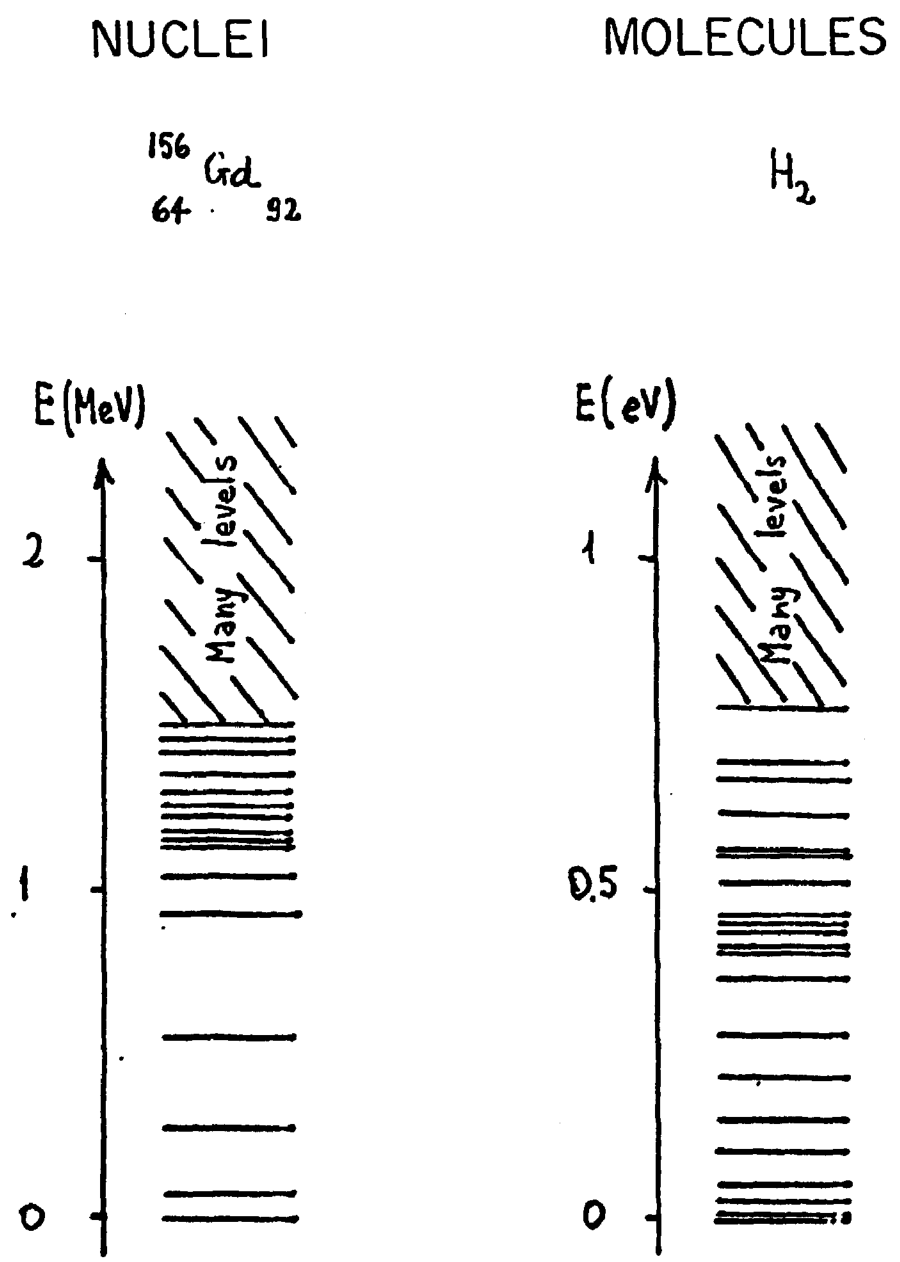
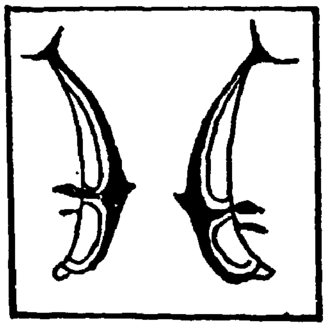
SYMMETRIES IN MOLECULAR AND NUCLEAR PHYSICS

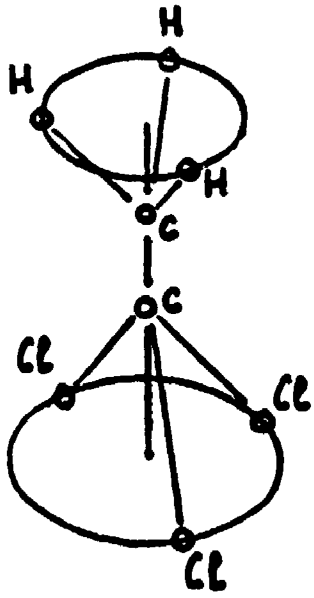
F. Iachello - Dept. of Physics, Yale University



$\Rightarrow$ Symmetries



FLOOR PATTERN FOUND AT THE MEGARON IN TYRINS, GREECE (ABOUT 1200 B.C.)



GEOMETRIC SYMMETRIES : THE TWISTED H_3C-CCl_3 MOLECULE (C_3 SYMMETRY)

Symmetries in physics

(i) Exact symmetries

(a) Poincarè invariance

$$\mathcal{L}(x) = \bar{\psi}(x)\left[i\gamma^{\mu}\frac{\partial}{\partial x^{\mu}} - m\right]\psi(x)$$

(b) Rotational invariance

$$H = \frac{p^2}{2m} + V(r)$$

(ii) Dynamic symmetries

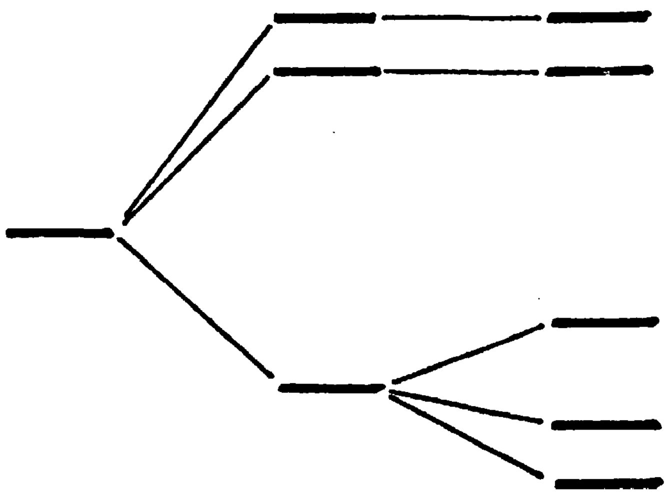
(a) The Hamiltonian has group structure G

(b) The Hamiltonian can be written in terms only of Casimir invariants of a chain of groups $G \supset G' \supset \ldots$

$\Rightarrow$ Properties of the system given in closed form in terms of quantum numbers

$\Rightarrow$ Classification scheme

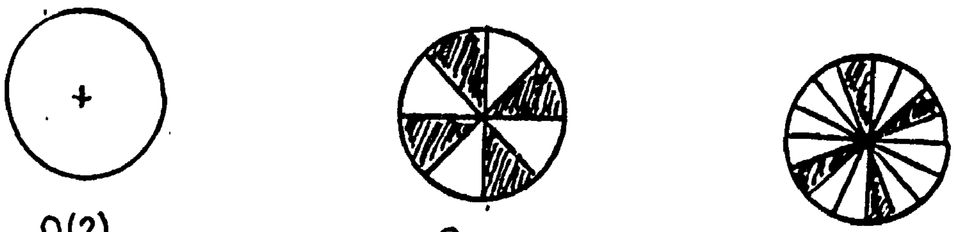
$\Rightarrow$ Splitting of the representations of G by successive addition of invariant operators



$$G \supset G' \supset G'' \supset \dots$$

$$H = \alpha C(G) + \alpha' C(G') + \alpha'' C(G'') + \dots$$

ILLUSTRATION OF DYNAMIC SYMMETRIES BY REGULAR BREAKING OF ROTATIONAL INVARIANCE

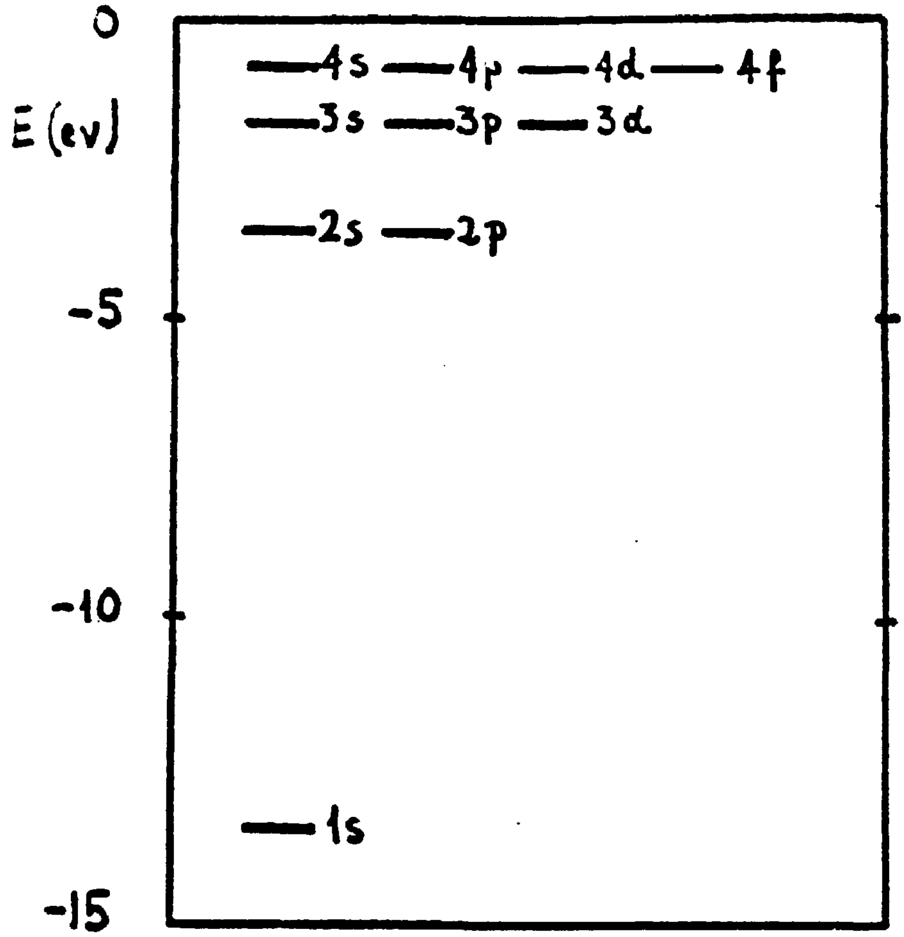


$O(2)$ $\qquad$ C_4 $\qquad$ C_2

Examples

(a) The hydrogen atom

$$E(n, \ell, m_\ell) = -A/n^2$$



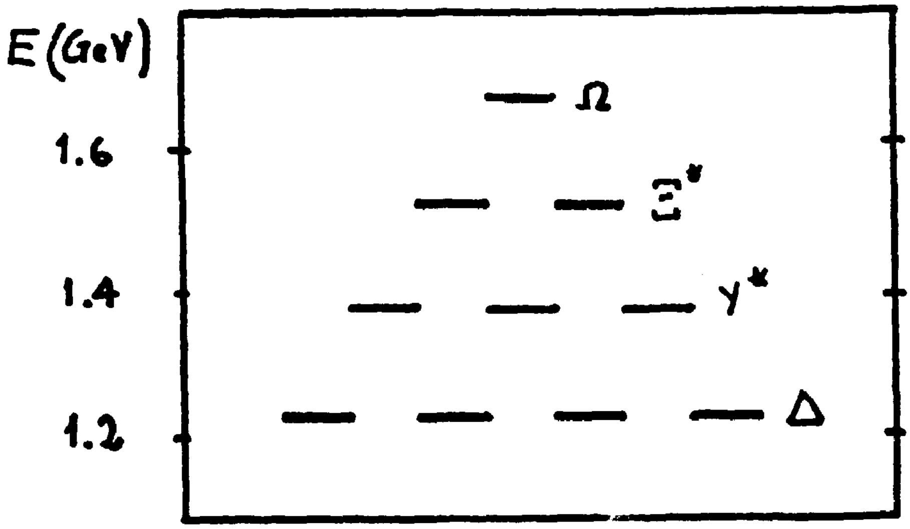
(b) Gell-Mann - Ne'eman SU(3)

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \text{ quarks}$$

$$\begin{array}{ccccccccc} G & & & G' & & & & G'' & \\ \downarrow & & & \downarrow & & & & \downarrow & \\ SU(3) & \supset & SU(2) & \otimes & U(1) & \supset & SO(2) & \otimes & U(1) \\ & & \uparrow & & \uparrow & & \uparrow & & \\ & & I & & Y & & I_3 & & \end{array}$$

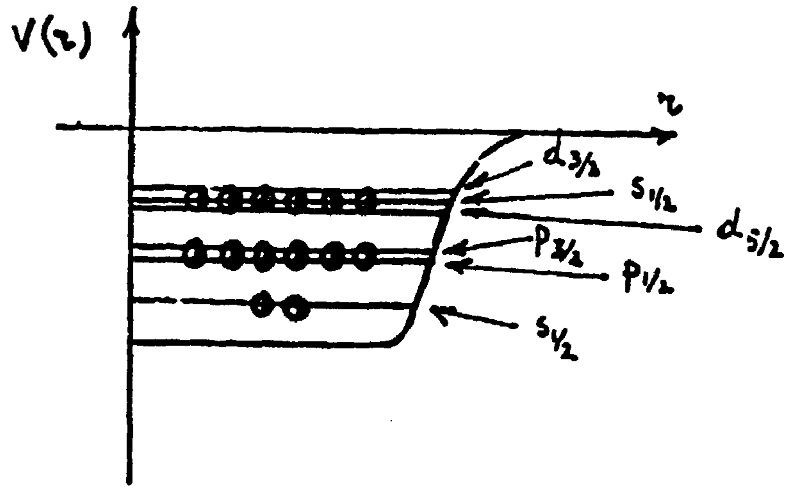
$\Rightarrow$ Energy formula (Gell-Mann - Okubo)

$$E(I, I_3, Y) = a + bY + c\left[\frac{1}{4}Y^2 - I(I+1)\right]$$

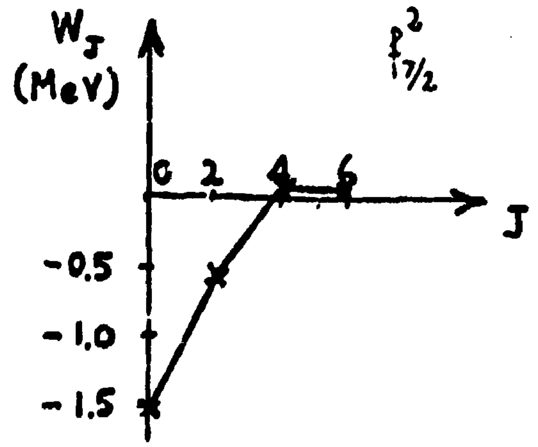


NUCLEAR STRUCTURE

CENTRAL FIELD



CORRELATIONS



COOPER PAIRS

S-BOSON

J=0

D-BOSON

J=2

INTERACTING BOSON MODEL

A. Arima and F. Iachello, Ann. Phys. (N.Y.) **99**, 253 (1976); **111**, 201 (1978); **123**, 468 (1979).

(a) Building blocks

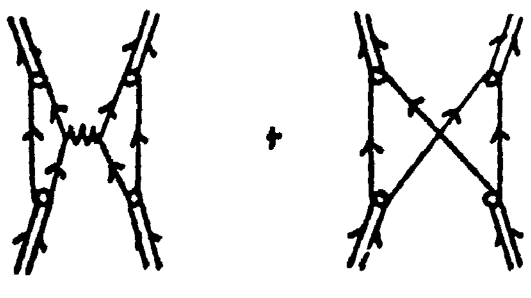
Boson operators

$$b^{\dagger}_{\alpha}\ (\alpha=1,\dots,6) \equiv s^{\dagger}, d^{\dagger}_{\mu}\ (\mu=\pm 2,\pm 1,0)$$

(b) Hamiltonian

$$H = E_0 + \sum_{\alpha\beta} \varepsilon_{\alpha\beta}\, b^{\dagger}_{\alpha} b_{\beta} + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} u_{\alpha\beta\gamma\delta}\, b^{\dagger}_{\alpha} b^{\dagger}_{\beta} b_{\gamma} b_{\delta}$$

Boson-boson interaction



(c) The interacting boson model $U(6)$

$$U(6) \nearrow U(5) \supset SO(5) \supset SO(3) \supset SO(2) \qquad \text{(I)}$$

$$U(6) \longrightarrow SU(3) \supset SO(3) \supset SO(2) \qquad \text{(II)}$$

$$U(6) \searrow SO(6) \supset SO(5) \supset SO(3) \supset SO(2) \qquad \text{(III)}$$

$\Rightarrow$ Energy formulas

$$E(N, n_d, v, n_\Delta, L, M_L) = \varepsilon n_d + \alpha n_d(n_d+4) + \beta v(v+3) + \gamma L(L+1) \qquad \text{(I)}$$

$$E(N, \lambda, \mu, K, L, M_L) = \kappa(\lambda^2+\mu^2+\lambda\mu+3\lambda+3\mu) + \kappa' L(L+1) \qquad \text{(II)}$$

$$E(N, \sigma, \tau, \nu_\Delta, L, M_L) = A\,\sigma(\sigma+4) + B\,\tau(\tau+3) + C\,L(L+1) \qquad \text{(III)}$$

$\Rightarrow$ Classification scheme for even-even nuclei

(d) Dual interpretation of the bosons

(i) Quantal (or particle)

Correlated pairs of nucleons with $J=0$ and $J=2$ (Cooper-like pairs)

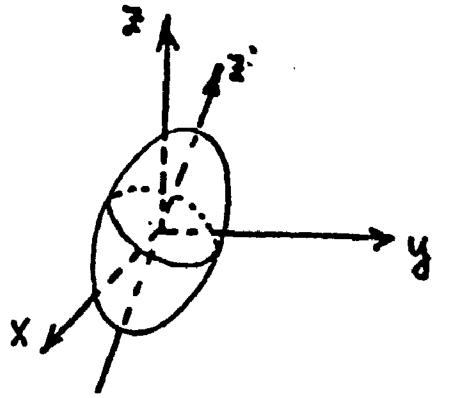
$$\Updownarrow$$

(ii) Classical (or geometric)

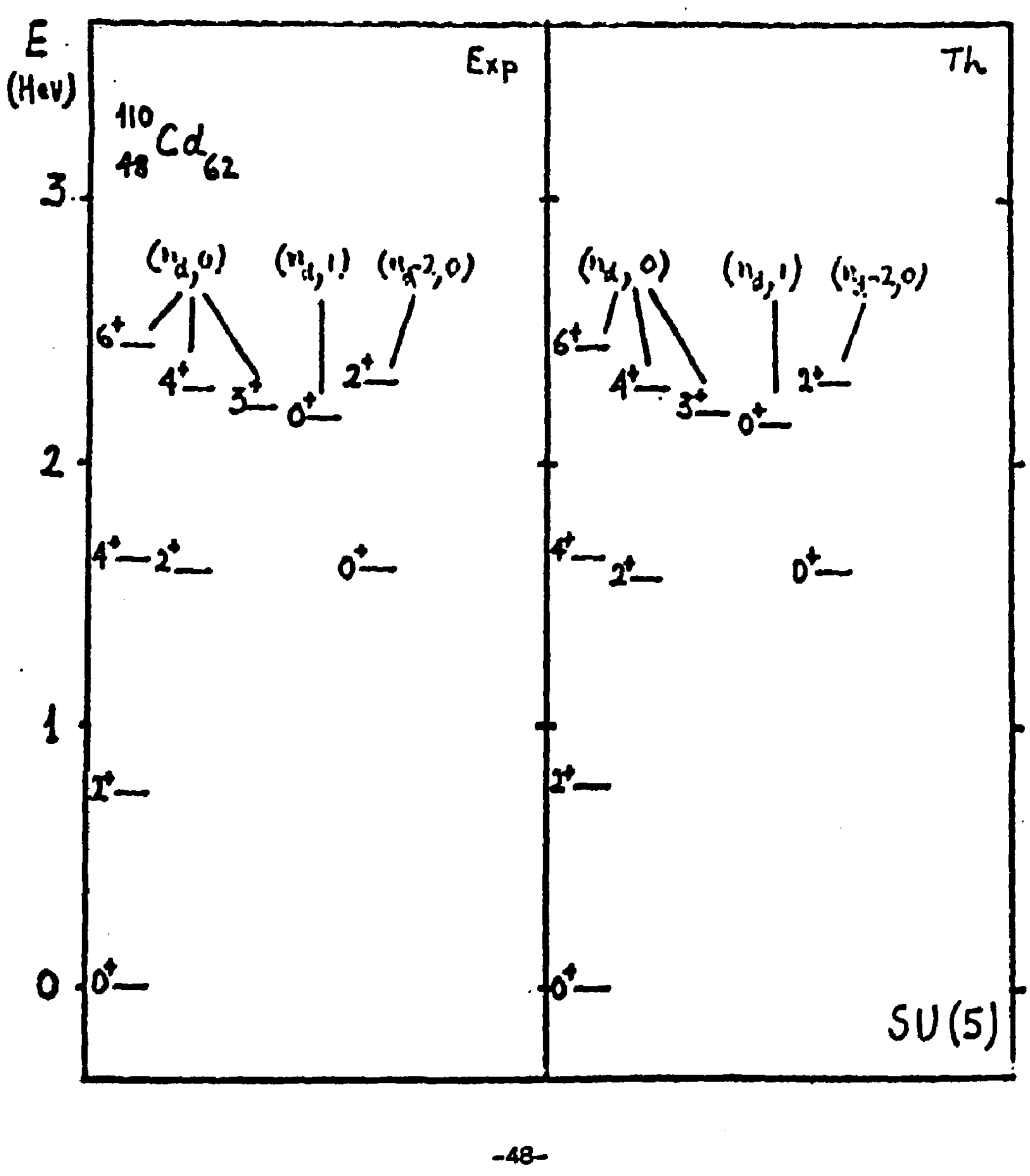
· Coherent state

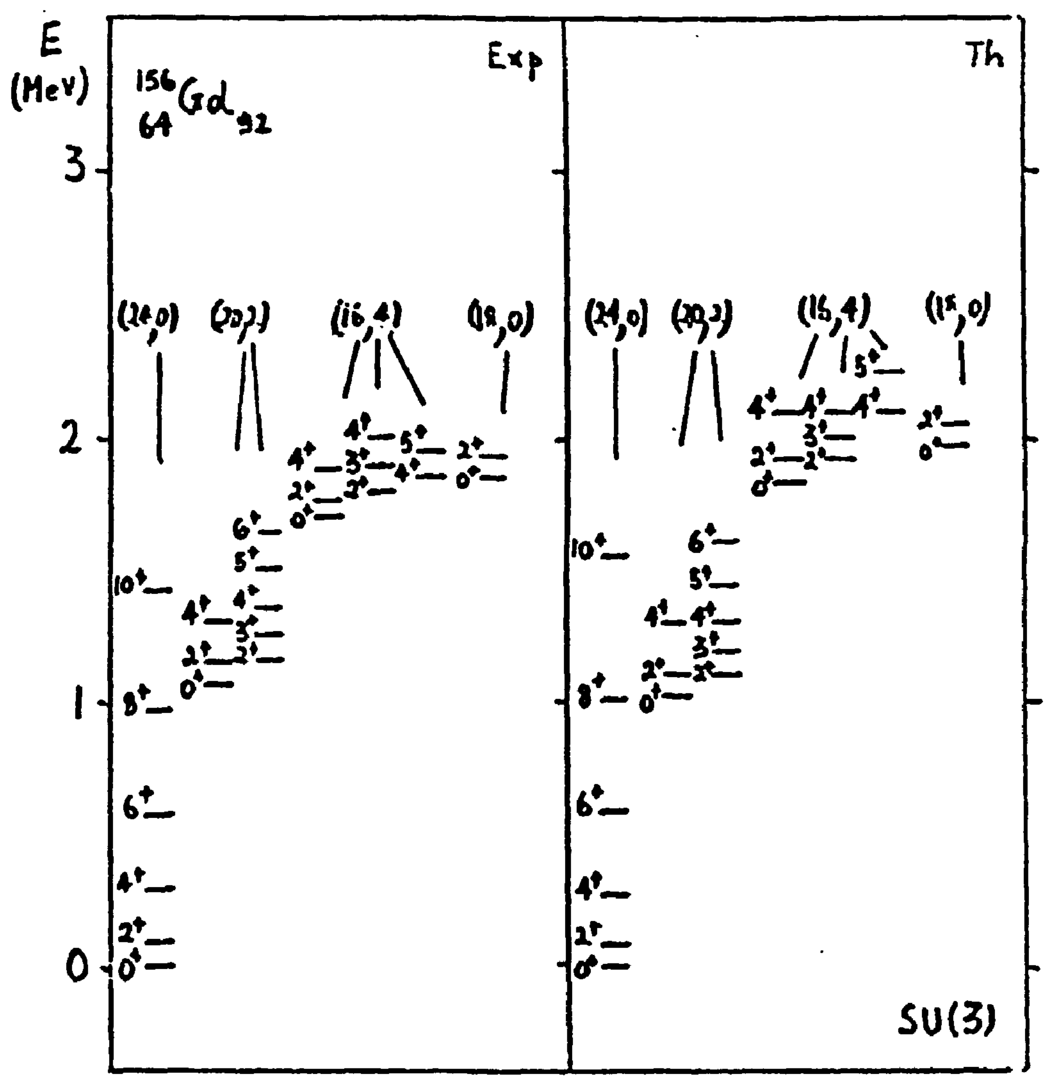
$$|N, \alpha_\mu\rangle = \left(s^\dagger + \sum_\mu \alpha_\mu d^\dagger_\mu\right)^N |0\rangle$$

Classical variables



$$R = R_0\left(1 + \sum_\mu \alpha_\mu Y_{2\mu}(\theta,\varphi)\right)$$





$\Rightarrow$ Classification of shapes of nuclei in terms of symmetry groups

(I) Spherical shape

(II) Axially symmetric deformed shape

(III) Non-axially symmetric deformed shape

CLASSIFICATION OF THE SHAPES OF CRYSTALS BY MEANS OF POINT GROUPS

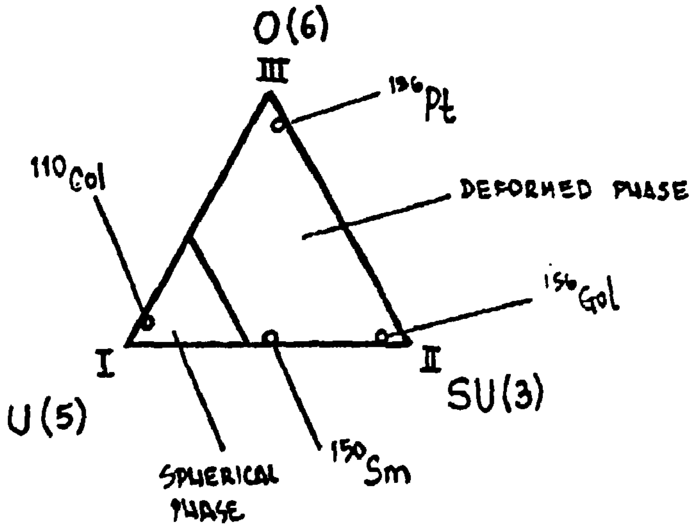
$\bar{O}(3)$ →	TRICLINIC	C_1, S_2	I)
	MONOCLINIC	C_{1h}, C_2, C_{2h}	II)
	ORTHORHOMBIC	C_{2v}, D_2, D_{2h}	III)
	TRIGONAL	C_3, S_6, C_{3v}, D_3, D_{3h}	IV)
	TETRAGONAL	S_4, D_{2d}, C_4, C_{4h}, D_4, D_{4h}	V)
	HEXAGONAL	C_6, C_{3h}, D_{3h}, C_{6h}, C_{6v}, D_6, D_{6h}	VI)
	CUBIC	T, T_h, T_d, O, O_h	VII)

(I) SPHERICAL SHAPE

(II) AXIALLY DEFORMED SHAPE

(III) γ-UNSTABLE DEFORMED SHAPE

SHAPE PHASE DIAGRAM



NORMAL SYMMETRIES

BOSONS $\longleftrightarrow$ BOSONS

OR

FERMIONS $\longleftrightarrow$ FERMIONS

SUPERSYMMETRIES

BOSONS $\longleftrightarrow$ FERMIONS

DYNAMIC SUPERSYMMETRIES

(i) THE HAMILTONIAN HAS SUPERGROUP STRUCTURE G^*.

(ii) THE HAMILTONIAN CAN BE WRITTEN IN TERMS ONLY OF INVARIANT OPERATORS OF A COMPLETE CHAIN OF GROUPS (OR SUPERGROUPS).

SUPERSYMMETRIES IN PHYSICS

(i) EXACT SUPERSYMMETRIES

(a) WESS-ZUMINO SUPERSYMMETRIES

$$\mathcal{L}(x) = -\frac{1}{2}\left(\partial_\mu A(x)\right)^2 - \frac{1}{2}\left(\partial_\mu B(x)\right)^2 - \frac{1}{2}\, i\, \bar{\psi}(x)\gamma^\mu \partial_\mu \psi(x)$$

$$-\frac{1}{2} m^2 A^2(x) - \frac{1}{2} m^2 B^2(x) - \frac{1}{2} i m \bar{\psi}(x) \psi(x)$$

$$- g m A(x) \left[A^2(x) + B^2(x)\right] - \frac{1}{2} g^2 \left[A^2(x) + B^2(x)\right]$$

$$- i g \bar{\psi}(x) \left[A(x) - \gamma_5 B(x)\right] \psi(x)$$

(ii) DYNAMIC SUPERSYMMETRIES

(a) THE HAMILTONIAN HAS SUPERGROUP STRUCTURE G^*

(b) THE HAMILTONIAN CAN BE WRITTEN IN TERMS ONLY OF CASIMIR OPERATORS OF A COMPLET CHAIN OF GROUPS (OR SUPERGROUPS)

$G^* \supset G' \supset G'' \supset \ldots$

MATHEMATICAL FRAMEWORK

Graded Lie groups

(a) Generators

$$\underset{\text{Bosonic}}{\underset{\uparrow}{G_\alpha}} \quad , \quad \underset{\text{Fermionic}}{\underset{\uparrow}{F_i}}$$

(b) Commutation relations

$$[G_\alpha, G_\beta] = c_{\alpha\beta}^{\gamma} G_\gamma$$

$$[G_\alpha, F_i] = f_{\alpha i}^{j} F_j$$

$$\{F_i, F_j\} = g_{ij}^{\alpha} G_\alpha$$

SUPERALGEBRAS $U(n|m)$

GENERATORS

$$G^{(B)}_{\alpha\alpha'} = b^\dagger_\alpha b_{\alpha'} \qquad n^2$$

$$G^{(F)}_{ii'} = a^\dagger_i a_{i'} \qquad m^2$$

$$F^\dagger_{\alpha i} = b^\dagger_\alpha a_i \qquad mn$$

$$F_{i\alpha} = a^\dagger_i b_\alpha \qquad mn$$

$$\overline{\qquad (m+n)^2}$$

FERMI SECTOR BOSE SECTOR

$$\left(\begin{array}{c|c} b^\dagger b & b^\dagger a \\ \hline a^\dagger b & a^\dagger a \end{array}\right)$$

REPRESENTATIONS

YOUNG SUPERTABLEAUX

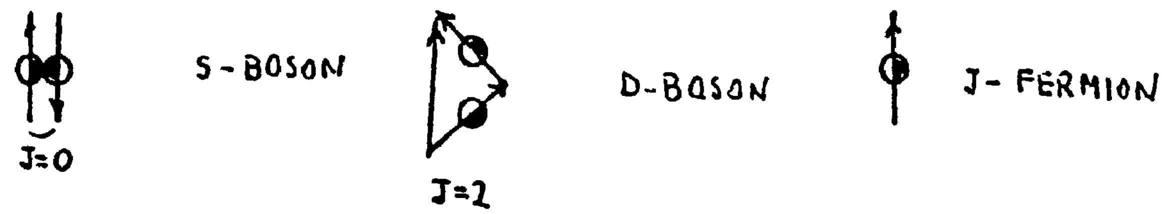
$$[N\} \equiv \overbrace{\boxtimes\ \boxtimes \ \ldots\ \boxtimes}^{N}$$

INTERACTING BOSON-FERMION MODEL

F. Iachello and O. Scholten, Phys. Rev. Lett. **43**, 679 (1979)

F. Iachello and S. Kuyucak, Ann. Phys. (N.Y.) **136**, 19 (1981)

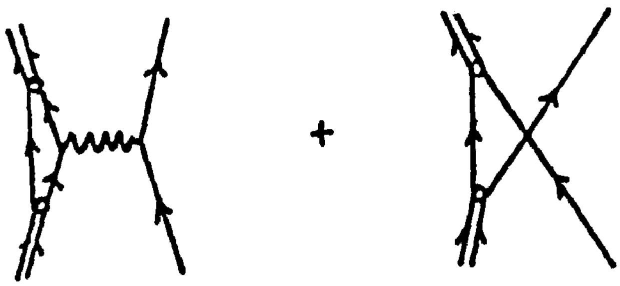
(a) Building blocks



(b) Hamiltonian

$$H = H_B + H_F + V_{BF}$$

Boson - fermion interaction

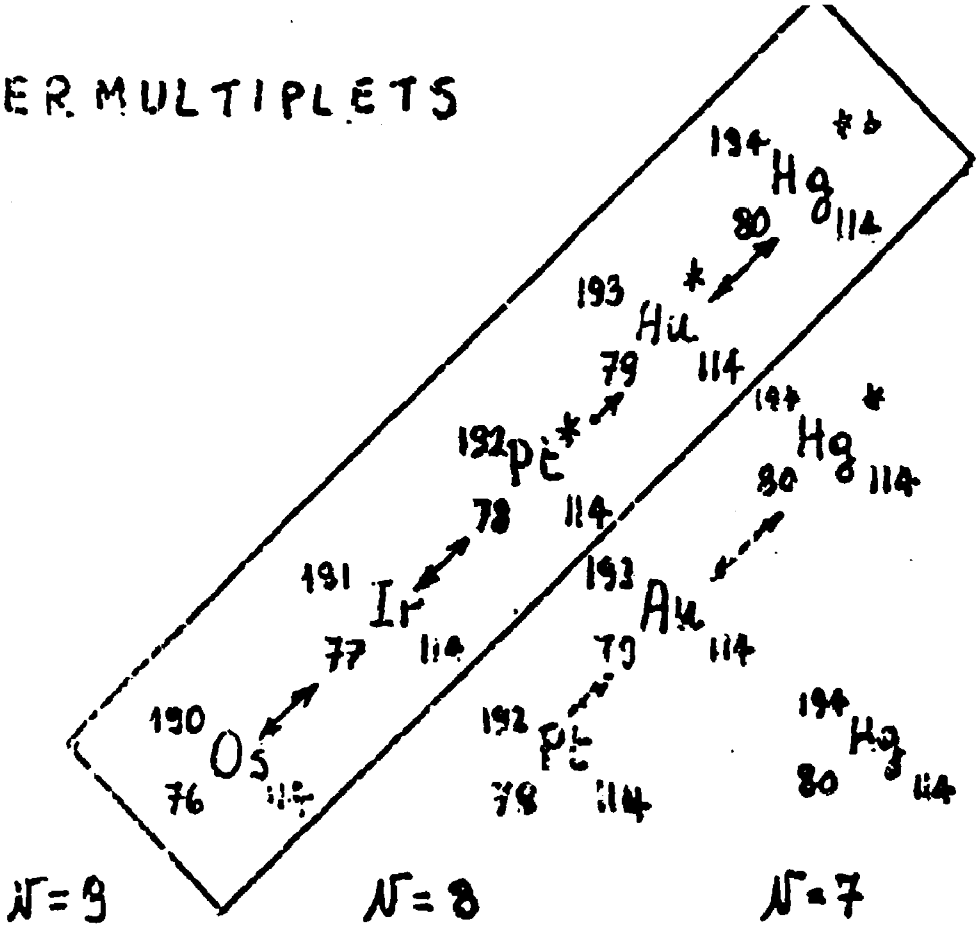


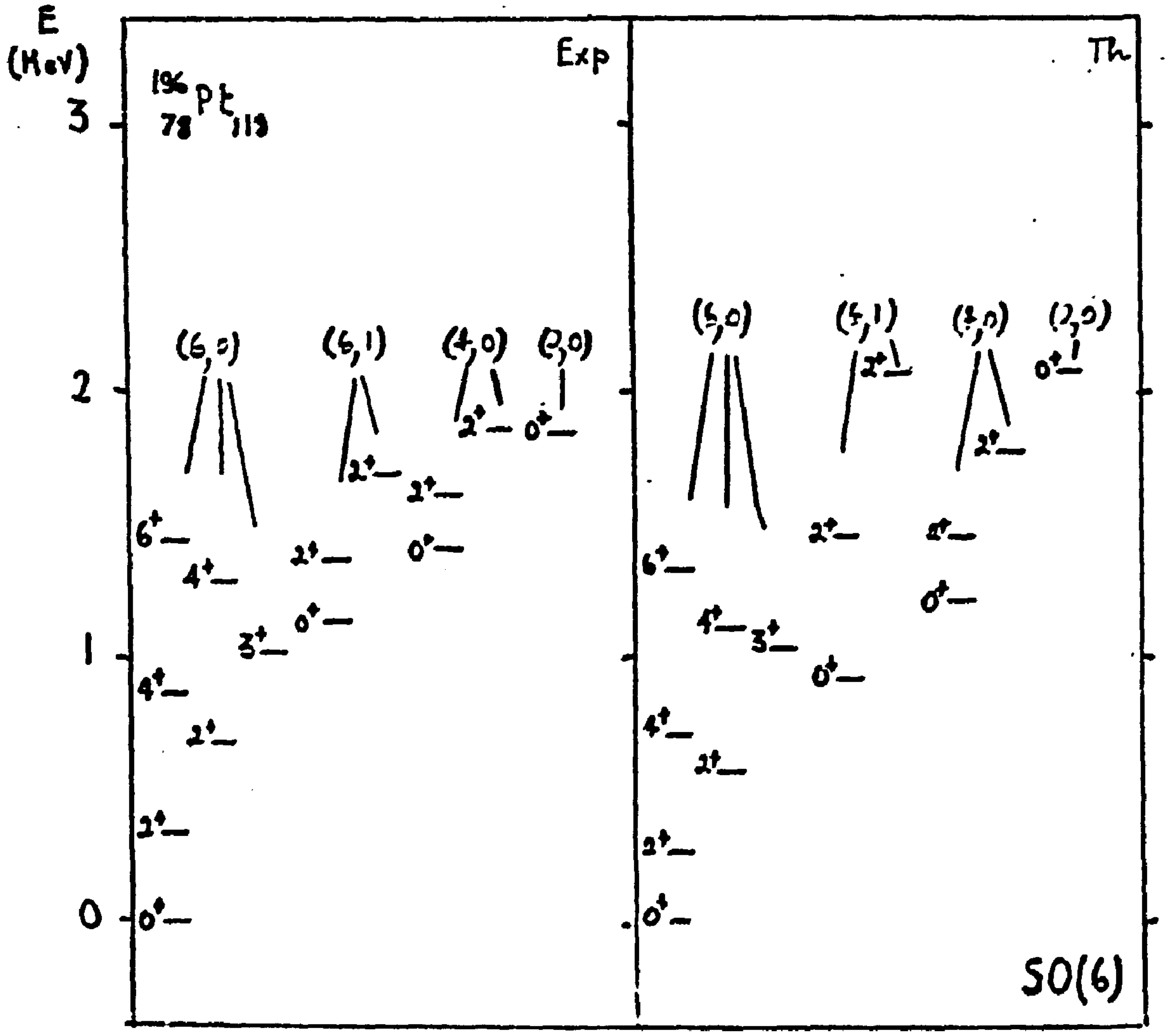
(c) Supersymmetries

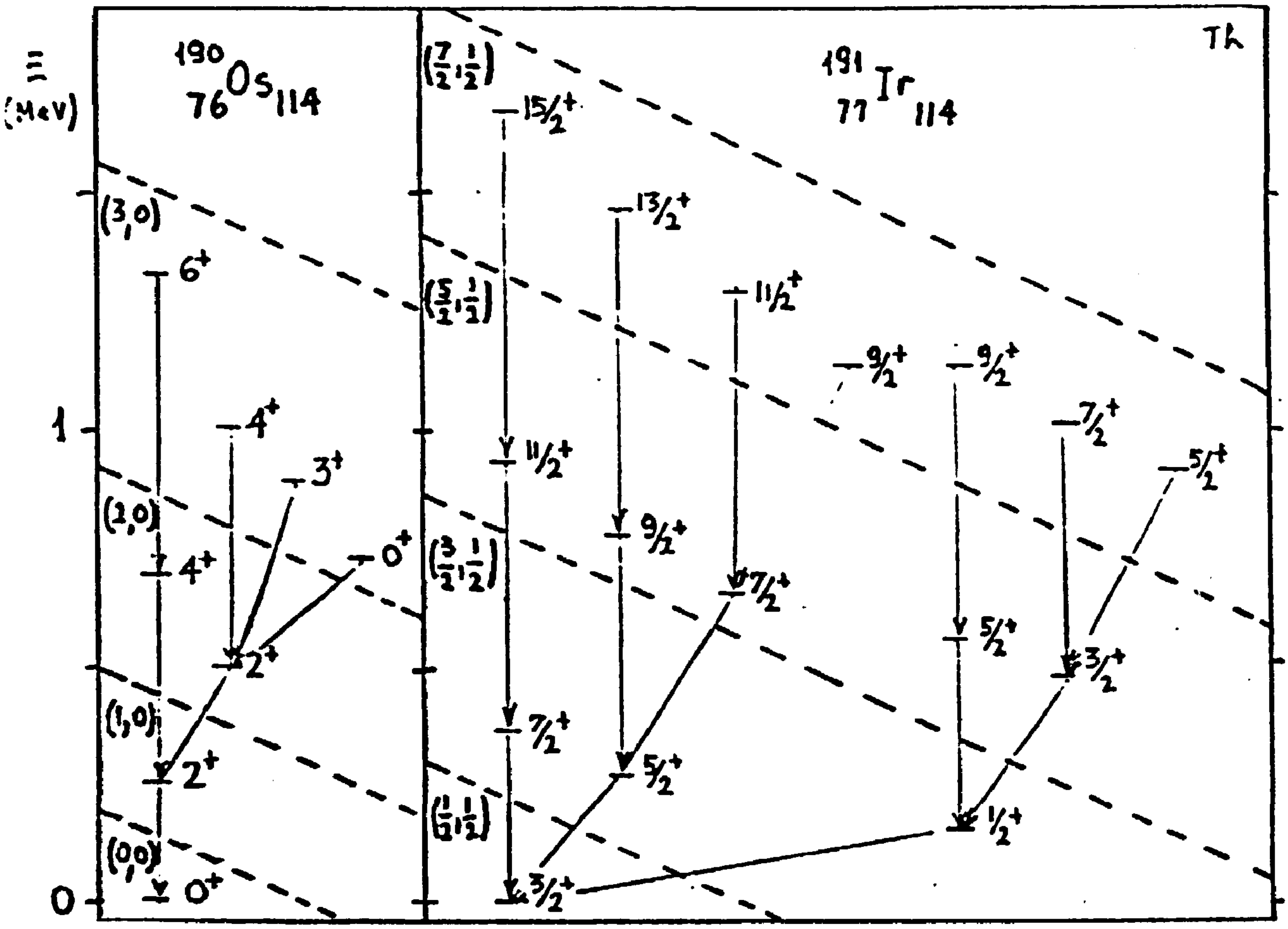
⇒ Many classes

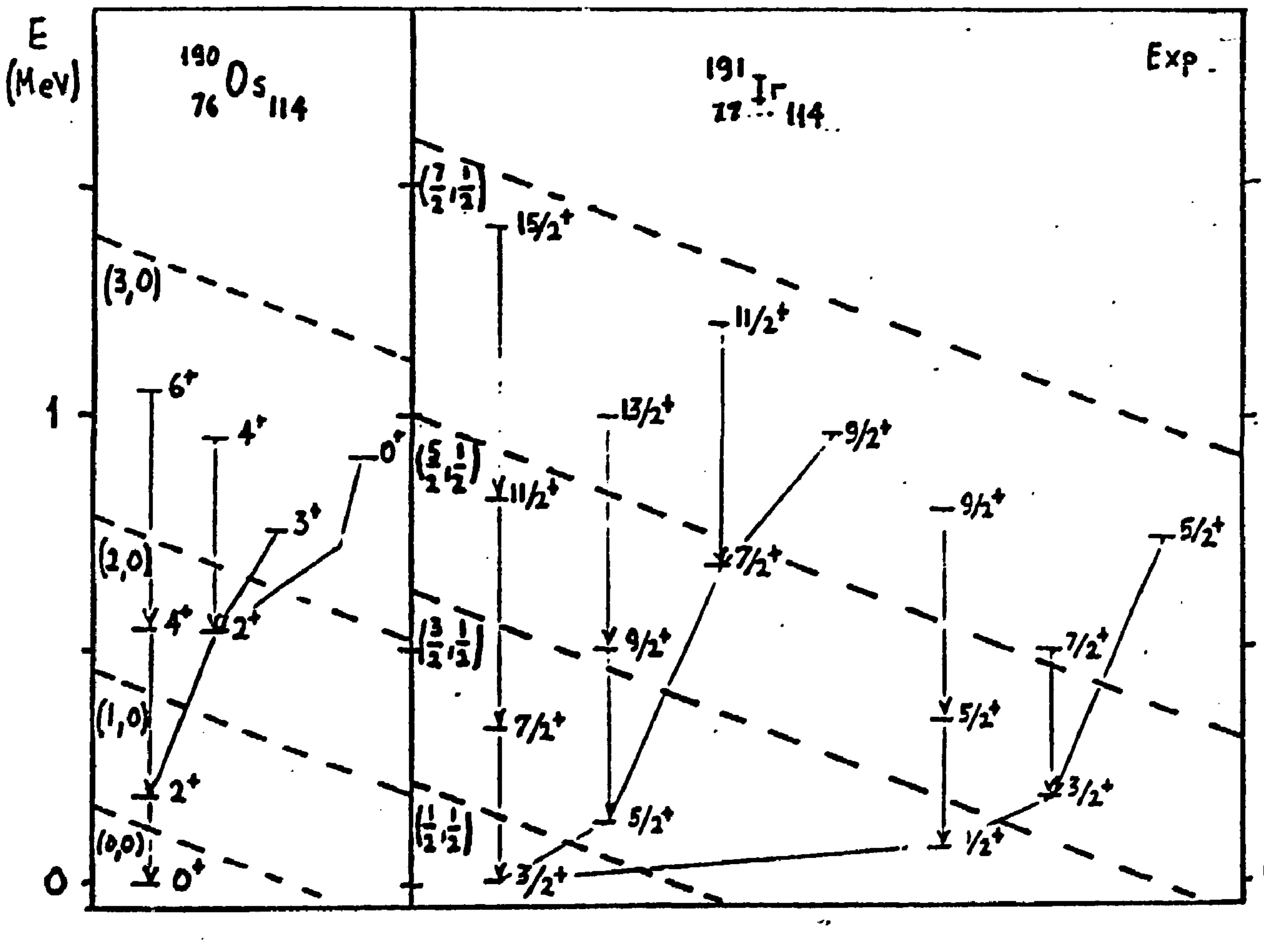
⇒ Provide a classification scheme for odd-even nuclei

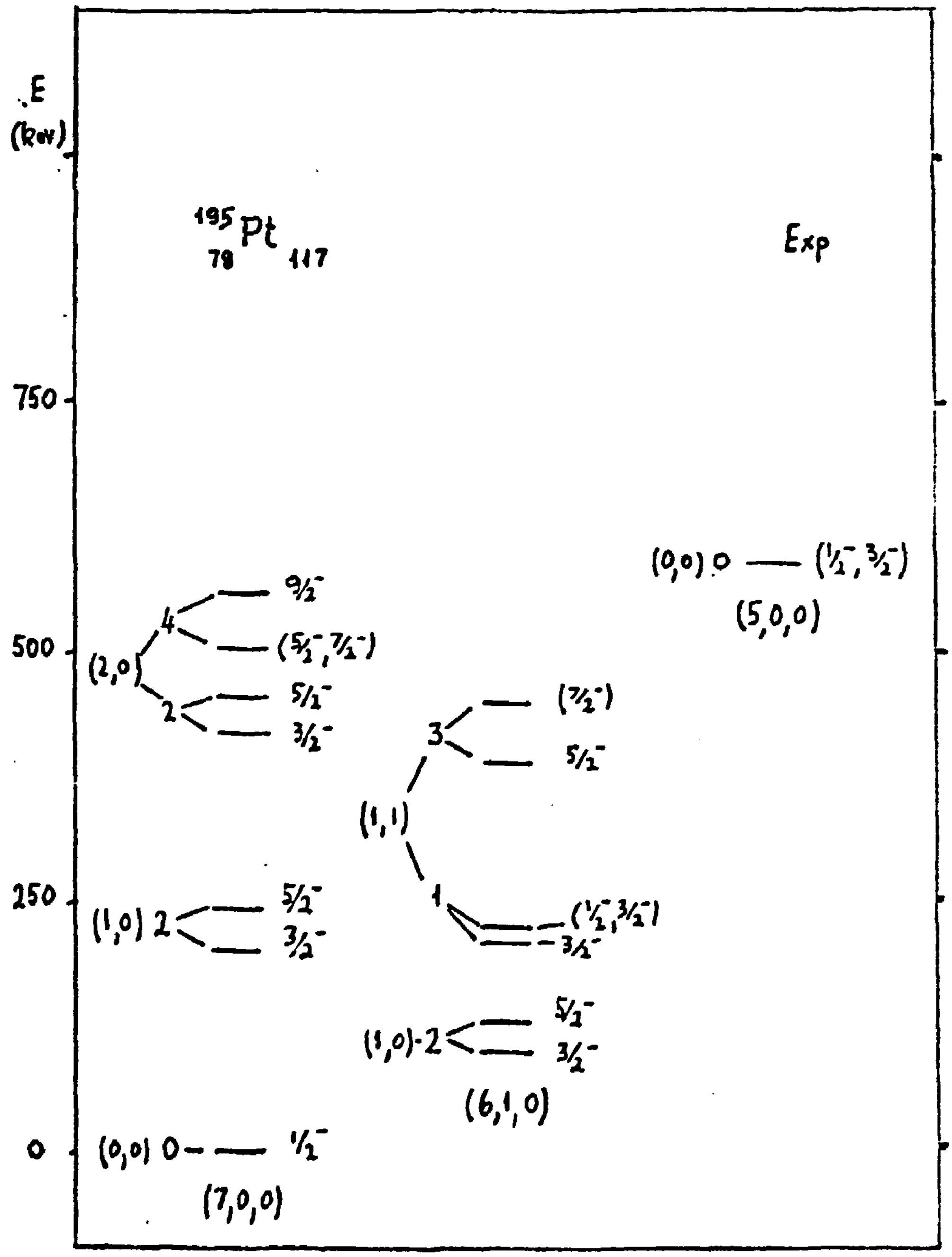
SUPERMULTIPLETS

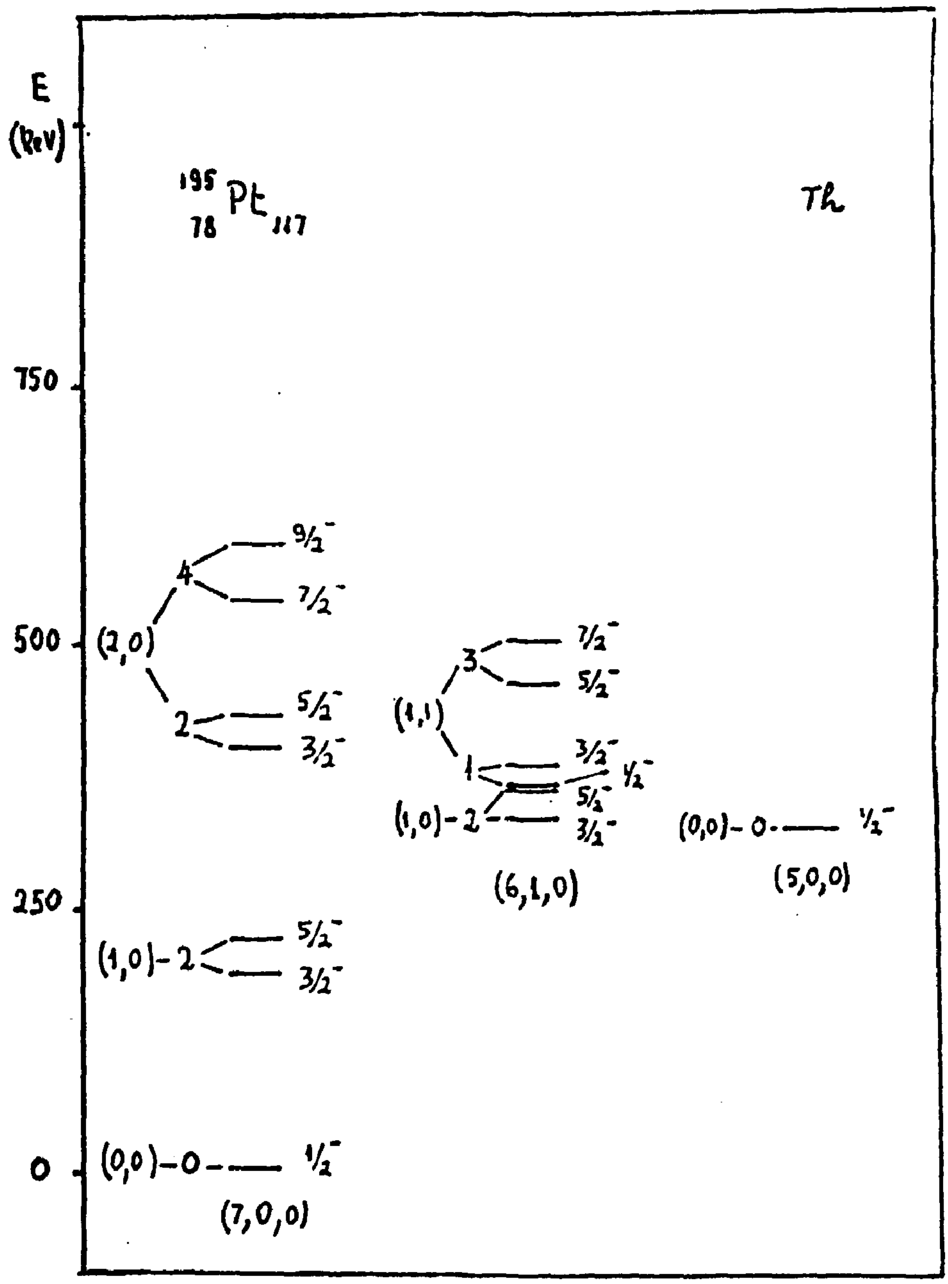





E
(MeV)
$^{190}_{76}Os_{114}$
$^{191}_{77}Ir_{114}$
Th
(3,0)
(2,0)
(1,0)
(0,0)
6+
4+
3+
0+
2+
2+
0+
1
0
(7/2,1/2)
(5/2,1/2)
(3/2,1/2)
(1/2,1/2)
15/2+
13/2+
11/2+
9/2+
9/2+
7/2+
5/2+
11/2+
9/2+
7/2+
5/2+
3/2+
7/2+
5/2+
1/2+
3/2+

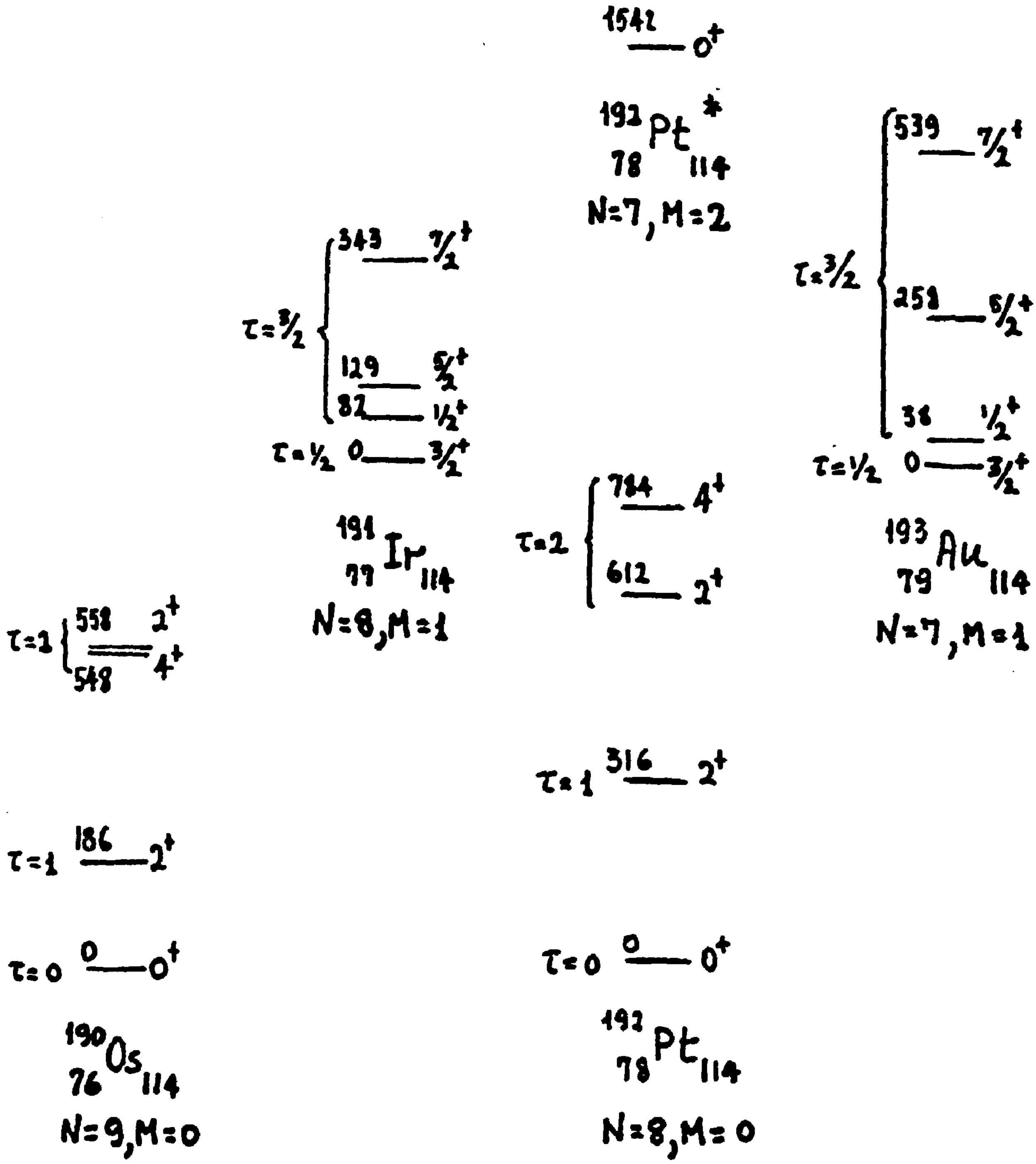

E
(MeV)
$^{190}_{76}Os_{114}$
$^{191}_{77}Ir_{114}$
Exp.
(3,0)
(2,0)
(1,0)
(0,0)
6+
4+
0+
3+
4+
2+
2+
0+
1
0
(7/2,1/2)
(5/2,1/2)
(3/2,1/2)
(1/2,1/2)
15/2+
11/2+
13/2+
9/2+
11/2+
7/2+
9/2+
9/2+
5/2+
7/2+
7/2+
5/2+
3/2+
1/2+
3/2+


E
(kev)
195 78 Pt 117
Exp
750
500
250
0
(0,0) 0 — ($1/2^-$, $3/2^-$)
(5,0,0)
$9/2^-$
4
($5/2^-$, $7/2^-$)
(2,0)
$5/2^-$
2
$3/2^-$
($7/2^-$)
3
$5/2^-$
(1,1)
(1,0) 2
$5/2^-$
$3/2^-$
1
($1/2^-$, $3/2^-$)
$3/2^-$
(1,0)-2
$5/2^-$
$3/2^-$
(6,1,0)
(0,0) 0
$1/2^-$
(7,0,0)

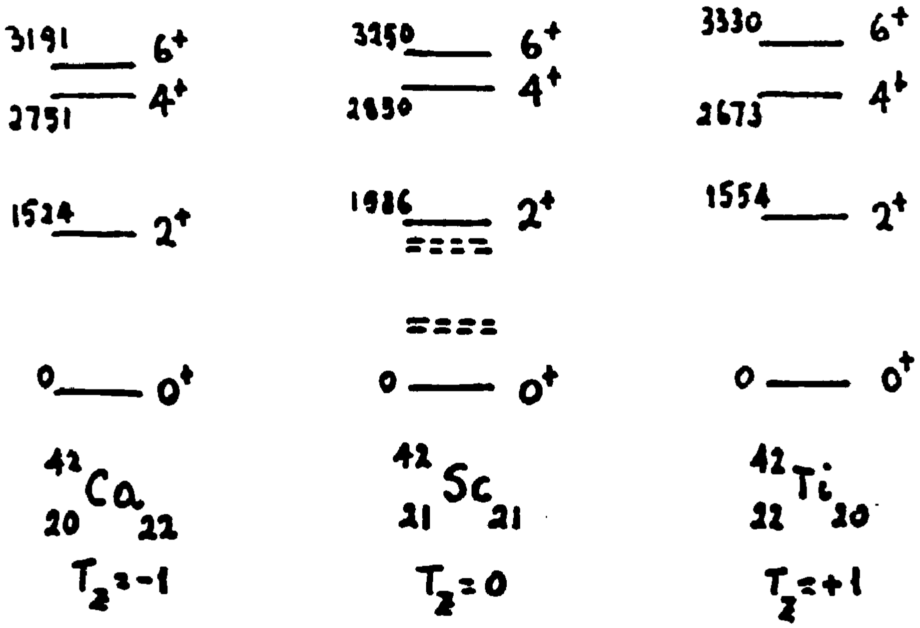

E
(keV)
750
500
250
0
$^{195}_{78}Pt_{117}$
Th
(2,0)
4
9/2⁻
7/2⁻
2
5/2⁻
3/2⁻
(1,0)-2
5/2⁻
3/2⁻
(0,0)-0
1/2⁻
(7,0,0)
(1,1)
3
7/2⁻
5/2⁻
1
3/2⁻
1/2⁻
5/2⁻
(1,0)-2
3/2⁻
(6,1,0)
(0,0)-0
1/2⁻
(5,0,0)

EXPERIMENTAL RESULTS

		U(6|4)	(p,p')	Coulex
^{194}Pt ($\sigma = 7$)	2^+_1	1.00	1.00	1.00
	2^+_2	0.00	<0.01	0.005
^{193}Ir ($\sigma = 15/2$)	$1/2^+_1$	0.11	≈0.08	0.068
	$5/2^+_1$	0.33	0.48	0.44
	$3/2^+_2$	0.00	0.07	0.052
	$7/2^+_1$	0.44	0.40	0.30
	$3/2^+_2$	0.00	0.019	0.017
	$7/2^+_2$	0.00	0.01	0.056
^{191}Ir ($\sigma = 17/2$)	$1/2^+_1$	0.14	0.04	0.051
	$5/2^+_1$	0.41	0.64	0.55
	$3/2^+_2$	0.00	0.08	0.068
	$7/2^+_1$	0.54	0.48	0.31
	$7/2^+_2$	0.00	0.07	<0.01



U(6|4) SUPERSYMMETRY



U(2) ISOSPIN SYMMETRY

CONCLUSIONS

⇒ The use of algebraic techniques (interacting boson and boson-fermion models) has lead to major advances in nuclear structure physics.

We are now able to predict properties of complex nuclei within 10-20% and do have a better understanding of the physics involved.

⇒ The exploitation of dynamic symmetries (and supersymmetries) has been of crucial importance in this development. We have found that even complex systems can display, in some cases, simple, easily recognizable, patterns.

Similar techniques have been used recently in molecular physics (Vibron Model)

F. Iachello and R.D. Levine, J. Chem. Phys. 77, 3066 (1982)
O.S. van Roosmalen, F. Iachello, R.D. Levine and A.E.L. Dieperink, J. Chem. Phys. 79, 2515 (1983)

(a) Diatomic molecules H_2, ...
(b) Triatomic molecules HCN, CO_2, ...
(c) Polyatomic molecules H_2O, ...

Properties studied:

(i) Structure (Vibration-Rotation)

(ii) Transition strengths

(iii) Electron scattering

It is hoped that the exploitation of dynamic symmetries in this field of physics will be as useful as in nuclear and particle physics.