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## HYPERFINE INTERACTIONS, THE KEY TO MULTIQUARK PHYSICS?\*

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### ABSTRACT

Clues in the search for a fundamental description of hadron physics based on QCD may be obtained from a phenomenological constituent quark model in which the color-electric force binds quarks into saturated color-singlet hadrons, and finer details of the spectrum and multiquark physics are dominated by the color-magnetic hyperfine interaction.

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MASTER

## 1. INTRODUCTION - HOW DOES QCD MAKE HADRONS?

We now all believe that QCD is the correct theory of strong interactions and that eventually hadron structure and spectroscopy will be explained by QCD just like atomic structure and spectroscopy are explained by QED. However, we still have a long way to go, and need clues for the right directions of research both from new experiments and from theoretical phenomenology. Multiquark physics is of particular interest, since there is total chaos in this area. While simple phenomenological constituent quark models have been remarkably successful in describing and predicting experimental results in meson and baryon spectroscopy there is still not a single prediction in the multiquark sector which has been confirmed by experiment<sup>1</sup>.

The best hope for getting experimental predictions from QCD at present is believed to be from lattice calculations. However, the difficulties involved in these calculations for the multiquark sector are illustrated by the existence of two lattice calculations for the  $H$  dibaryon<sup>2</sup>, one concluding that the  $H$  is unbound<sup>3</sup> and the other that it is strongly bound<sup>4</sup>. One hears arguments on both sides<sup>5</sup> criticizing both calculations, and no one can state authoritatively from QCD whether the  $H$  will eventually be found as a bound state in experiments.

Experimental information on the possible existence of bound multiquark states would be very useful in helping theorists understand how QCD binds quarks and gluons together to make hadrons. Guidance from theorists would be very useful in helping experimenters know where to look for possible bound exotics. But experimenters should note that the Greek word for theory ( $\Theta$  E  $\Omega$  P I A) is completely unrelated to the Greek word for god ( $\Theta$  E O  $\Sigma$ ); one is spelled with an *Omega*, the other with an *Omicron*<sup>6</sup>. So theorists are not gods and experimenters should not treat theory as if it were theology.

Recently there has been some preliminary experimental evidence for exotic states<sup>7,8,9</sup>, and also theoretical arguments like that for the  $H$  suggesting other configurations like an anticharmed strange baryon, denoted by  $P_{\bar{c}}$ , with the constituents  $\bar{c}suud$  or  $\bar{c}sudd$  where the net hyperfine interaction is attractive and may produce a bound state<sup>10,12,11,13</sup>. Hopefully we have seen the beginning of a new era.

The aim of this talk is to explore the possibility of examining the properties of multiquark systems using a simple phenomenological constituent quark model which has been very successful in describing the properties of single hadrons, and which suggests that the hyperfine interaction plays a crucial role in multiquark physics.

In trying to understand how QCD makes hadrons out of quarks and gluons, the questions often arise why there are no strongly bound multi-quark exotic states like a dipion with a mass less than two pion masses or a dibaryon bound by 100 MeV, and why nuclei seem to be composed of three-quark clusters called nucleons rather than a quark gas, quark bags, a quark shell model<sup>14,15</sup> or a quark-gluon plasma. The constituent quark model gives a very simple answer. The color-electric force gives saturation and produces no strong attractive forces between color singlet hadrons<sup>16</sup>. The color-magnetic or “hyperfine” interaction plays a dominant role in the behaviour of possibly loosely bound multi-quark systems by introducing short-range color-spin correlations into a multi-quark wave function which forces the decomposition into simple color singlet clusters.

## 2. TWO EXAMPLES OF HYPERFINE CONTRIBUTIONS IN HADRON SPECTROSCOPY

### 2.1 The $\Lambda$ Magnetic Moment

One very interesting example of the success of experimental predictions based on the role of hyperfine interactions is the prediction of the  $\Lambda$  magnetic moment by DeRujula, Georgi and Glashow<sup>18</sup>. We reexamine their derivation to use only the minimum number of assumptions necessary to obtain their result. These are:

1. Hadron spin splittings are entirely due to a two-body hyperfine interaction proportional to the product of the color-magnetic moments of the constituent quarks. Thus

$$\frac{M(\Sigma^*) - M(\Sigma)}{M(\Delta) - M(N)} = \frac{\mu_s^{col}}{\mu_d^{col}} \quad (1a)$$

where  $\mu_f^{col}$  denotes the color magnetic moment of a quark of flavor  $f$ .

2. The electromagnetic magnetic moments of constituent quarks are proportional to the product of their electric charges and color magnetic moments; e.g..

$$\frac{\mu_s^{EM}}{\mu_d^{EM}} = \frac{\mu_s^{col}}{\mu_d^{col}} = \frac{m_d}{m_s} \quad (1b)$$

where  $\mu_f^{EM}$  and  $m_f$  denote the electromagnetic moment and mass of a quark of flavor  $f$ . The original derivation assumed Dirac moments for the quarks and introduced the quark mass ratio in this way<sup>18</sup>. In the present derivation we see that this assumption is not needed to obtain the successful prediction. However, other predictions use this assumption relating color magnetic moments to effective quark masses, thereby raising the interesting question of what is meant by the effective mass of a constituent quark. It is therefore convenient to introduce the quark mass assumption at this stage. This point is discussed in detail below.

3. Baryon magnetic moments are obtained by vector addition of the magnetic moments of the constituent quarks, using the unique spin coupling required by fermi statistics of colored quarks in a color singlet state with zero orbital angular momentum. Note: although this gives the angular momentum couplings of the “ $SU(6)$  wave function” there is no assumption of any  $SU(3)$  or  $SU(6)$  symmetry other than the assumption that the quark magnetic moments do not vary from one baryon to another.

Thus

$$-0.61 = \mu_\Lambda = (-1/3)\mu_p \cdot \frac{\mu_s^{EM}}{\mu_d^{EM}} = (-1/3)\mu_p \cdot \frac{M_\Sigma^* - M_\Sigma}{M_\Delta - M_N} = -0.61. \quad (2)$$

This exact agreement was predicted *before* the *subsequently* measured moment.

## 2.2 Charmonium Wave Functions at the Origin

Another example of the role of the hyperfine interaction in hadron physics arises in ratios like  $\Gamma(\eta_c \rightarrow \gamma\gamma)/\Gamma(J/\psi \rightarrow \mu^+\mu^-)$  of the widths of two-photon decays of quarkonium pseudoscalar mesons and leptonic decays of corresponding vector mesons<sup>19</sup>. These are generally assumed to be proportional to the square of the wave function at the origin, which in the simplest approximation<sup>20</sup> is the same for both states and cancels in the ratio. Corrections have been introduced and calculated to account for relativistic effects<sup>21,22,23</sup>, QCD corrections<sup>23,24</sup>, changes in the wave function from spin-dependent forces<sup>21</sup>, and gluon condensates<sup>24</sup>. The results are in a confused state at present with different treatments differing not only in the magnitude but also in the direction of the correction to the simple model.

The hyperfine interaction is attractive in the pseudoscalar state and repulsive in the vector. It therefore enhances the wave function at the origin of the pseudoscalar relative to the vector. We now show how a general model-independent estimate of this effect is obtained using experimental masses as input and no free parameters from the simple general assumption that the entire mass splitting results from a short range interaction proportional to the square of the wave function at the origin. Such an interaction has been shown to give large coherent effects from comparatively small differences in the wave function in cases where the wave function at the origin enters in two different ways.<sup>25</sup> In this case we obtain an enhancement of over 25% for  $\Gamma(\eta_c \rightarrow \gamma\gamma)/\Gamma(J/\psi \rightarrow \mu^+\mu^-)$  over the prediction which assumes the same wave function for both the  $J/\psi$  and  $\eta_c$ .

This interaction mixes only states having the same angular momentum quantum numbers; i.e. different radial excitations of the pseudoscalar or vector states. We therefore use a basis of radially excited s-wave states in the unperturbed basis which is the same for vector and pseudoscalar states, with masses denoted by

$M_{nS}$ . The matrix elements of such an interaction between two states  $|\alpha\rangle$  and  $|\beta\rangle$  are given by

$$\langle\beta|V|\alpha\rangle = v\psi_{\beta}^*(0)\psi_{\alpha}(0) \quad (3)$$

where  $v$  is a spin-dependent interaction strength parameter.

The value of the ground state wave function at the origin denoted by  $\psi_g(0)$  is given to first order in perturbation theory by

$$\begin{aligned} \psi_g^{(1)}(0) &= \psi_{1S}(0) + \sum_{n \neq 1} \frac{v\psi_{nS}^*(0)\psi_{1S}(0)}{M_{1S} - M_{nS}} \psi_{nS}(0) = \psi_{1S}(0) \left( 1 + \sum_{n \neq 1} \frac{v|\psi_{nS}(0)|^2}{M_{1S} - M_{nS}} \right) = \\ & \psi_{1S}(0) \left( 1 - \sum_{n \neq 1} \frac{(\Delta_{hyp}M)_g}{(\Delta_{rad}M)_{mS}} \cdot \frac{|\psi_{mS}(0)|^2}{|\psi_{1S}(0)|^2} \right) \end{aligned} \quad (4)$$

where  $(\Delta_{hyp}M)_x$  and  $(\Delta_{rad}M)_x$  denote the hyperfine shift and the radial excitation energy above the the ground state of the state  $x$ , and we use the first order hyperfine shift and the zero order radial excitation to obtain the wave function to first order. Then to first order in the perturbation, the hyperfine correction to the ratio of the ground state pseudoscalar and vector wave functions at the origin is given by

$$\frac{P_g(0)}{V_g(0)} - 1 = \sum_{n \neq 1} \frac{M_{Vg} - M_{Pg}}{M_{nS} - M_{1S}} \cdot \frac{|\psi_{nS}(0)|^2}{|\psi_{1S}(0)|^2} = \sum_{n \neq 1} \frac{M_{Vg} - M_{Pg}}{M_{nS} - M_{1S}} \cdot \left( \frac{M_{nS}}{M_{1S}} \right)^2 \cdot \frac{\Gamma(\psi_{nS} \rightarrow e^+e^-)}{\Gamma(\psi_{1S} \rightarrow e^+e^-)} \quad (5)$$

where we have used the well-known relation between the wave functions at the origin and the leptonic decay widths.

The first term in the positive definite sum gives a lower bound. For the case of charmonium, all the quantities in this term are known experimentally to give

$$\frac{|P_g(0)|^2}{|V_g(0)|^2} > \left( 1 + \frac{M_{J/\psi} - M_{\eta_c}}{M_{\psi'} - M_{J/\psi}} \cdot \left( \frac{M_{\psi'}}{M_{J/\psi}} \right)^2 \cdot \frac{\Gamma(\psi' \rightarrow e^+e^-)}{\Gamma(J/\psi \rightarrow e^+e^-)} \right)^2 = 1.26 \quad (6)$$

This first order result without any assumptions about the specific potential model is easily generalized to obtain an expression valid to all orders in the perturbation<sup>19</sup> and leads to the same qualitative conclusion. Enhancements of the order of 25% can result from the perturbation of the wave function by the hyperfine interaction, and errors introduced by neglecting these effects may be considerably greater than relativistic and QCD corrections. A full QCD calculation also includes these hyperfine effects on the wave function which arise from the spin-dependent part of gluon exchange. But QCD corrections are not easily included together with the phenomenological wave function effects considered here, as there is danger of double counting. The exact relation of these results with QCD sum rules is also

unclear, in particular the role of the gluon condensate which is dominant in the sum rule calculations<sup>24</sup> and disregarded elsewhere.

These examples show how a simple phenomenological constituent quark model can give meaningful experimental predictions at a stage where going beyond the simple phenomenology to introduce next order QCD corrections are still nearly hopelessly complicated. We therefore explore the consequences of this simple model further in both single hadron and multi-quark physics.

### 3. THE SAKHAROV-ZELDOVICH MODEL — A NUCLEAR PHYSICS APPROACH

#### 3.1 The Mass Formula

One of the earliest and still valid theoretical investigations which anticipated nuclear chromodynamics and realized the importance of the hyperfine interaction used a “nuclear physics approach to hadrons”. The matrix elements for effective interactions are obtained from the 2-body problem. These are then used for predictions in the N-body problem<sup>26,27</sup> with dramatic success for predicting the baryon spectrum from the meson spectrum. In 1966 Sakharov and Zeldovich obtained the mass relation<sup>26</sup>

$$m_s - m_u = M_\Lambda - M_N = 177 \text{ MeV} = \frac{3}{4} (M_{K^*} - M_\rho) + \frac{1}{4} (M_K - M_\pi) = 180 \text{ MeV}. \quad (7)$$

This striking evidence that mesons and baryons are made of the same quarks is obtained from a very simple mass formula which pinpoints the crucial role of the hyperfine interaction and forces the choice of the particular linear combinations of masses in eq. (7) in which the hyperfine interaction cancels out.

$$M = \sum_i m_i + \sum_{i>j} \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j} v_{ij}. \quad (8)$$

where  $m_i$  is an effective quark mass,  $\vec{\sigma}_i$  is a quark spin operator and  $v_{ij}$  is a hyperfine interaction. Note that this formula is completely relativistic, since it contains no explicit nonrelativistic dynamics and all relativistic effects are included in the effective masses and effective interactions. The remarkable success of this formula has recently been noted in a derivation of the sum of the nucleon magnetic moments<sup>1</sup> whose agreement with experiment with no free parameters is impressive.

The mass formula (8) can be interpreted in a language familiar to nuclear physicists as the sum of single-particle energies (called effective masses in this case) and two-body interactions. By using a flavor-dependent hyperfine interaction for the two-body interaction and assuming that all other contributions are included in the effective mass terms Sakharov and Zeldovich *anticipated* QCD and obtained

the relation (7) between meson and baryon masses<sup>28</sup> in surprising agreement with experiment and also other relations; e.g.

$$1.53 = \frac{M_{\Delta} - M_N}{M_{\Sigma^*} - M_{\Sigma}} = \frac{M_{\rho} - M_{\pi}}{M_{K^*} - M_K} = 1.61 = \frac{m_s}{m_u} \quad (9)$$

An equivalent formula for the quark mass ratio was noted by Sakharov<sup>29</sup>, along with the comment that the masses are of course effective masses. This Sakharov-Zeldovich model was later rediscovered and refined by De Rujula, Georgi and Glashow<sup>13</sup> and by Cohen and Lipkin<sup>30</sup> who explained the small discrepancy in eq. (9) by a refinement of the model involving the different sizes of baryon and meson wave functions<sup>30</sup>.

### 3.2 Magnetic Moments and Effective Quark Masses

DeRujula et al<sup>18</sup> added input from QCD that the hyperfine interaction is produced by one gluon exchange and inversely proportional to the same quark masses appearing in the magnetic moments, explained the sign of the  $\Delta - N$  and  $\rho - \pi$  mass splittings, and obtained the prediction (2) for  $\mu_{\Lambda}$ .

Assuming that the parameter  $m_i$  appearing in both terms in eq. (8) was the same gave another independent good prediction for  $\mu_{\Lambda}$ <sup>31</sup>

$$-0.61 = \mu_{\Lambda} = -\frac{M_p}{3m_s} = \frac{-2M_p}{M_N + M_{\Delta} + 6(M_{\Lambda} - M_p)} = -0.58. \quad (10)$$

The basic physics expressed by these successful relations is in the three different roles of the quark mass parameter  $m_i$ .<sup>30</sup> The same value is used for mesons and baryons in both terms in eq. (8) and also for the Dirac moment of the constituent quark. Why this should be valid remains to be explained by QCD. Some indications of the underlying physics has been given in one simple model<sup>30</sup>, which showed that the same effective mass parameter appears in all three roles and includes to a good approximation both the relativistic corrections and the contributions from flavor and spin-independent effective two-body interactions whose effective matrix elements satisfy the well known relation<sup>16</sup>

$$\langle q(x_1)q(x_2); 3^* | V | q(x_1)q(x_2); 3^* \rangle = (1/2) \langle q(x_1)\bar{q}(x_2); 1 | V | q(x_1)\bar{q}(x_2); 1 \rangle \quad (11)$$

where  $|q(x_1)q(x_2); 3^*\rangle$  and  $|q(x_1)\bar{q}(x_2); 1\rangle$  denote respectively quark-quark and quark-antiquark states with the color antitriplet ( $3^*$ ) and color singlet ( $1$ ) couplings at the points  $x_1$  and  $x_2$ . This relation has been justified both by handwaving theoretical arguments and by its success in hadron spectroscopy<sup>32</sup>, and small corrections due to scaling the wave functions between mesons and baryons explain both the corrections to the simple mass formula (8) and the ratios (9).

## 4. THE SATURATION PROBLEM AT DIFFERENT LEVELS

### 4.1 Why Nuclei and Hadrons Do Not Collapse

In the old days of nuclear physics, when the proton and neutron were believed to be the fundamental constituents of nuclei and the nucleon-nucleon force was believed to be a two-body force attractive in all channels, it was difficult to see why the binding energy of a nucleus should not increase like the number of nucleon pairs, leading to a collapse of the nucleus. The Yukawa theory of meson exchange led to a universal attraction from pion exchange, but no saturation. Various attempts were made to introduce different kinds of forces to produce saturation, but without much success.

The saturation problem was then clarified by the experimentalists, who found a short-range repulsive core in the nucleon-nucleon interaction necessary to explain their scattering experiments. But then it became clear that nucleons were not elementary and were made of quarks which also had a universal attractive interaction. The saturation problem now arose at a deeper level in the absence of exotics. This problem was solved by the introduction of the color degree of freedom<sup>16</sup> which led to saturation at the quark-antiquark and three-quark levels, with mesons and baryons behaving like neutral atoms and having no color-electric force between them, and hadron-hadron interactions analogous to molecular interactions. However, there was at this stage no simple description of the repulsive core in the nucleon-nucleon interaction. The key to further progress was found in the hyperfine interaction.

The importance of the color-magnetic hyperfine interaction in the binding of multi-quark states has been discussed in the context of the MIT bag model.<sup>2,33</sup> A more general and nearly model-independent point of view, applying to Harvard<sup>18</sup> as well as to MIT, follows from the observation that hyperfine (color magnetic) energies like the  $\Delta - N$  mass difference are much larger than nucleon-nucleon interaction energies like the deuteron binding energy<sup>34</sup>,

$$M_{\Delta} - M_N \gg M_n + M_p - M_d. \quad (12)$$

This suggests that in any model where the  $\Delta - N$  mass difference is attributed to the hyperfine interaction, this interaction can be assumed to be dominant in examining the possible binding of multi-quark systems.

### 4.2 Hyperfine Interactions - the Pauli-Fermi-Heisenberg Repulsion

One might expect that an effective nucleon-nucleon repulsion at short distances could arise from the Pauli principle alone, when two nucleons overlap sufficiently so that identical quarks in the two nucleons feel the effect of the Pauli principle.

But Pauli alone is not enough. Since nonstrange quarks come in three colors and two flavors and have two spin states, it is possible to place twelve nonstrange quarks at the same point in space without violating the Pauli principle. However, we see that three  $u$  quarks with spin “up” at the same point are required by Pauli to have a color-antisymmetric wave function and therefore be in a color singlet state. This gives a color singlet state with  $T=3/2, S=3/2$  and is just a  $\Delta$ . The same argument applies to the three  $d$  quarks with spin “up”, the three  $u$  quarks with spin “down”, and the three  $d$  quarks with spin “down”. Thus Pauli blocking does not prevent pushing all twelve quarks close together, but it forces them into a  $4\Delta$  configuration which costs energies of the order of four times the  $N - \Delta$  mass splitting (12).

For a “pedestrian” picture of this effect let us consider a pair of  $d$  quarks looking for a path to enter a bound state. The most promising path, leading to a totally symmetric wave function in space, color and spin, is closed by the Pauli principle. The next path, leading to a spatially symmetric wave function which allows them to be close together and which is overall antisymmetric in color and spin in order to be allowed by Pauli is blocked by the Fermi hyperfine interaction, which is repulsive in this state and demands a high price in energy, of the order of the  $N - \Delta$  mass splitting. The third path, which is antisymmetric in space and satisfies the Pauli principle while avoiding the repulsive Fermi interaction requires the the wave function to vanish at very short distances. Now the Heisenberg uncertainty principle requires a high price in kinetic energy if the wave function changes rapidly enough to become large at reasonably short distances.

The two  $d$  quarks can find their way around this effect, which has been called the “Pauli-Fermi-Heisenberg repulsion”<sup>35,36</sup>, by finding a friend of a different flavor, a  $u$  quark, which can pull them past the Fermi barrier because the  $ud$  pairs are not restricted by Pauli and can be in the attractive symmetric color-spin state. This picture, with the two  $d$  quarks with negative electric charge repelling one another while being held together by a positively charge  $u$  quark, also explains the observed structure of the resulting bound state, the neutron, which has an overall electric charge of zero, but a nonvanishing electric charge radius arising from a positive charge density at the center and a negative charge density toward the surface.

This simple argument shows how the hyperfine interaction due to one gluon exchange between constituent quarks<sup>18,2</sup> introduces color-spin correlations into a nonstrange multiquark wave function which force it to dissociate into three-quark clusters. The hyperfine interaction is always repulsive between a pair of quarks of the same flavor. In two-flavor systems a net attraction at short range occurs only in the three-body system which has one pair of identical quarks and two pairs of quarks with different flavor. For systems with six or more nonstrange quarks the

ratio of the number of ( $ud$ ) pairs to the number of identical pairs is never more than 3:2 and there is an overall repulsion. For comparing this effect in different systems, it is convenient to define the “Pauli-Fermi-Heisenberg ratio”, denoted by  $R_{PFH}$  as the ratio of the number of nonidentical quark pairs in the system which have no restrictions from the Pauli principle to the number of identical pairs which are required to be in antisymmetric states.

$$R_{PFH}(N) = 2/1 = 2 \quad (13a)$$

$$R_{PFH}(d) = 9/(3 + 3) = 3/2 \quad (13b)$$

$$R_{PFH}(\alpha) = 36/(15 + 15) = 6/5 \quad (13c)$$

$$R_{PFH}(H) = (4 + 4 + 4)/3 = 4 \quad (13d)$$

$$R_{PFH}(P_{\bar{c}s}) = (2 + 2 + 1)/1 = 5 \quad (13e)$$

We thus see that nonstrange multiquark systems gain a more favorable value of  $R_{PFH}$  by breaking up into nucleons, while the  $H$  and the  $P_{\bar{c}s}$  have more favorable values than the nucleon and are tentatively seen as good candidates for possible bound multiquark exotics.

## 5. CALCULATIONS OF HYPERFINE BINDING

### 5.1 A Color-Spin Algebra Calculation

The above pedestrian “flavor-counting” argument has been made rigorous and quantitative by explicit calculations using Jaffe’s color-spin algebra<sup>2</sup>. The stability against breakup of an exotic multiquark system can be examined by checking whether hyperfine energy can be gained by recoupling the color and spins of the lowest lying two-hadron threshold<sup>37</sup>. We use a variational approach with a wave function in which the two-body density matrix is the same for all pairs as in a baryon, and can then use the experimental  $N - \Delta$  mass splitting to determine the strength of the hyperfine interaction energy<sup>37</sup>. This approach has also been used to search for possible multiquark bound states in configurations with wave functions in which the two-body density matrix is the same for all pairs as in some known hadron. The masses of such wave functions can be estimated from known hadron masses without assuming an explicit model for the forces, by using the refined and updated Sakharov-Zeldovich model<sup>30</sup> and assuming that the dominant interactions are the same effective two-body forces that have been used successfully in meson and baryon spectroscopy and which satisfy the relation (11)<sup>32,35,36</sup>.

A simplified form of the color-spin hyperfine interaction<sup>2</sup> can be used for systems containing only quarks and no active antiquarks:

$$V = -(v/2)[C_6 - C_3 - (8/3)S(S + 1) - 16N] \quad (14)$$

where  $v$  is a parameter defining the strength of the interaction,  $C_6$  and  $C_3$  denote the eigenvalues of the Casimir operators of the  $SU(6)$  color-spin and  $SU(3)$  color groups respectively,  $S$  is the total spin of the system and  $N$  is the number of quarks in the system.

The hyperfine interaction (14) is easily evaluated for the states of interest<sup>38</sup>: to give

$$V(N) = -8v \quad (15a)$$

$$V(\Delta) = 8v \quad (15b)$$

$$V(\Lambda) = -8v \quad (15c)$$

$$V(H) = -24v \quad (15d)$$

$$V(d) = +(8/3)v \quad (15e)$$

$$V(\alpha) = +96v \quad (15f)$$

$$V(\text{diquark}) = -8v \quad (15g)$$

$$V(T = A = 2) = +24v \quad (15h)$$

$$V(P_{\bar{s}}) = -16v \quad (15i)$$

The states  $d$  and  $\alpha$  denote a “quark-matter” deuteron and  $\alpha$  particle with 6 and 12 quarks respectively in the lowest shell. We have included the  $\Delta$ , the  $H$  dibaryon and the  $P_{\bar{s}}$  in the flavor  $SU(3)$  symmetry limit with no correction for the smaller hyperfine interaction of the strange quark without including the hyperfine interaction of the charmed antiquark in the  $P_{\bar{s}}$ . We have also included the “diquark” in the most attractive symmetric state, which is a color antitriplet ( $3^*$ ) with spin zero. The state ( $T = A = 2$ ) denotes a state with 6 nonstrange quarks coupled to isospin 2. A state with these quantum numbers has been suggested as a possible  $\pi^-nn$  bound state<sup>39</sup> and preliminary experimental evidence has been reported for such a state<sup>40</sup>. We then obtain

$$M_{\Delta} - M_N = V(\Delta) - V(N) = 16v \quad (16a)$$

$$V(d) - 2V(N) = (56/3)v = (7/6)[M_{\Delta} - M_N] \quad (16b)$$

$$V(\alpha) - 4V(N) = 128v = 8[M_{\Delta} - M_N] \quad (16c)$$

$$V(H) - 2V(\Lambda) = -8v = -(1/2)(M_{\Delta} - M_N) \quad (16d)$$

$$V(\text{diquark}) - V(N) = 0 \quad (16e)$$

$$V(P_{\bar{s}}) - V(\Lambda) = -8v = -(1/2)[M_{\Delta} - M_N] \quad (16f)$$

$$3V(\text{diquark}) - 2V(N) = -8v = -(1/2)(M_{\Delta} - M_N) \quad (16g)$$

$$V(T = A = 2) - 2V(N) = 40v = (5/2)[M_{\Delta} - M_N] \quad (16h)$$

where we assume that  $M_{\Delta} - M_N$  is entirely due to the hyperfine energy difference.

The quark model  $\alpha$  particle is seen to be unbound by  $8(M_{\Delta} - M_N)$ ; i.e. it costs this enormous hyperfine energy to squeeze four nucleon together so that all

12 quarks are in the same s-wave shell model orbit. The quark model deuteron is seen to be unbound by a much smaller amount,  $(7/6)(M_\Delta - M_N)$ , but it is still unbound. The gain in hyperfine interaction for the  $P_{\pi\pi}$  over the  $NF$  or  $\Lambda D$  threshold (degenerate in this symmetry limit) is equal to the gain for the  $H$  over the relevant  $\Lambda\Lambda$  threshold and is just half the  $\Delta - N$  mass splitting. At this level it appears that the  $P_{\pi\pi}$  is an equally attractive candidate for hyperfine binding as the  $H$  dibaryon<sup>10,12</sup>. The state ( $T = A = 2$ ) is seen to be unbound by  $(5/2)[M_\Delta - M_N] - M_\pi$  relative to the relevant  $\pi^-nn$  threshold. Thus if the preliminary experimental evidence for a bound state with these quantum numbers<sup>40</sup> is confirmed, it cannot be a simple six-quark state, but may be the suggested  $\pi^-nn$  bound state<sup>39</sup>.

This calculation uses "deuteron" and "alpha" wave functions with the same radius as the nucleon. The repulsion will be less if we allow the wave function to occupy a larger volume. This effect has been calculated by assuming that the range of the hyperfine interaction is much smaller than the nucleon radius and scaling the wave function to give a hyperfine energy inversely proportional to the volume<sup>41</sup>. The hyperfine energy for a six-quark or twelve-quark bag or shell model is considerably higher than that of separated nucleon clusters for any scale of the six or twelve-quark wave function.

## 5.2 Clustering — No Diquarks in Baryons and Dibaryons; Possible Molecules

The diquark has strong hyperfine binding, but one can easily see that diquark correlations are not introduced into baryons by the hyperfine interaction. The hyperfine energy of the normal proton configuration can be seen from eq. (15g) to be exactly equal to the hyperfine energy of a single ( $ud$ ) pair in the most attractive symmetric state. Thus there is no gain in hyperfine energy in the proton wave function if one ( $ud$ ) pair is placed in this attractive antisymmetric state and the remaining  $u$  quark is removed far enough away so that its hyperfine interaction vanishes. However, the quark-diquark wave function will have a higher color-electric energy since in any potential or string model, the color electric potential energy is minimized in the spatially symmetric configuration. This is also borne out by experiment which shows that the nucleon magnetic moments are described by the symmetric configuration.

Eq. (16g) shows that there is a gain in dibaryon hyperfine energy exactly equal to the gain in the  $H$  (16d) from using a wave function with three separated diquarks rather than two baryons. However, here the Pauli principle forces a price in the color electric and kinetic energies. Separated color antitriplet diquarks behave like spinless bosons and must have an overall symmetric wave function. The color singlet state of three antitriplets is totally antisymmetric in color.

For a "demon deuteron"<sup>42</sup> with deuteron-like quantum numbers the lowest

allowed state of three identical nonstrange diquarks is spatially antisymmetric and has two units of orbital angular momentum. Although the energy required for such orbital excitation is model dependent, a crude approximation is obtained from orbital excitation energies in the baryon spectrum, which are much larger than the hyperfine energy gain of half the  $\Delta - N$  splitting. For the doubly-strange  $H$  the  $(ud)$ ,  $(ds)$  and  $(su)$  diquarks are classified in a triplet of flavor  $SU(3)$  and can be in a flavor-antisymmetric singlet state with a spatially symmetric wave function. However, this flavor-spin coupling is forbidden by the Pauli principle at the quark level when there is an appreciable overlap between the wave functions of pairs of quarks of the same flavor in different diquarks. Keeping the diquarks sufficiently separated will cost color electric energy. This point has also been noted by Karl<sup>43</sup>.

The color-magnetic interaction can also give molecular type wave function extending over a distance large compared with the range of the hyperfine interaction. Such models have been proposed for the  $\delta$  and  $S^*$  mesons<sup>44,45</sup>. A rough estimate of the binding is obtained from a description by a two-body Schroedinger equation with a short range effective potential with a strength proportional to the gain in hyperfine energy by recoupling color and spin<sup>45</sup>. This equation has a bound state if the strength of the effective potential is greater than a critical value depending upon hadron masses. If the unknown proportionality factor between the hyperfine energy gain and the effective potential is determined by comparison with the  $\delta$  and  $S^*$  mesons under the assumption that the latter are barely bound  $K\bar{K}$  molecular states<sup>45,44</sup> the  $H$  and  $P_2$ , have been shown to be more strongly bound than the  $\delta$  and  $S^*$  mesons<sup>17</sup>. Thus the  $H$  and  $P_2$ , are excellent candidates for weakly bound molecular states.

### 5.3 Guides to Theoretical Investigations and Experimental Searches for Bound States

So far there has been no real experimental test of this description in multiquark systems, as there is as yet no convincing candidate for a multiquark bound state or resonance. If indeed the repulsive core in the nucleon-nucleon force is so strong that it prevents observation of short-range interactions in all multiquark systems, then there can be no real experimental test. For this reason the experimental discovery and verification of a multiquark bound state or resonance would be a significant breakthrough for our understanding of multiquark spectroscopy and of how QCD operates in multiquark systems, and a bridge between particle and nuclear physics.

The above hyperfine argument suggests that there can be a net attractive interaction in some systems with more than two kinds of components, either multiquark systems with three or more flavors or systems containing both quarks and antiquarks. However, it is not yet clear whether other forces may introduce ad-

ditional repulsions and prevent binding. Although Jaffe's original calculation for the  $H$  dibaryon<sup>2</sup> and subsequent work<sup>46</sup> indicate a gain in hyperfine interaction energy by recoupling color and spins in the six quark system over the two- $\Lambda$  system, a lattice gauge calculation<sup>3</sup> indicates that the  $H$  is unbound and well above the  $\Lambda\Lambda$  threshold. Furthermore, although hyperfine binding calculations<sup>46</sup> indicate sensitivity of the hyperfine energy to flavor- $SU(3)$  symmetry breaking, the lattice results are insensitive to the strange quark mass and  $SU(3)$  breaking<sup>3</sup>.

This difference in the effects of  $SU(3)$  breaking suggests that different physics dominates lattice gauge and bag or potential model calculations. Which calculations include the important physics is not obvious. However, the lattice calculation shows a repulsive  $\Lambda$ - $\Lambda$  interaction generated by quark exchange<sup>5</sup> which is not included in bag model calculations and could well prevent the six quarks from coming close enough together to feel the additional binding of the short range hyperfine interaction.

It is therefore of interest to look for other cases of hyperfine binding where such a repulsive exchange force may not be present; e.g. the anticharmed strange baryon  $P_{cs}$ , which is shown by eqs. (16) to have a hyperfine binding roughly equal to that of the  $H$ , but which has no possibility of a quark exchange force in the lowest decay channel  $FN$ , now called  $D_s N$ <sup>17</sup>. If this five quark system breaks up into an  $F$  (or  $D_s$ ) and a nucleon, there is no possible quark exchange between the two hadrons without flavor exchange, and therefore no diagonal matrix element of the one-gluon-exchange interaction that could give rise to a short range repulsion.

There is therefore interest both in experimental searches for the  $P_{cs}$ , and in lattice gauge calculations. The simplest lattice calculation with an infinitely heavy charmed antiquark and four light quarks  $uuds$ , can easily be done in parallel with the more complicated  $H$  calculation both in the symmetry limit where all light quarks have the same mass and with  $SU(3)$  symmetry breaking. Comparing the results for these cases may provide considerable insight into our understanding of the physics of QCD in multiquark systems even if the  $P_{cs}$  is not found as a physical bound state in experiment. There is however a difficulty in treating loosely bound molecular states on the lattice, since these are sensitive both to the details of the short range hyperfine interaction and the long range part of the wave function. A proper treatment of both these effects may require a rather large lattice.

## 6. PAULI REPULSION VS. OMEGA EXCHANGE

The question of the relation between the conventional nuclear physics description of nucleon-nucleon interactions with meson exchange and a more fundamental description in terms of quark variables remains open. It is clear that a long range interaction due to pion exchange exists and is not easily described in terms of

quark variables as long as there is no satisfactory description of the pion wave function in terms of quarks. However, the situation with regard to heavier mesons is unclear. One can worry about whether short range effects described by quark exchange and meson exchange effects with a similar range are describing the same physics, and whether it is justified to consider heavy meson exchange at all with an arbitrary truncation of the meson spectrum.

There have been suggestions that meson exchange and quark exchange have the same topology when described by quark diagrams and therefore describe the same physics. This however does not seem to be valid. In the analogous case of the force between hydrogen atoms, the analog of quark exchange is the electron exchange which produces molecular binding, while the analog of meson exchange is positronium exchange. The two appear to be very different, and illustrate the dangers of considering only topology without examining the kinematic variables.

The same questions arise in discussing the repulsive core in the nucleon-nucleon interaction. The conventional nuclear physics description is in terms of  $\omega$  exchange. The question then arises of the relation between this  $\omega$ -exchange picture and the Pauli-Fermi-Heisenberg description at the quark level, where the Pauli blocking between identical quarks plays a crucial role. Are these different ways of seeing the same physics at different levels or are they different, with one right and the other wrong? One can look for experimental tests to distinguish between the two cases by examining interactions in channels where there is no Pauli blocking, but the  $\omega$  is coupled and should give repulsion.

It has been pointed out that there is no Pauli blocking<sup>35</sup> in the interaction of a  $\Delta^{++}$  and a  $\Delta^{-}$ , since one consists entirely of  $u$  quarks and the other of  $d$  quarks, and similarly between two  $\Delta$ 's with completely antiparallel spins. This has led to the suggestion that there may be weakly bound  $\Delta^{++}\Delta^{-}$  bound states with quantum numbers  $T=3, J=0$  or  $T=0, J=3$ . However, the isoscalar  $\omega$  couples to the  $\Delta$  equally in all charge states and its dominant coupling is spin independent. Thus  $\omega$  exchange is expected to give an equal  $\Delta^{++}\Delta^{-}$  repulsion in all charge and spin states. This suggests that the existence of  $\Delta^{++}\Delta^{-}$  bound states with quantum numbers  $T=3, J=0$  or  $T=0, J=3$  might support the quark Pauli model and rule out  $\omega$  exchange.

There are also the isospin-exchange scattering reactions  $NN \rightarrow \Delta\Delta$  and  $NN \rightarrow N\Delta$  which can proceed via Pauli blocking at short distances, where the  $\Delta$  is mixed into the nucleon by the same effect that produces the repulsion. Such reactions cannot proceed via  $\omega$  exchange.

One must, however, be careful before jumping to such conclusions. In any model with  $\omega$  exchange there is also  $\rho$  exchange which couples to nonstrange quarks with the same strength as the  $\omega$ . The contribution of  $\rho$  exchange to the

nucleon-nucleon interaction is smaller than that of the  $\omega$  by a factor of nine because the  $\omega$  couples additively to all quarks while the  $\rho$  couples to isospin and the isospin of one quark in the nucleon is antiparallel to the other two. This gives a factor of three favoring the  $\omega$  at each vertex and a factor of nine in the interaction.

In the  $\Delta$ , however the isospins of the three quarks are parallel and the  $\rho\Delta\Delta$  and  $\omega\Delta\Delta$  couplings are equal. They exactly cancel for the  $\Delta^{++}\Delta^-$  interaction and give the same result as the quark-Pauli model. However, this cancellation does not occur for the  $\omega\Delta\Delta$  and  $\rho\Delta\Delta$  couplings for the same charge and opposite spin. This can be seen by choosing as basic vector meson states the linear combinations of the  $\omega$  and  $\rho$  which are pure  $u\bar{u}$  and  $d\bar{d}$  in the quark model. These then couple only to  $u$  quarks and  $d$  quarks respectively, just as the  $\phi$  couples only to strange quarks. The exchanges of vector mesons are thus seen to give nonvanishing contributions only to interactions between quark pairs of the same flavor, but the dominance of a spin-independent interaction remains.

Thus the  $\omega$  exchange model predicts qualitatively different behavior for the  $T=3, J=0$  and  $T=0, J=3$  channels, giving the same absence of repulsion as the quark-Pauli model for  $T=0, J=3$  and normal repulsion for  $T=3, J=0$ .

One might also worry about the  $\rho$  exchange contribution to the isospin exchange reactions  $NN \rightarrow \Delta\Delta$  and  $NN \rightarrow N\Delta$ . Here a detailed calculation is needed to see if this exchange can give a contribution similar to that of Pauli blocking. One would guess that this is not the case, because there is no obvious symmetry principle present to make them equal.

## REFERENCES

1. Harry J. Lipkin, Nucl. Phys. **A478**, 307c (1988).
2. R. L. Jaffe, Phys. Rev. Lett. **38**, 195 (1977).
3. P. MacKenzie and H. Thacker, Phys. Rev. Letters **55**, 2539 (1985).
4. Y. Iwasaki et al, Phys. Rev. Letters **60**, 1371 (1988).
5. H. Thacker, private communication.
6. N. Samios, private communication.
7. M. Bourquin et al., Phys. Lett. **B172**, 1133 (1986).
8. S. I. Bityukov et al, Phys. Lett. **B188**, 383 (1987).
9. D. Alde et al., Phys. Lett. **B205**, 397 (1988).

10. C. Gignoux, B. Silvestre-Brac and J. M. Richard, in *The Elementary Structure of Matter, Proceedings of the Workshop, Les Houches, France*, Edited by J. M. Richard, E. Aslanides and N. Boccara, Springer Proceedings in Physics 26, Springer Verlag (1987) p. 42
11. Harry J. Lipkin, in *The Elementary Structure of Matter, Proceedings of the Workshop, Les Houches, France*, Edited by J. M. Richard, E. Aslanides and N. Boccara, Springer Proceedings in Physics 26, Springer Verlag (1987) p. 24
12. C. Gignoux, B. Silvestre-Brac and J. M. Richard, *Phys. Lett.* **B193**, 323 (1987).
13. Harry J. Lipkin, *Phys. Lett.* **195B**, 484 (1987).
14. Igal Talmi, *Phys. Lett.* **205**, 140 (1987).
15. A. Arima, K. Yazaki and H. Bohr, *Phys. Lett.* **183**, 131 (1987).
16. H.J. Lipkin, *Phys. Lett.* **45B**, 267 (1973).
17. Harry J. Lipkin, Argonne preprint, ANL-HEP-CP-87-51, to be published in *Hadrons, Quarks and Gluons Proceedings of the XXII<sup>nd</sup> Rencontre de Moriond*.
18. A. De Rujula, H. Georgi and S. L. Glashow, *Phys. Rev.* **D12**, 147 (1975).
19. Harry. J. Lipkin, Weizmann Preprint WIS-88/13/Mar-PH to be published in the *Proceedings of the VIII International Workshop on Photon-Photon Collisions, Jerusalem (1988)*.
20. F. Close, *An Introduction to Quarks and Partons* (Academic Press, New York, (1978), Chapter 16.
21. Stephen Godfrey and Nathan Isgur, *Phys. Rev.* **D32**, 189 (1985).
22. L. Bergstrom et al, *Phys. Lett.* **82B**, 419 (1979).
23. D. Silverman and H Yao, *Phys. Rev.* **D36**, 3392 (1987).
24. L.J. Reinders, H. Rubinstein and S. Yazaki, *Physics Reports* **127**, 1 (1985).
25. Isaac Cohen, Nathan Isgur and Harry J. Lipkin *Phys. Rev. Lett.* **48**, 1074 (1982).
26. Ya. B. Zeldovich and A.D. Sakharov, *Yad. Fiz* **4**, 395 (1966); *Sov. J. Nucl. Phys.* **4**, 283 (1967).

27. P. Federman, H.R. Rubinstein and I. Talmi, *Phys. Lett.* **22**, 203 (1966);  
H.R. Rubinstein, *Phys. Lett.* **22**, 210 (1966).
28. Harry J. Lipkin, in *Gauge Theories, Massive Neutrinos and Proton Decay* (Proceedings of Orbis Scientiae 18th Annual Meeting, Coral Gables, Florida, 1981) edited by Behram Kursonoglu and Arnold Perlmutter (Plenum, N.Y., 1981), p. 359.
29. A. D. Sakharov, private communication; Harry J. Lipkin, *Annals of the New York Academy of Sciences* **452**, 79 (1985), and *London Times Higher Education Supplement*, January 20 (1984), p.17
30. I. Cohen and H. J. Lipkin, *Phys. Lett.* **93B**, 56 (1980).
31. H.J. Lipkin, *Phys. Rev. Lett.* **41**, 1629 (1978).
32. H. J. Lipkin, *Phys. Lett.* **171B**, 293 (1986).
33. K. Johnson, in *Fundamentals of Quark Models*, Proc. Seventeenth Scottish University Summer School in Physics, 1976, Edited by I. M. Barbour and A. T. Davies, Scottish University Summer School in Physics (1977) p. 245
34. Harry J. Lipkin, *Phys. Lett.* **70B**, 113 (1977).
35. Harry J. Lipkin, In *Intersections Between Particle and Nuclear Physics*, Proc. Conf. on The Intersections Between Particle and Nuclear Physics, Lake Louise, Canada, 1986 Edited by Donald F. Geesaman AIP Conference Proceedings No. 150, p. 657.
36. H. J. Lipkin, *Proceedings of the Workshop on Nuclear Chromodynamics*, Santa Barbara, August 12-23, 1985, ed. by S. Brodsky and E. Moniz, p. 328.
37. N. Isgur and H. J. Lipkin, *Phys. Lett.* **99B**, 151 (1981).
38. H. Högaasen and P. Sorba, *Nucl. Phys.* **B145**, 119 (1978).
39. Humberto Garcilazo, *Phys. Rev.* **C26**, 2685 (1982).
40. D. Ashery et al, Los Alamos preprint, to be published in *Physics Letters*.
41. Harry J. Lipkin, *Phys. Lett.* **198B**, 131 (1987).
42. Harry J. Lipkin, *Phys. Lett.* **117b**, **117B**, 457 (1982).
43. G. Karl, private communication.

44. John Weinstein and Nathan Isgur, Phys. Rev. Lett. **48**, 659 (1982); Phys. Rev. **D27**, 588 (1983).
45. Harry J. Lipkin, Phys. Lett. **124B**, 509 (1983).
46. J. L. Rosner, Phys. Rev. **D33**, 2043 (1986).
47. Harry J. Lipkin, Phys. Lett. **B74**, 399 (1978).