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Department of Plasma Physics and Fusion Research
Royal Institute of Technology
S-10044 Stockholm,Sweden

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The Royal institute of Technology,S-10044,Stockholm,Sweden

ABSTRACT

The method of the dynamic stabilization is invoked to explain the enhanced stability of a Z-pinch in EXTRAP configuration [Phys.Scr.10. 139.(1974); Phys.Scr.16,147,(1977)].The oscillatory motion is assumed to be forced on EXTRAP due to self-similar oscillations of a Z-pinch.Using a scaling for the net energy loss with plasma density and temperature typical for divertor configurations,a new analytic,self-similar solution of the fluid equations is presented.Strongly unharmonic oscillations of the plasma parameters in the pinch arise.These results are used in a discussion on the stability of EXTRAP,considered as a system with a time dependent internal magnetic field.The effect of the dynamic stabilization is considered by taking estimates.

1.Introduction

An ordinary Z-pinch is proved both theoreticaly and experimentally to be a violently unstable configuration on the MHD time scale.The most unstable modes are either the m=1 kink mode without a toroidal field or in a presence of a weak toroidal field the sausage m=0 and the screw m=1 modes.A large variety of higher-order modes should become unstable as well.However,since early days of fusion research there is little doubts that perturbations with high mode numbers and/or short wavelengths in a classical Z-pinch are efficiently stabilized by the FLR effects.On the other hand,it is also well known that these effects do not affect rigid global modes with low mode numbers and long wavelengths at any significant rate.

Generally,the variety of methods,capable to provide for a significant stabilization of unstable configurations is very sparse.However,the suggestion that bounding a Z-pinch by a purely transverse (without any toroidal field),octupole magnetic field might strongly enhance the plasma stability of a Z-pinch [1] seems to be confirmed experimentally[2].

The resulting configuration is called EXTRAP (Fig.1).Stable discharges were attained in linear EXTRAP configurations for around 50 msec,which corresponds to around 50 Alfven transit times.Experimental and theoretical studies then concentrated on searching after the reasons that might provide stability on this timescale.The answer to this question is not available so far.

One possible way of improving the stability of a Z-pinch at a significant rate is by employing dynamic techniques[3-4].However,the fact that the octupole field is generated by d.c.currents renders it difficult to apply the mechanism of dynamic stabilization to EXTRAP.In the present study it is assumed that the oscillatory motion is forced on the system by imposing an equilibrium,shown to cause a highly non-linear oscillations of the magnetic field.Although these oscillations have been shown to arise in a Z-pinch before[5-8],we reconsider this effect for an arbitrary form of the magnetic field profile, including the energy loss,typical for divertor configurations.

2.Dynamic Equilibrium in EXTRAP.

Starting from a static equilibrium,which may be unstable,we use Maxwell and transport equations for a cylindrically symmetric Z-pinch.

Considering Maxwell equations in the form:

$$\partial(rB)/r\partial r=4\pi j_z/c \quad (1)$$

$$\partial E_z/\partial r=\partial B/c\partial t \quad (2)$$

and Ohm´s law

$$E_z+(\underline{v}\times\underline{B})/c=0 \quad (3)$$

where v is the mass flow velocity. Here,resistive effects have been neglected,because we are interested in processes taking place on the MHD time scale.

Turning over to transport equations,we get for the particle balance:

$$\partial n/\partial t+\partial\{nrv)/r\partial r=0 \quad (4)$$

.The momentum balance reads if viscous effects are neglected:

$$nm_i dv/dt=-\partial p/\partial r+j_z\times B/c \quad (5)$$

Finally the energy balance in cylindrical geometry reads:

$$3(n_e+n_i)dT/2dt+p\partial(rv)/r\partial r+\partial(rq)/\partial r=j^2/\sigma - L \quad (6)$$

where $d/dt=\partial/\partial t+v\partial/\partial r$ is the substantial derivative.

In EXTRAP L,defined as the energy loss,is very small during the build-up phase of the discharge,until the pinch radius R reaches the separatrix.The term L then drastically increases and the expansion of the discharge is terminated[9].In the classical Z-pinch just the opposite is valid.The discharge starts at the periphery and during the start-up phase the Ohmic heating is exactly compensated by wall losses.

We assume that the plasma in EXTRAP has been preheated to an initial state,which is different from the Bennet equilibrium.Thus the pinch current contributes only to the generation of the magnetic field and the confinement,but not to the further heating.

Therefore in the present study we consider Eq.(6),simplified as follows.The term at the right hand side of Eq.(6) scales as

$$A=nT/\tau \quad (7)$$

where τ is essentially the energy confinement time in the octupole field.It is assumed to be a constant.The energy is assumed to be transported primarily by convection, owing to the X-points and the open field line structure of the magnetic field in

EXTRAP outside the separatrix.

Then the energy balance takes the form:

$$2nTd[\ln(T^{3/2}/n)]/dt=nT/\tau \qquad (8)$$

We are looking for self-similar solutions of the system of Eqs. (1-8). The self similar variable is chosen as $x=r/R(t)$,where R is the pinch radius. It is a function of time,even during the flat-top phase of the discharge.The convection velocity is taken as $v=rdR/Rdt$.Then all physical parameters of the problem are functions of x and t and the factorization of the system becomes possible.It means that all physical parameters are cast in the form f(x) F(t).All distributions are self-similar in time,although the scale length of profiles might change according to the motion of the pinch radius.

Solving Eqs.(5 and 8) we obtain:

$$n=n_0R^{-2}(t)\, f_n(x)\,;\; B=B_0R^{-1}(t)f_B(x) \qquad (9)$$

Factorizing also the temperature $T=T(t)f_T(x)$ and requiring that:

$$\partial(f_nf_T)/\partial x=-A_1xf_n(x) \qquad (10)$$

$$f_B\partial(f_Bx)/\partial x=A_2x^2f_n(x) \qquad (11)$$

we get from the momentum balance:

$$d^2R/dt^2=T_0(2\,A_1T/T_0-\,A_2B_0^2/4\pi T_0n_0)/m_iR \qquad (12)$$

where A_1, A_2 and T_0 are arbitrary constants to be determined later.

It is important to emphasize that the number of equations(10-11) is less than the number of profile functions.Hence one of them is arbitrary and may be chosen either as a measured profile or from some other constraints for example from the stability considerations.The condition,required to provide for stability,is the increase of the magnetic field toward the periphery.

However,in the present study in order to compute the values of constants A_1 and A_2,we assume the simplified profiles for plasma parameters,taken as $n=N/\pi R^2(t)$, $B=2Ix/cR(t)$ and $T=T(t)(1-x^2)$.It yields $A_1=2$ and $A_2=2$.It should be pointed out that our conclusions are not very sensitive to the exact values of these constants.

Introducing a new parameter $T_1=I^2/2Nc^2$, we obtain from Eq.(12):

$$d^2R/dt^2=4\,T_1(T/T_1-1)/m_iR \qquad (13)$$

Considering now the energy balance we get:

$$T(t)=T_0[R_0/R(t)]^{4/3}(\exp-2t/3\tau) \qquad (14)$$

where T_0 and R_0 are the plasma temperature and the pinch radius at t=0.

Substituting now Eq.(14) into Eq.(13) we get the equation describing strongly non-linear oscillations of the pinch boundary with damping:

$$d^2R/dt^2=4T_1[(T_0/T_1)(R/R_0)^{-4/3}\{\exp-2t/3\tau)-1]/m_iR \qquad (15)$$

Equation(15) was solved numerically with the following boundary conditions $R(t=0)=R_0$ and $dR/dt(t=0)=0$.The result is shown in Figs.2 and 3.

If τ>>t i.e the confinement time of the octupole field is much longer than the pulse duration,these oscillations are almost unattenuated and the solution describes strongly non-linear adiabatic oscillations. If τ decreases the oscillations are damped more efficiently and the pinch collapses after a small number of oscillation periods. However the scaling of the energy losses will probably change as well and the whole problem has to be reconsidered with some other scaling.

It should be emphasized that most of the results obtained above are rather sensitive to the form of the energy loss scaling.The liberty to choose arbitrary one of the plasma profiles vanishes,which follows from Eqs.(10-11),and therefore the physical picture might change if some other loss channels are taken into account.

The amplitude of the oscillations is a strong function of initial conditions.The boundary of the pinch and the maximum and minimum plasma temperature at a given observation point can easily vary by a factor 3-4 for departures from the Bennet equilibrium larger than 50%.

Linearizing Eq.(15) we easily find that the oscillation period is either less or about a few transit times of an isothermal sound wave across the initial plasma radius.For the case of EXTRAP this implies a frequency of about 1MHz.This value lies within the MHD frequency range.It is worthwhile to mention that to obtain such high frequencies by means of an external circuit presents severe technical difficulties[3,4]. Another interesting feature due to the strongly non-linear nature of the oscillations is a broad frequency spectrum generated by them.Both these facts affect the strength of the stabilizing effect exercised by the oscillations.

There is a number of experimental observations,which quite clearly indicate the existence of the dynamic equilibrium in a Z-pinch.For example in a well diagnosed experiment[4],the stabilization of the Z-pinch with currents up to 100kA was obtained by the high frequency quadrupole field.By the way,the stabilization of the Z-pinch by a constant quadrupole field has not been reported there.If the stabilization remained unattained, it seems hard to reconcile this result with EXTRAP experiments.

On the other hand it has been observed that the cross-section of a Z-pinch exhibited oscillations,induced by the oscillating quadropole field.These oscillations grew rapidly in amplitude until the plasma reached the tube wall.This observation provides a direct confirmation of our assertion about the link between the self-similar oscillations, described above, and the behavior of a Z-pinch immersed in an external field.

Moreover,the experiment with an expanding wire plasma[10] has shown that fluid profiles in this case look nearly self-similar.

Although the Z-pinch in EXTRAP is the ideal object for the experimental verification of the non-linear oscillations owing to its extremely long lifetime there is only circumstantial evidence that they do appear in current EXTRAP experiments.The temperature measurements at three selected times during the discharge in EXTRAP,made by Thomson diagnostic technique[11],have shown the strong variations by factor 2-3 of the plasma temperature along the same line of sight.We tried to model this situation by our theory.Results are shown in Figs.2,3 For these cases the departure from the static equilibrium is kept at a rather modest level of about 30%.This is certainly in good agreement with the experimental results,mentioned above.Secondly,in EXTRAP by the same diagnostics it has been shown that the total number of electrons increases dramatically during the discharge.This rise may be caused by either the direct contact of the plasma column with the liner or by the release of molecular compounds from the wall by peripheral r.f.discharges,induced by the oscillations.Release of one molecular gas layer($\approx 10^{16}$ atoms/cm^2) would be more than enough to account for the observed increase if this gas is ionized and enters the core of the discharge.

Concluding this Section,we mention that the link between the self-similar oscillations and the dynamic equilibrium of a Z-pinch seems to be adequately established both theoretically and experimentally.The main reason for this may be a well-established universality of the self-similar solutions.Even solutions of non-linear systems that are not initially self-similar often converge to the self-similar form on a rather short time scale.This bears a certain similarity with the principle of the "profile consistency"[12],experimentally observed in Tokamaks.

3.The stabilizing effect of the dynamic equilibrium.

Dynamic stabilization is a well established idea in plasma physics.This idea stems from the analogy with the stable inverted pendulum on an oscillating suspension.

Considering the possible stabilizing effect of these oscillations it is important to keep in mind that the main reason for the collapse of the discharge in a classical Z-pinch on a very short time scale is the decrease of the magnetic field with increasing distance from the sharp boundary.In our model,owing to the scaling chosen by us,the magnetic field profile at the periphery remains arbitrary and may be chosen to fulfill this constraint(see Eqs.10&11).The stability criteria as derived by Osovets[13],using the analogy with the inverted pendulum read :

$$\omega>(4\pi v_s/\lambda)[(\ln(\lambda/\pi R)]^{1/2} \qquad (16)$$

$$\partial B/\partial r>(2I/c)(2\pi/\lambda)^2\ln\lambda/\pi R \qquad (17)$$

Here ω is the frequency of oscillations,I is the pinch current,λ is the wavelength of the perturbation ,R is the pinch radius,v_s is the ion sound speed and B is the strength of the oscillating magnetic field.

For the EXTRAP pinch Eq.(16) implies that the most dangerous kink instabilities with wavelengths larger or approximately equal to the half length of the pinch of about 20cm have to be stabilized by this effect.The accuracy of this estimate is limited by the fact that Eq.(16) has been derived under the assumption of a stabilizing field with only one harmonic ω. The non-linear oscillations have a broad frequency spectrum with a large half width.Therefore they might provide the stabilizing effect,stabilizing even kink modes with shorter wavelengths than given by our simple estimate.On the other hand it is clear that this effect is not sufficient to provide the stabilization of the short wavelength kinks with wavelengths of the order of the pinch radius around 2cm.As shown by Forman et al.in [4],the growth rate of these modes is reduced by only approximately 30% due to this effect.Although there is no experimental evidence that these modes are really stabilized in EXTRAP(only the global stability without measurements of wavelengths of the stabilized modes has been reported in [2]),this decrease of the growth rate is insufficient to account for the observed stabilization.Possibly some other stabilizing effect such as the line-tying of the Z-pinch on the unmagnetized plasmas around X-points as discussed in[14] may provide for their stability.

In the theory used for the stability estimates,only first order terms were included.Probe measurements in a similar system reported in [3] have shown the induction of a quasi-stationary longitudinal field of a complex, azimuthally periodic structure due to the external quadrupole field.They suggest that these fields are generated by Hall and drift currents.This quasi-stationary toroidal field affects strongly the growth rate of the sausage m=0 instability.Our conjecture is that the same phenomenon might occur in EXTRAP too and therefore accounts for the observed stability on this timescale.

Thinking along these lines,we conclude that whereas EXTRAP consists of a high frequency internal magnetic field and a constant octupole field,the principle in the other dynamically stabilized schemes is just the reverse.This fact alleviates the usual severe constraint on the quality factor Q of the a.c.generator,providing an external field.On the other hand,there is no need for the r.f.power supply of a Z-pinch due to these intrinsic high frequency oscillations.The magnetic field varies at a rate comparable to the growth rates of the most devastating instabilities.To generate these oscillations it is sufficient to put a pinch into a position different from the Bennet´s equilibrium.

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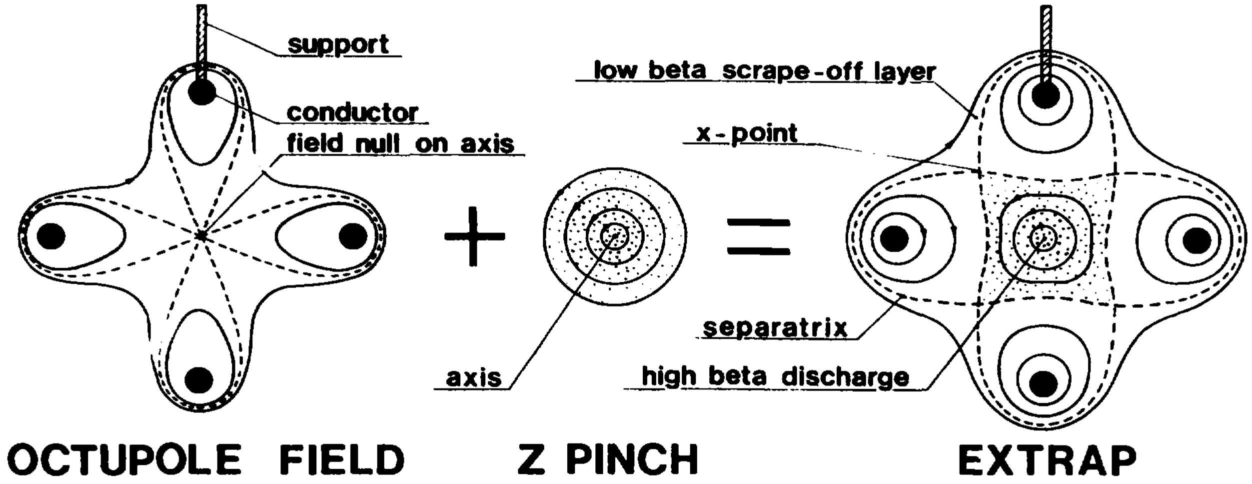
Figure captions.

Fig.1 A linear EXTRAP configuration is produced by a Z-pinch,generated along the axis of an octupole vacuum field,induced by currents in four rods.antiparallel to the current in the pinch.The resulting configuration is bounded by a separatrix defined by four X-points nulls.The high-ß is located around the 0-axis and the low-ß plasma,contained by an octupole field is located at the periphery.

Fig.2 Self-similar oscillations of the pinch radius(solid lines) and temperature(dashed lines) normalized to their initial values as functions of time normalized to a time constant $t_1= R_0/(4T_1/m_i)^{1/2}$ in EXTRAP for a)$T_0/T_1=1.3$

b) $T_0/T_1=0.7$ and without damping.

Fig.3 Self-similar oscillations of the pinch radius(solid lines) and temperature(dashed lines) normalized to their initial values as functions of time normalized to a time constant $t_1= R_0/(4T_1/m_i)^{1/2}$ for a)$T_0/T_1=1.3$ b) $T_0/T_1=0.7$ and with damping $\tau=34$ t_1.


support
conductor
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low beta scrape-off layer
x-point
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high beta discharge
OCTUPOLE FIELD
Z PINCH
EXTRAP

Fig.1

Fig. 2

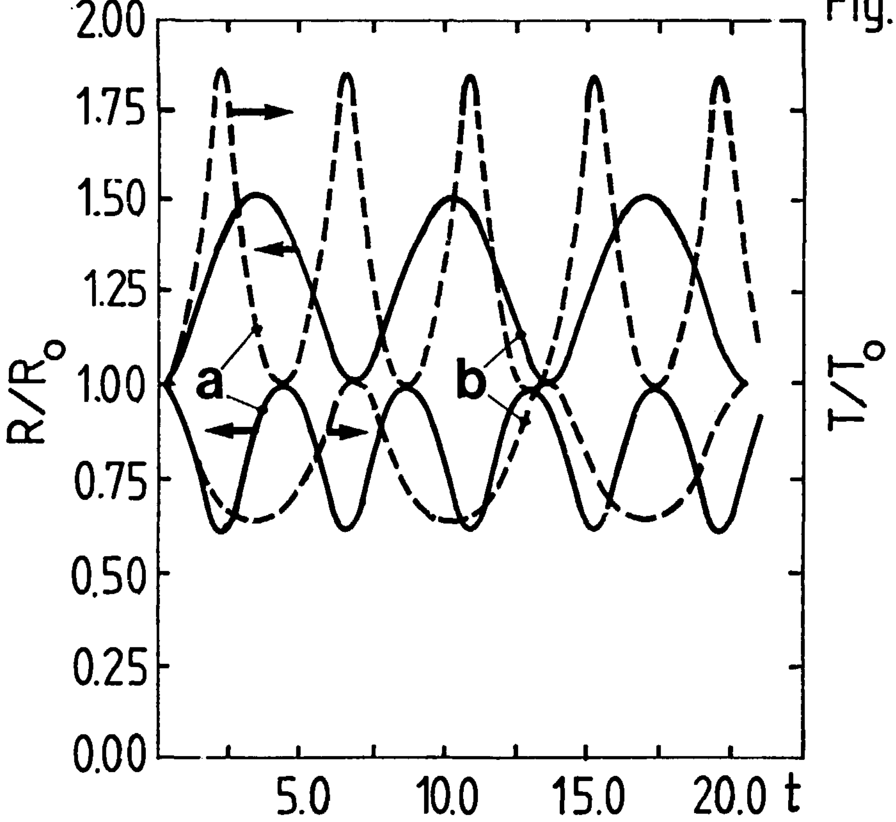

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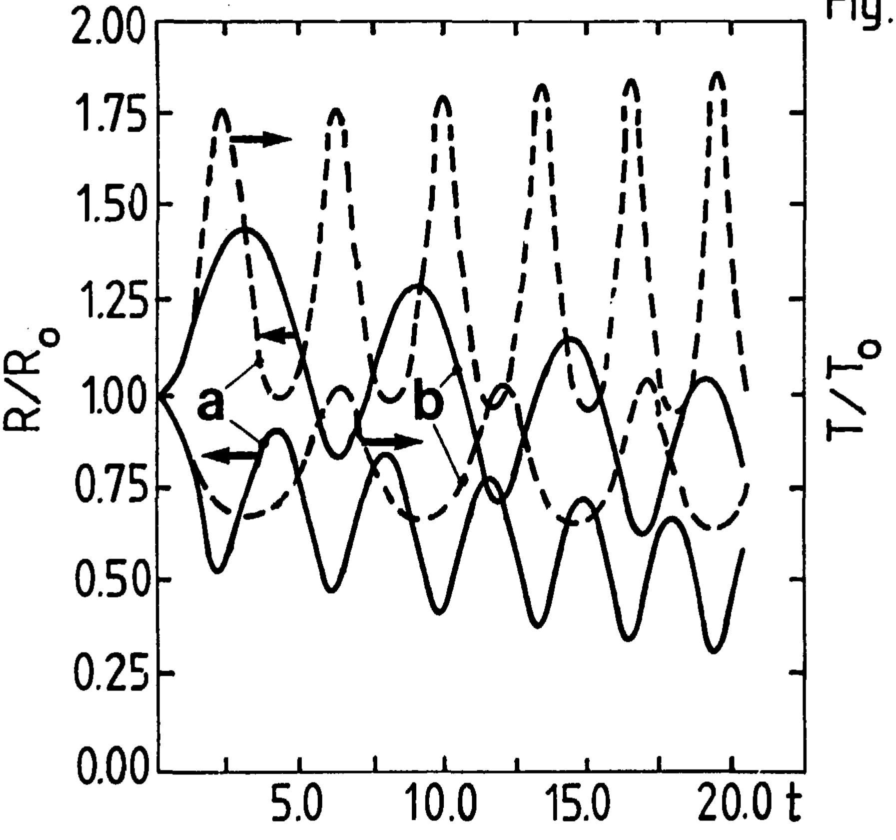
Fig. 3



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Key words Z-pinch,EXTRAP,self-similarity,dynamic stabilization,profile consistency