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STOCHASTIC NUCLEAR REACTION THEORY: BREIT-WIGNER NUCLEAR NOISE

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INTRODUCTION

Our present understanding of neutron-nucleus interaction is largely based on Bohr's statistical compound nucleus model.[1] However, theoretical developments based on quantum corrections[2] to Bohr's model, together with considerable improvements in neutron spectroscopy have predicted and revealed the presence of marked fluctuations in the neutron cross sections of both fissile and fertile nuclei. These fluctuations are wider than the sharp resonances associated with the compound nucleus levels and narrower than the broad structure due to the energy dependence of the neutron penetration coefficients. They, in fact, represent departures from the statistical compound nucleus model in localized energy regions, leading to an intermediate structure which is not predicted by Bohr's model. The observed enhancement of the neutron cross section is due to the presence of doorway states[3,4] in the neutron channel or in the fission channels in fissile nuclei. The understanding and detection of this intermediate structure is of great relevance both in nuclear reaction theory and for the calculation of nuclear reactor parameters.

The purpose of this paper is the application of various statistical tests for the detection of the intermediate structure, which lies immersed in the Breit-Wigner "noise" arising from the superposition of many compound nucleus resonances. To this end, neutron capture cross sections are constructed by Monte-Carlo simulations of the compound nucleus,[5] hence providing the "noise" component. In a second step intermediate structure is added to the Breit-Wigner noise. The performance of the statistical tests in detecting the intermediate structure is evaluated using mocked-up neutron cross sections as the statistical samples. Afterwards, the statistical tests are applied to actual nuclear cross section data.

GENERAL THEORY

The doorway states are special states with a decay width to the continuum. These states have the following properties: (1) their level widths are larger than the average width of the compound nucleus states, (2) they are not eigenfunctions of the total nuclear Hamiltonian, but of a slightly different Hamiltonian which differs from the

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former by a residual interaction, and (3) these special states satisfy the boundary conditions of R-matrix theory.

The introduction of the special levels provides a particularly simple device to interpret the intermediate structure. Endowed with a large level width and with a residual interaction potential, they interact with the compound nucleus levels, which share the strength of the special state. This process leads to enhancements of the cross section in localized energy regions which cannot be explained within the framework of the statistical nuclear model. It can be shown,[6,7] that the intermediate structure is introduced by energy "resonant" reaction widths in the usual Breit-Wigner resonance formula. Hence the Breit-Wigner noise is not additive to the ordered intermediate structure.

MONTE-CARLO SIMULATIONS

Neutron capture cross sections for the ^{238}U nucleus were constructed by sampling from the statistical distribution functions for level widths[8] and level spacings,[9] in accordance to the statistical compound nucleus model. The intermediate structure was simulated by replacing the neutron widths in the $J = \frac{1}{2}, \ell = 1$, ^{238}U compound nucleus state, by their "resonant" counterparts. In every instance the average resonance parameters were adjusted to reproduce the measured average neutron cross section.

STATISTICAL TESTS

Statistical samples were generated by averaging the cross sections over energy bins of variable length, W. Thus, by varying the bin length, W, one obtains sets of cross sections, containing different levels of information.

For instance, by setting the value of W larger than the average spacing of the intermediate structure (a few keV), one erases from the statistical sample both the Breit-Wigner noise and the intermediate structure contributions, leaving only the long range energy dependence of the average cross section on the neutron penetration factors.

The Wald-Wolfowitz[10] (WW) distribution-free statistics test was applied to mocked-up (simulated) ^{238}U capture cross sections with and without the intermediate structure, as well as to sets of measured ^{238}U capture cross sections section data.

The WW test deals with the number of runs, R, of consecutive values which lie above or below a given reference line (in our case the average cross section is taken as the reference line). The WW statistics provides the number of runs, $E(R)$, to be expected from random statistical data, as well as the standard deviation $\sigma(R)$. It can also be shown that the ratio

$$\epsilon_R = \frac{[R - E(R)] - \frac{1}{2}}{\sigma(R)}$$

approximates a normal probability distribution, $P(\epsilon_R)$. Low values for the ratio, ϵ_R, indicate that the statistical sample approximates the statistics of a random sample. The distribution $P(\epsilon_R)$ gives the probability that the tested sample behaves as a random data set.

RESULTS

The WW test for the search of intermediate structure in the ^{238}U neutron capture cross section was applied to two sets of measurements,[8] set I and set II, and for two sets of simulated cross-section data: model I, constructed on the basis of the statistical compound nucleus model, and model II, containing the intermediate structure.

To evaluate the performance of the WW test, this methodology was applied to a set of random data (white noise), as well as to models I and II, and to the experimental data set I and set II. Figure 1 shows the number of runs (R), versus the averaging interval W. For averaging intervals up to 10 keV the results for model I (i.e., the cross section computed according to the statistical compound nucleus model) fall within the error bands of the number of runs expected for white noise. For values of W up to 2 keV, model II as well as sets I and II show deviations from the results for the random set of data. For W=10 keV, the intermediate structure appears to have been washed out and all the tested sets of data behave in a similar fashion. The ratios ϵ_R and the probability $P(\epsilon_R)$ obtained for each cross-section data set are given in Table 1.

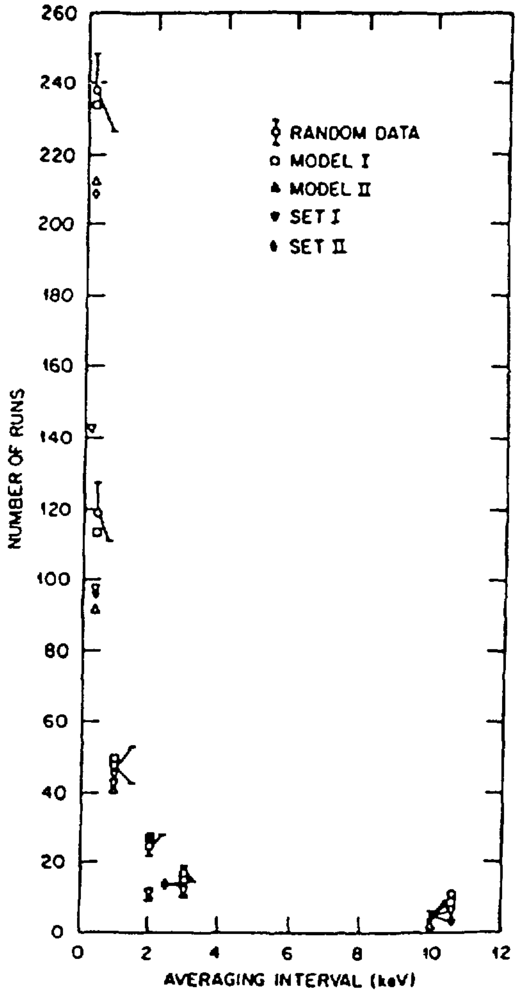

Figure 1. The Wald-Wolfowitz runs test for various simulated ^{238}U neutron capture cross sections and the experimental data, compared with the expected number of runs from a set of randomly distributed objects.

These results indicate that the experimental cross-section data behave similarly to the data constructed on the basis of the statistical nuclear model modified by the inclusion of intermediate structure. For W=400 eV, the significance level for the structure in the ^{238}U capture cross section is equal to that corresponding to about three standard deviations for a normal distribution. The effectiveness of the WW test for the detection of intermediate structure is shown in Table 2.

Table 1. Results of the Wald-Wolfowitz runs test for the ^{238}U capture cross-section measurements (sets I and II) and comparison with the results obtained for model II

	Set I		Set II		Model II	
Δ^a (keV)	ϵ_R	$P(\epsilon_R)$ (%)	ϵ_R	$P(\epsilon_R)$ (%)	ϵ_R	$P(\epsilon_R)$ (%)
0.2	8.72	$<10^{-15}$	2.65	0.4	2.20	1.4
0.4	2.81	0.2	2.99	0.1	3.41	0.03
1.0	1.03	15.0	0.42	33.0	1.37	8.0
2.0	2.34	0.9	0.30	38.0	2.49	0.65
3.0	0.98	16.0	0.36	36.0	1.05	14.7
10.0	0.4	34.0	0.40	34.0	1.41	7.8

[a]Energy averaging interval.

Table 2. Results of the Wald-Wolfowitz runs test for the mockup ^{238}U capture cross-section with and without intermediate structure

	Model I[b]		Model II[c]	
Δ^a (keV)	ϵ_R	$P(\epsilon_R)$ (%)	ϵ_R	$P(\epsilon_R)$ (%)
0.2	0.30	38.0	2.20	1.4
0.4	0.74	23.0	3.41	0.03
1.0	0.09	46.0	1.37	8.0
2.0	0.89	18.0	2.49	0.65
3.0	1.08	14.0	1.05	14.7
10.0	0.68	25.0	1.41	7.8

[a]Energy averaging interval.
[b]Mockup cross section without intermediate structure.
[c]Mockup cross section with intermediate structure.

The results pertaining to model I, show that on the average there is a 27% probability of obtaining larger or equal ratios, ϵ_R, from a sample of random data. In the presence of intermediate structure this figure goes down to about 3% for values of the energy averaging interval between 0.2 keV and 2 keV. For larger intervals, $P(\epsilon_R)$ increases again, indicating a progressive washout of the intermediate structure. Hence, the WW test indicates the persistence of intermediate structure effects up to energy intervals as large as 3 keV. For the 400-eV capture cross-section averages, there is a factor of five increase in the ratio, ϵ_R, for the model II case. Clearly, the WW test appears to be a sensitive test for the detection of ordered structures buried in Breit-Wigner noise.

From the present study we draw the following conclusions: (1) the Wald-Wolfowitz runs test is a suitable tool for the detection of intermediate structure in neutron cross-section data, and (2) the application of this test to simulated and measured cross-section data indicates with a high confidence level, the presence of substantial departures from the statistical compound nucleus model.

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