preserve a fluid character. In the present context this has the implication that we sufficiently keep off the axis $r=0$.
* $\lambda_{ii}/r_L = \Omega_i\tau_{ii} >> 1$ so that the collisionless transport coefficients can be used. Here λ_{ii} and τ_{ii} denote ion-ion collision mean free path and -time, respectively. We denote the ion Larmor frequency by Ω_i.
* $T_i = T_e$, i.e. equal ion and electron temperatures.

Finally to obtain the *dispersion relationns* we assume

* Cylindrical geometry.
* Linearized analysis applicable.
* Constant particle density, implying that the electron diamagnetism term of Ohm's law vanish.
* No equilibrium flow.
* kr >> 1, which substantially reduces the complexity of the equations and $\lambda > r_L$ for physical soundness. Consequently the wavelength regime being studied here is $r_L < \lambda \leq a$.

6. REFERENCES

[1]Lehnert, B., Phys. Scripta 10(1974)139. See also Lehnert, B., Nucl. Instr. Methods 207(1983)223.

[2]Lehnert, B., Phys. Fluids 4(1961)525.

[3]Lehnert, B., Phys. Fluids 4(1961)847.

[4]Rosenbluth, M.N., Krall, N.A., and Rostoker, N., Nucl. Fusion Suppl. 1(1962)143.

[5]Roberts, K.V., and Taylor, J.B., Phys. Rev. Letters 8(1962)197.

[6]Tayler, R.J., Proc. Phys. Soc. B70(1957)31.

[7]Scheffel, J., and Faghihi, M., Royal Institute of Technology, Stockholm, TRITA-PFU-86-09.

[8]Spies, G., and Faghihi, M., Phys. Fluids 30(1987)1724.

[9]Kadomtsev, B.B., **Reviews of Plasma Physics**, M.A. Leontovich ed., Consultants Bureau, New York (1966), Vol.2.

[10]Scheffel, J. and Faghihi, M.; Royal Institute of Technology, Stockholm, TRITA-PFU-87-05, To appear in Journal of Plasma Physics.

[11]Braginskii, S.I., **Reviews of Plasma Physics**, M.A. Leontovich ed., Consultants Bureau, New York (1966), Vol.1.

[12]Oraevskii,V., Chodura, R. and Feneberg, W., Plasma Phys. 10(1968)819.

[13]Thompson, W.B., **An Introduction to Plasma Physics**, Pergamon Press (1962).

[14]MacMahon, A., Phys. Fluids 8(1965)1840.

[15]Chhajlani, R.K., and Bhand, S.C., J. Plasma Phys. 23(1980)205.

[16]Mikhailovskii, A.B., and Tsypin, V.S., Beitr. Plasmaphys. 24(1984)335.

[17]Goedbloed, J.P., and Sakanaka, P.H., Phys. Fluids 17(1974)908.

TRITA-PFU-87-09

Royal Institute of Technology, Department of Plasma Physics
and Fusion Research, Stockholm, Sweden

FINITE LARMOR RADIUS EFFECTS ON Z-PINCH STABILITY

J. Scheffel and M. Faghihi, 21p. in English.

The effect of finite Larmor radius (FLR) on the stability of m=1 small axial wavelength kinks in a z-pinch with purely poloidal magnetic field is investigated. We use the Incompressible FLR MHD model; a collisionless fluid model which consistently includes the relevant FLR terms due to ion gyroviscosity, Hall effect and electron diamagnetism.

With FLR terms absent, the Kadomtsev criterion of ideal MHD $2rdp/dr+m^2B^2/\mu_0 \geq 0$ predicts instability for internal modes unless the current density becomes singular at the centre of the pinch. The same result is obtained in the present model, with FLR terms absent. When the FLR terms are included, a normal mode analysis of the linearized equations yields the following results. Marginally unstable (ideal) modes are stabilized by gyroviscosity. The Hall terms have a damping, however not stabilizing, effect, in agreement with earlier work. Specifying a constant current and particle density equilibrium, the effect of electron diamagnetism vanishes. For a z-pinch with parameters relevant to the EXTRAP experiment, the m=1 modes are then fully stabilized over the cross-section for wavelengths $\lambda/a \leq 1$, where a denotes pinch radius.

As a general z-pinch result a critical line density limit $N_{max} = 3\text{-}5 \cdot 10^{18}$ m^{-1} is found, above which gyroviscous stabilization near the plasma boundary becomes insufficient. The result holds for wavelengths close to, or smaller than, the pinch radius and for realistic equilibrium profiles. This limit is far below the required limit for a reactor with contained alpha particles, which is in excess of 10^{20} m^{-1}.

Key words Z-pinch, FLR effects, Finite Larmor Radius, gyroviscosity, Hall effect, stability.