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Structual Instability of Sheath Potential Distribution and Its Possible Implications for the L/H Transition in Tokamak Plasmas

Zensho Yoshida and Hiroshi Yamada

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Structural instability of sheath potential distribution and its possible implications for the L/H transition in tokamak plasmas

Zensho Yoshida[a] and Hiroshi Yamada

Institute of Plasma Physics, Nagoya University
Chikusa-ku, Nagoya 464, JAPAN

The Bohm equation of electrostatic potential distributions in one-dimensional plasmas has been studied for various Mach numbers and plasma potentials. Solvability and structural stability have been discussed using the Sagdeev potential. Implications of the structural stability for the L/H transitions in tokamak plasmas has been also discussed.

Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya 464-01, Japan

[a] Visiting Member of IPP-Nagoya. Permanent address: Department of Nuclear Engineering, The University of Tokyo, Bunkyo-ku, Tokyo 113, JAPAN.

I. INTRODUCTION

We consider an electrostatic plasma in the one-dimensional half space $(0, +\infty)$, and study the structural stability of the electric-field distributions. The problem is related to a simple model of electric fields in the edge region of fusion plasmas. Since we consider one-dimensional problems, we are studying electric fields parallel to magnetic fields, if they exist, and are neglecting the transport across magnetic-field lines.

Recently the electric fields in the edge region of tokamak plasmas are attracting much interest in their relations to the so-called L-mode/H-mode transition, which is a discontinuous change in the magnitude of particle fluxes (leak) in the edge region.[1] The electrostatic field is considered to be one of key candidates that change the transport drastically. Experimentally, there are observations of changes in the floating potential of the edge plasma on the occasion of the L/H transition.[2] Itoh and Itoh[3] calculated the neoclassical transport with taking account of radial electric fields, and found that two different potentials are solutions of their model, which are considered to be corresponding to both L and H states.

In this paper, we study the spatial distribution of the electric field for various plasma potentials and Mach numbers, and discuss the structural stability of the field distributions. The Mach number is discussed as a key parameter that governs the bifurcation and the stability of the electrostatic field parallel to the magnetic-field lines. Their implications for the L/H transition are also discussed.

II. BOHM SHEATH EQUATION

We consider a simple model of electrostatic plasmas; electrons are thermally relaxed and ions are collisionless. The model was firstly studied by Bohm[4] for the sheath potential of plasmas contacting with walls. The model is also related to the ion acoustic shock. Sagdeev[5] introduced the so-called Sagdeev potential to study oscillating solutions for the model equation. We call the model equation the Bohm equation:

$$L\psi = (1 - 2M^{-2}\psi^*)^{-1/2} - e^{\psi^*}, \quad (1)$$

where

$$\psi^* := \psi - P,$$

ψ is the electrostatic potential normalized by the thermal energy of electrons, and $L = -d^2/dx^2$ is an elliptic differential operator of one dimension. The coordinate x is normalized by the Debye length. The Bohm equation has two independent parameters; M is the Mach number defined by

$$M := [(\text{kinetic energy of ion})/(\text{thermal energy of electrons})]^{1/2},$$

and P is the potential deep inside the plasma.[6] The first and the second terms in the right-hand-side of Eq.(1) correspond to the densities of ions and electrons, respectively. Using the Sagdeev potential, we write Eq.(1) as

$$L\psi = V'(\psi^*), \tag{1'}$$

where the Sagdeev potential $V(\psi^*)$ is defined by

$$V(\psi^*) = 1-e^{\psi^*}+M^2[1-(1-\frac{2}{M^2}\psi^*)^{1/2}]. \tag{2}$$

A certain generalization is easily done when we consider the velocity distribution of ions and/or the spatial distribution of the electron temperature. In the present work, however, we keep the simplicity of formulation, and consider a monochromatic velocity of ions and a uniform electron temperature.

The Bohm equation is a nonlinear elliptic differential equation. We consider boundary-value problems for the equation. We set

$$\psi(0) = 0, \tag{3}$$

$$\lim_{n\to\infty} \psi(x_n) = P, \tag{4}$$

where the limit is taken for a certain sequence $\{x_n\ ; n=1,2,\cdots\}$ which satisfies $\lim_{n\to\infty} x_n = +\infty$. This limit is generaly dependent on the choice of the sequence. The condition (4) implies that the potential $\psi(x)$ should not deviate from the value P, but may oscillate around P. Physically P corresponds to the potential inside the plasma where the ion is originated. Therefore, the value $\psi^* = \psi - P$ is the potential difference that the ion feels. We do not consider that the ion is originated at the mathematical infinity. The weak boundary condition (4) permits a large variety of solutions for the model equation (see Sec.III).

III. MATHEMATICAL BACKGROUND

In this section, we prepare mathematical background and note some non-trivialities. The Bohm equation is a nonlinear elliptic differential equation, so the solvability, the uniqueness, bifurcations of solutions, and the stabilities are subjects of mathematical considerations. Let us start with reviewing the simplest solution for the Bohm equation, that is the Bohm-sheath solution, and then discuss mathematical problems concerning the structural stability of the sheath solutions.

When we linearize the Bohm equation, we have

$$L\psi = \alpha(\psi - P), \qquad (\alpha := M^2 - 1). \tag{5}$$

Equation (5) has different characters for $M>1$ and $M<1$ regimes. For $M>1$, Eq.(5) has a unique solution for every $P \in \mathrm{R}$:

$$\psi(x) = P[1 - e^{-\sqrt{\alpha}x}]. \tag{6}$$

A mathematical question is the solvability of the original nonlinear equation (1) for given P and $M>1$. Another question is the structural stability of the solution. The solution (6) for the linearized equation (5) satisfies the boundary condition (4) in a stronger sense; viz., the function asymptotically converges to P as $x \to +\infty$. Since we set a weaker condition, we may expect a wider class of solutions for the nonlinear equation. The ion-acoustic-shock solutions are given by the structural instability of the asymptotic solutions. These points will be discussed in the next section.

In the region $M<1$, the linearized equation (5) is of the Helmholtz type, and the equation has non-trivial solutions for $P=0$. This implies the possibility of bifurcation of solutions for the original nonlinear equation (1). To answer the above-mentioned questions, the Sagdeev method using a formulation of initial-value problems is useful.

Although the formulation is essentially a boundary-value problem, we may take the advantage of one-dimensional differential equations, and convert the boundary-value problem to an initial-value problem (IVP) for an ordinary differential equation. By this technique, we easily find structurally unstable solutions; see section 4.

We consider initial values at $x = 0$;

$$\psi(0) = 0,$$

$$\psi'(0) = \nu,$$

where ν is a certain number that should be determined to meet the boundary condition (4). Using the analogy of Newton's equation (x: time, ψ: position), $V(\psi)$: potential energy, $\psi'^2/2$: kinetic energy), we may easily find ν that matches the boundary condition (3). This IVP technique for solving the Bohm equation has been given by Sagdeev to find oscillating solutions. In the next section, we will study the mathematical structure of the Bohm equation using the IVP method.

IV. SOLVABILITY AND STRUCTURAL STABILITY OF BOHM EQUATION

The IVP method has an advantage in studying the structural stability for the Bohm equation. Figure 1 shows the numerically calculated Sagdeev potential. We can construct solutions by starting from x(considered to be the time) = 0 with $\psi^*(0)$(considered to be the initial position) = $-P$ and $\psi^{*\prime}(0)$(considered to be the initial velocity) = ν. The boundary condition (4) should be finally satisfied. Therefore, the curve $\psi^*(x)$ should stay around 0 in the sense of the weak convergence of the condition (4). The initial value ν is chosen to satisfy this condition. The analogy of one-dimensional Newtonian dynamics easily explains the method to get the solution; see Fig.2.

First let us consider the case of $M \geq 1$. We should set

$$\nu^2/2 = V(-P), \tag{7}$$

to get the asymptotic convergence of $\psi^*(x)$ to 0 at $x \to +\infty$. Figure 3(a) shows the asymptotic solution that corresponds to the Bohm sheath. The asymptotics $\psi^* = 0$, however, is top of the potential, so that the structural instability may give bifurcated solutions. When we start with a little bit larger velocity v, we get oscillations in the region of $\psi^* \geq 0$, if we may include some dissipative structure (see Sec.VI); Fig.3(b). The oscillating solution is the ion-acoustic shock, which has been given by Sagdeev.

Next, let us discuss the regime of $M<1$. When $M<1$, the position $\psi^* = 0$ is the bottom of the Sagdeev potential. Because of this structural change in the Sagdeev potential, we see a drastically different behavior of solutions. There is no asymptotic solution. Only oscillating solutions may exist. The weak boundary condition (4) retains such pathological solutions. Figure 3(c) shows a typical oscillatory solution in the $M<1$ regime.

Figure 4 shows the solvability and classification of solutions for the Bohm equation. The Sagdeev potential is not defined in the regime $\psi^* \geq \psi^*_c := M^2/2$. The potential difference $\psi^* \geq \psi^*_c$ is large enough to stop the transit motion of ions. When ions are stopped by the potential barrier, positive charge accumulates, so that steady solutions do not exist. The no-solution region 'C' in Fig.4 is given by the positive potential barrier. In the region 'A', the Bohm equation (1) has normal positive-ion-sheath solutions (Fig.3(a)) and ion-acoustic-shock solutions (Fig.3(b)). In the region 'B', only oscillating solutions (Fig.3(c)) exist.

V. IMPLICATIONS FOR THE L/H TRANSITION

Phenomenologies of electrostatic potential distributions and their bifurcations have some implications for the L/H transition in tokamak plasmas. Causals of the L/H transition are considered to be fairly complex. Phenomonological arguments are therefore useful to understand the correspondences between the observed parameter dependences of the phenomena and some basic physical models such as the present Bohm equation. Here we discuss the Mach number and the structural stability of the sheath potential which is considered to be related to the L/H transition.

The electric field across magnetic field lines is considered to be a key candidate that gives the drastic change in the particle transport in the L and H modes. A theoretical model that considers the cross-field potential (zero-dimensional) succeeded to explain two different potentials and the corresponding two different transports;[3] the lower (negative) potential gives smaller particle fluxes. The distributions of the potential is the next question. We have to study the *structure* of the potential in one or two dimensions. The solvability and the structural stability for the differential equation should be checked for the parameter P in Eq.(1) that is obtained by the zero-dimensional (algebraic) arguments.

In this paper, we study the potential distribution parallel to the magnetic field (one dimensional). When the magnetic field lines are open to the wall (see Fig.5), the spatial distribution of the potential is strongly influenced by the parallel-field dynamics of electrons and ions. Implications of the analyses given in the former sections are as follows.

In the parameter space of M and P, there are no-solution region (marked by 'C' in Fig.4) for the steady-state parallel-field model. The parallel-field particle leak is stopped, so that the potential and the particle transport are dominated by cross-field particle fluxes. This easily happens when $M<1$; see Fig.3.

In the region of $M<1$, normal positive ion sheath is not established. In the region 'B' of Fig.3, potential dips are formed because of the following reason. A drop of potential decreases the electron density. It also decreases the ion density because ions are accelerated by the potential drop so that they pass there rapidly. When $M<1$, the decrease of ion density is larger than that of electrons, because hot electrons are insensitive to the change of the potential. Therefore, the drop of the potential is enhanced.

In contrast to the low M regime, in the region 'A' of Fig.4 ($M \geq 1$),normal positive sheath is stably established. Some positive peaks may appear (Fig.3(b)),which correspond to the ion acoustic shock.

In summary, when $M<1$ (viz. electrons are hot relative to ions), the potential tends to take smaller values due to the parallel-field dynamics.

Some plasmas whose electrons in the edge region are strongly heated show improvements in the particle/energy confinement times. Concerning the

L/H transition, it is known that the transition is triggered by a sawtooth instability which acts to heat up the edge region. The ECR heating is also known to reduce the threshold value of the heating power for the transition.[7] In contrast, ion-heated plasmas, such as NBI or ICRF heated plasmas, show degradation in the confinement. These general tendency suggests that the Mach number has much relation to the confinement characteristics in the edge region.

A possible interpretation is that the electric-field distributions parallel to the magnetic field has a key role for the confinement, and the electric-field is sensitive to the Mach number. Considering the structural stability of the potential *distribution*, the negative potential that the neoclassical calculations consider is realistic, when $M<1$. These points seem to be consistent to the experimental observations.

VI. DISCUSSIONS

Dissipative structure of a nonlinear dynamical system is mathematically formulated by nonlinear partial differential equations (PDE's) with hyperbolic characteristics. The Hamilton-Jacobi system is a classical example of hyperbolic PDE system, which is purely hyperbolic. Non-trivial steady state of modern interest is related to partially hyperbolic systems of PDE's. When an equilibrium equation has a hyperbolic characteristics, boundary conditions are not sufficient to determine the equilibrium; characteristic functions are necessary to be supplied by considerations of the dissipative dynamics that the system passes through to reach the equilibrium.

There are some examples of partially hyperbolic equilibrium problems in plasma physics. The magnetohydrodynamic equilibrium equation

$$(\mathrm{rot}\,\boldsymbol{B})\times\boldsymbol{B} = \mathrm{grad}\,p, \quad \mathrm{div}\,\boldsymbol{B} = 0$$

is a system of two-hyperbolic and two-elliptic PDE's. Two characteristics are the distributions of the pressure and the force-free field. When we consider two-dimensional ($\partial/\partial z = 0$) problems, the magnetostatic equation reduces to the Grad-Shafranov equation:

$$L\phi = (I(\phi)^2)' + p(\phi)'$$

where L is an elliptic differential operator (Laplacian), ϕ is the flux function, $I(\phi)$ is the distribution of the z-component of the magnetic field, and $p(\phi)$ is the distribution of the pressure.

Another important example is the ion-sheath and ion-acoustic-shock problems of electrostatic plasmas. When we consider that the dynamics of ions is fully conservative and the dynamics of electron is fully relaxed (dissipative), and when we consider one-dimensional problems, the self-consistent ion-sheath equation reduces to the Bohm sheath equation (1'). The equation has two parameters: the Mach number M and P. Both are subject to the dissipative structure of the system. The parameter M^2 represents the ratio of the kinetic energy of ions and the thermal energy of relaxed electrons. The parameter P is the potential of the plasma that is related to the global transport (particle fluxes).

Formal analogy of both equations is worthwhile noting. Structures are subject to the characteristic functions; for the Grad-Shafranov equation, $p(\phi)$ and $I(\phi)$, and for the Bohm equation, $V(\psi^*)$. These characteristic functions are related to the dissipative structure of the systems. Structural instabilities are studied by analyzing the deformations of the solutions due to perturbations for the characteristic functions. Using the Sagdeev potential, we got some different type of solutions which show catastrophic changes in the potential distributions. We note that oscillatory solutions in the $M<1$ regime are hard to be found, if we use the usual boundary-value scheme to solve the Bohm equation. Also in time-evolutional simulations, such oscillating solutions are difficult to be stably calculated. However, these solutions are related to the structural instability of the electrostatic plasma systems, so that they have much physics interest.

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[6] Local Mach number in the sheath region ($0 < x < 1$) is known to be greater than 1, when the normal positive ion sheath is formed and the ion is accelerated in the pre-sheath region; see T. Takizuka *et al* ., J. Nucl. Mater. **128 & 129**, 104 (1984). The Mach number we use here has a different meaning; which is defined by the velocity of ions at the potential $\psi = P$ inside the plasma.

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FIGURE CAPTIONS

Fig.1 Sagdeev potentials for (a)$M = 1.2$ and (b)$M = 0.7$.

Fig.2 Schematics of the IVP method to construct solutions for the Bohm equation. We shoot a ball from $\psi^* = -P$ with an initial velocity to place the ball at the top of the hill.

Fig.3 Solutions for the Bohm equation.
(a) Asymptotic solution for $M = 1.2, P = 1$.
(b) Oscillating solution for $M = 1.2, P = 1$.
(c) Oscillating solution for $M = 0.7, P = 1$.

Fig.4 Solvability and classification of solutions for the Bohm equation in the M/P plane.
Region A : asymptotic solutions (Bohm-sheath solutions) and oscillating solutions (ion-acoustic-shock solutions) exist.
Region B : oscillating solutions exist.
Region C : no solution exists.

Fig.5 Schematics of the edge plasmas in tokamaks. (a) X-point configuration. (b) Simplified model of the edge plasma.

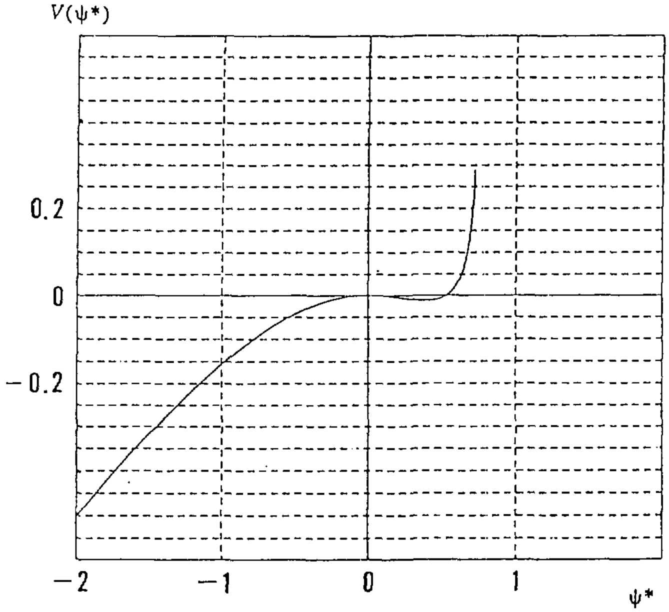


Figure 1(a)

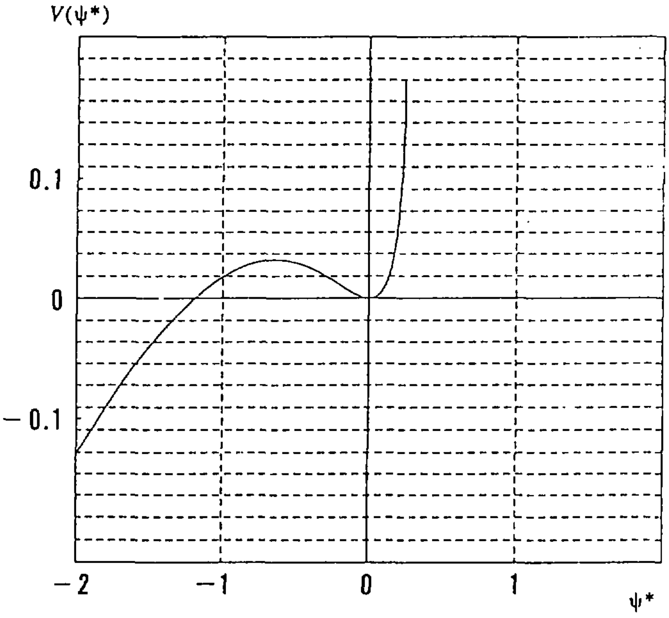


Figure 1 (b)

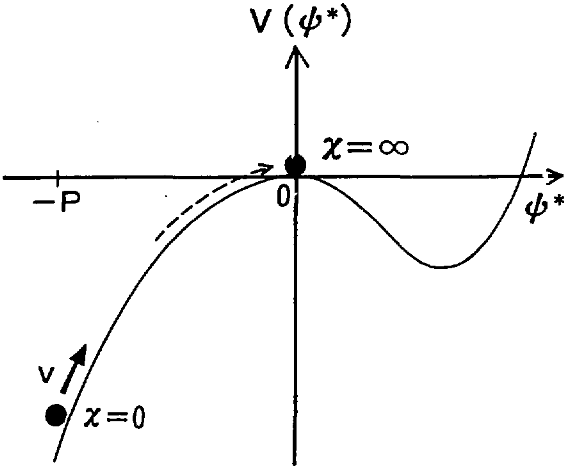
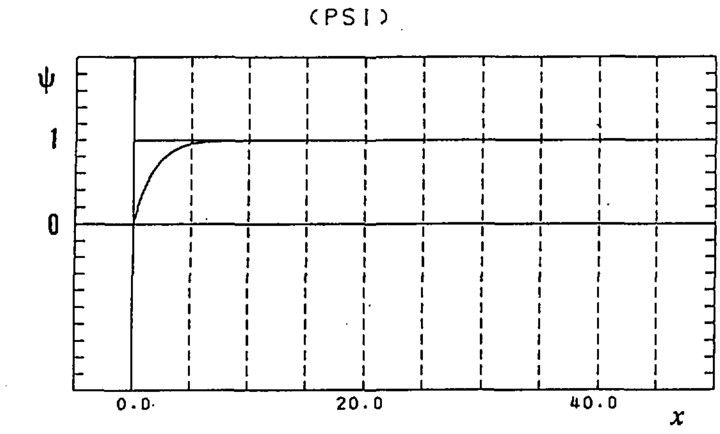


Figure 2



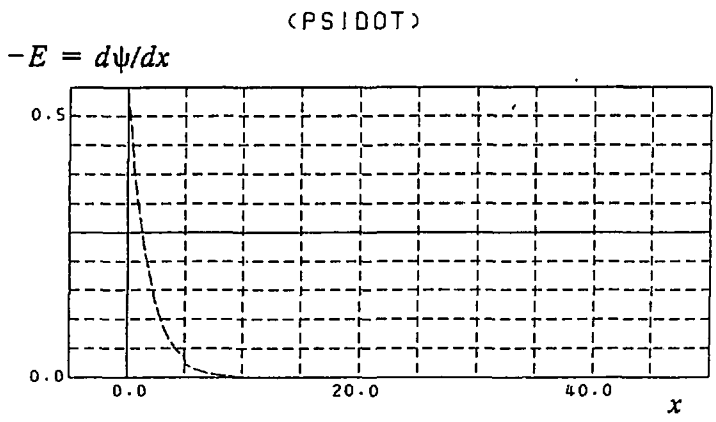
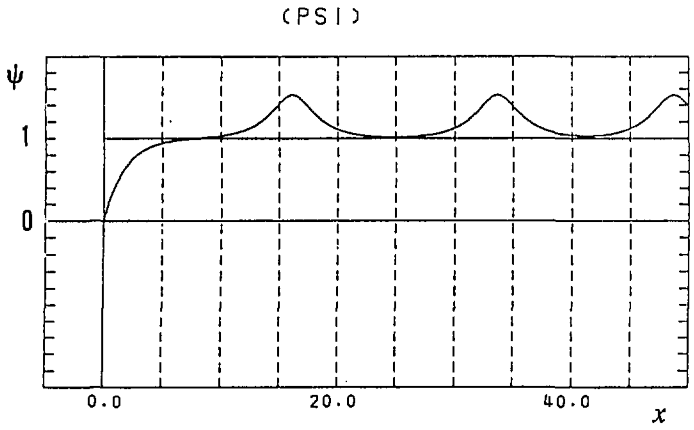
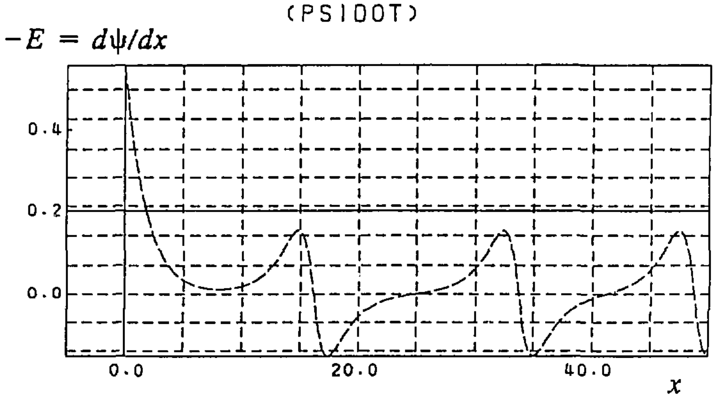


Figure 3(a)





F i g u r e   3 (b)

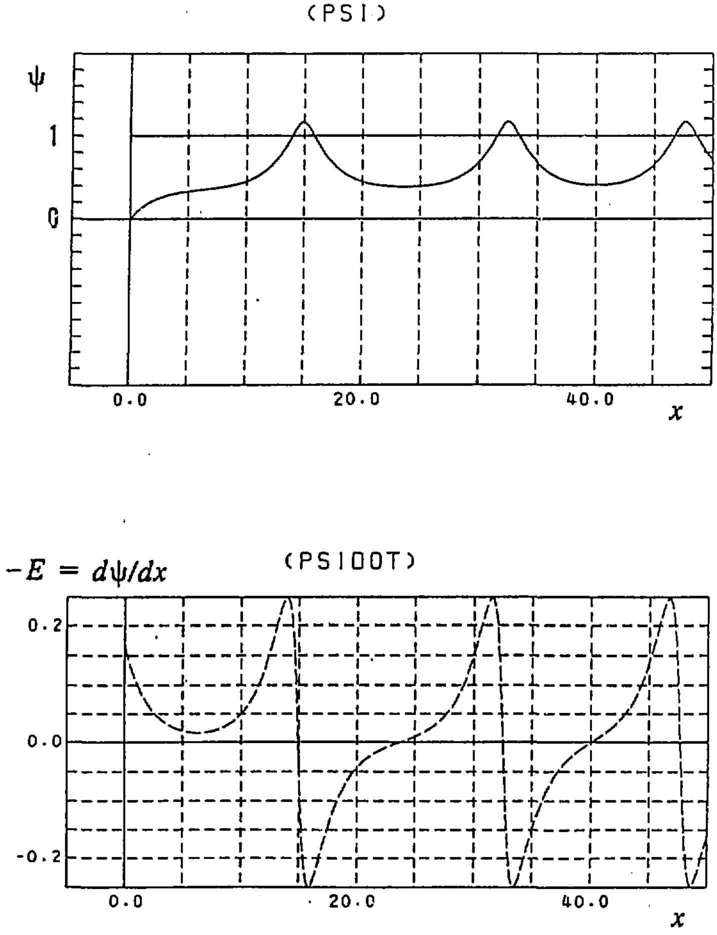


Figure 3 (c)

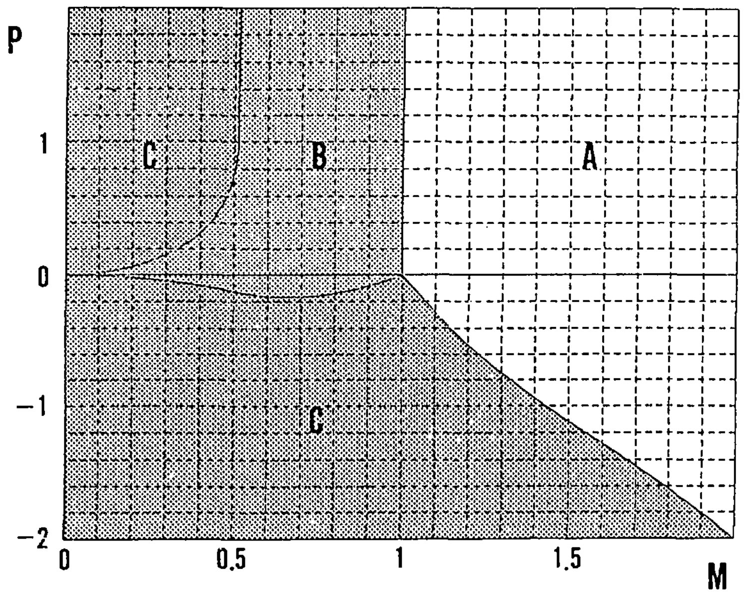
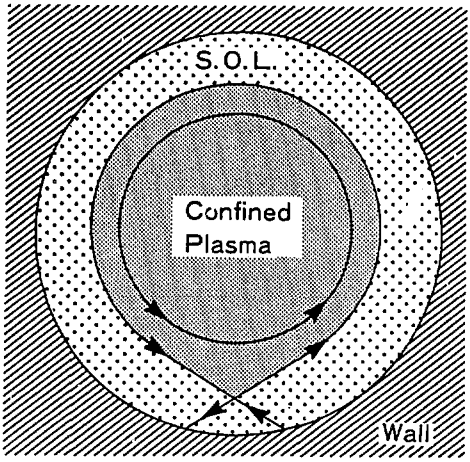


Figure 4


S.O.L.
Confined
Plasma
Wall

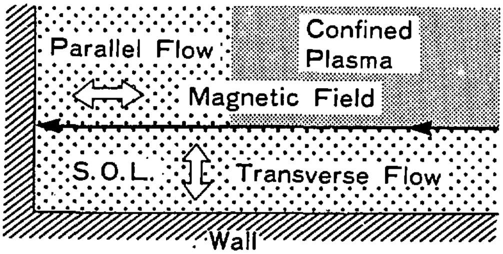

Confined
Plasma
Parallel Flow
Magnetic Field
S.O.L.
Transverse Flow
Wall

Figure 5