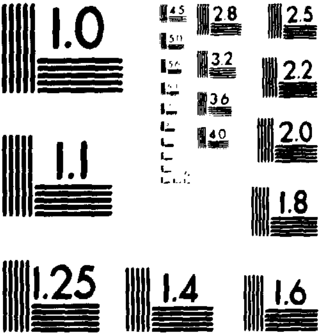

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ISSN/0393-6287

RT/FUS/88/6

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COMITATO NAZIONALE PER LA RICERCA E PER LO SVILUPPO DELL'ENERGIA NUCLEARE E DELLE ENERGIE ALTERNATIVE

ISSN/0393-6287

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DELL'ENERGIA NUCLEARE E DELLE ENERGIE ALTERNATIVE
Associazione EURATOM-ENEA sulla Fusione

CONFINEMENT AND DIFFUSION IN TOKAMAKS

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RT/FUS/88/6

Testo pervenuto nel luglio 1988
Progetto Enea: Fisica della fusione (DA)

This report has been prepared by: Servizio Studi e Documentazione - ENEA, Centro Ricerche Energia Frascati, C.P. 65 - 00044 Frascati, Rome, Italy.

Abstract The effect of electric field fluctuations on confinement and diffusion in tokamaks is discussed. Based on the experimentally determined cross-field turbulent diffusion coefficient,

$$D_\perp \approx 3.7 \frac{cT_e}{eB} \left(\frac{\delta n_i}{n_i}\right)_{rms}$$

which is also derived by a simple theory, the cross-field diffusion time, $\tau_p = a^2/D_\perp$, is calculated and compared to experimental results from 51 tokamaks for standard Ohmic operation.

Diffusion, transport, and confinement times are central issues in the efforts to achieve the goal of thermonuclear fusion in tokamak devices. It is thought commonly that one source for transport across magnetic fields is fluctuating electric fields in the plasmas. In the operation of specialized devices like tokamaks, frequently it is difficult to study separately the causes of the electric field fluctuations and the effects of these fluctuations. As a result, complex scaling laws have been developed for guiding the operation of present tokamaks and the planning of future devices. It is the purpose of this paper to help separate effect from cause and thus to aid in understanding the physics of confinement times and diffusion in tokamaks. Based on experiment and a simple theory, the effect of electrostatic fluctuations on transport and confinement in tokamaks is predicted. The cause of these fluctuations is not addressed and confinement scaling laws are not deduced here.

Plasmas are rarely in a state where classical collisional processes are observed to dominate cross-field transport. Bohm [1] was one of the first to recognize this and predict non-classical transport. Spitzer [2] made simple agruments about transport and it is in the spirit of his approach that the present calculations are made, though the conclusions are somewhat different. Notable papers giving recent insight into experimental observations of fluctuations and confinement times in tokamaks are by Hugill [3], Surko and Slusher [4], and work from TFR [5] and TEXT [6]. On the theory side Dupree [7], Hinton and Hazeltine [8], and Liewer [9] have given considerable attention to the theoretical efforts attempting to understand turbulence and confinement. Direct, non-perturbing measurements of cross-field transport of majority species ions have been rare. McWilliams and Okubo [10] reported such measurements which were consistent with classical transport predictions after having gone to some trouble to reduce the normalized plasma density fluctuation level to 2×10^{-3}. Other work by McWilliams, et al [11] at the University of California, Irvine showed that fluctuations of the order of a few percent caused anomalous cross-field transport. Further work [12 - 14] by this group has lead to clear, direct observations of the dependence of cross-field transport on electrostatic turbulence for fluctuations levels has lead to clear, direct observations of the dependence of cross-field transport on electrostatic turbulence for fluctuation levels up to 4×10^{-2}. Additional work at higher fluctuation levels has been performed by the group, but has not been presented yet. A brief review of this work [10-14] forms the basis of the present predictions.

Experiments [10-14] at Irvine using laser induced fluorescence (LIF) diagnostics observed circumstances where low frequency turbulence was seen to produce an autocorrelation time of about one third of an ion gyroperiod and spatial interferograms showed a wave phase decorrelation in about one third of a wavelength for waves which were expected to have $k_\perp \rho_i \sim 1$ ($k_\perp$ is the wavenumber perpendicular to the magnetic field and ρ_i is a thermal ion gyroradius). The turbulent waves were thought to be homogeneous and isotropic across the magnetic field. It appears that for the low frequency wave turbulence an ion will move about one third of a wavelength (about $3\rho_i/2$) across the field and then will lose the velocity given to it by the wave electric field, resulting in a diffusion coefficient of (in c.g.s. Gaussian units with Boltzmann's factor in T_e)

$$D_\perp \approx 3.7 \frac{c\,T_e}{e\,B}\left(\frac{\delta n_i}{n_i}\right)_{rms} \tag{1}$$

The numerical factor is related to the autocorrelation time, the fluctuation level shows that the electric fields are responsible for the diffusion, the electron temperature enters because at low frequecies and $T_i \approx T_e$ the electrons respond quickly to the fields, and the magnetic field shows how plasmas are in some sense locked to the field lines. The local ion fluctuation level is $<(\delta n_i/n_i)^2>^{\frac{1}{2}}_t$ but for ease is written as $(\delta n_i/n_i)_{rms}$ in equation (1). The density fluctuation level enters by assuming a Boltzmann response to the potential fluctuation level. When this is violated, the correct re-introduction of the potential fluctuations must be done in equation (1). Following the method of Spitzer [2] one may argue that the coefficients involved in the autocorrelation time and response to the electric field turbulence are not precisely correct for all experiments, but it is likely that these coefficients would differ only by factors of order unity from the above prediction.

In the Irvine experiments of McWilliams, et al [10 - 14] it was observed that, except for the small and previously studied [10] classical diffusion offset, that agreement with equation (1) was obtained, especially noting the experimental linear dependence of the diffusion coefficient on the fluctuation level. For fluctuation levels less than 1% it was possible (within the experimental uncertainty) to fit $D_\perp$ with a quadratic dependence on fluctuation level, but a linear fit described well the entire range studied.

Equation (1) predicts diffusion coefficients. Figure 1 shows the predicted $D_\perp$ versus magnetic field for several fluctuation levels. From the graph, for example, using $T_e\,(\delta n_i/n_i)_{rms} \approx 2$ eV, then $D_\perp$ (3 T) $\approx 2.5\ m^2/s$ and $D_\perp$ (6T) $\approx 1.25\ m^2/s$ These values are in the range of the inferred perpendicular diffusion coefficients for experiments with these parameters.

It follows from equation (1) that the time for a majority species ion to diffuse a distance, a, across the magnetic field is (when all variables are independent of position and time)

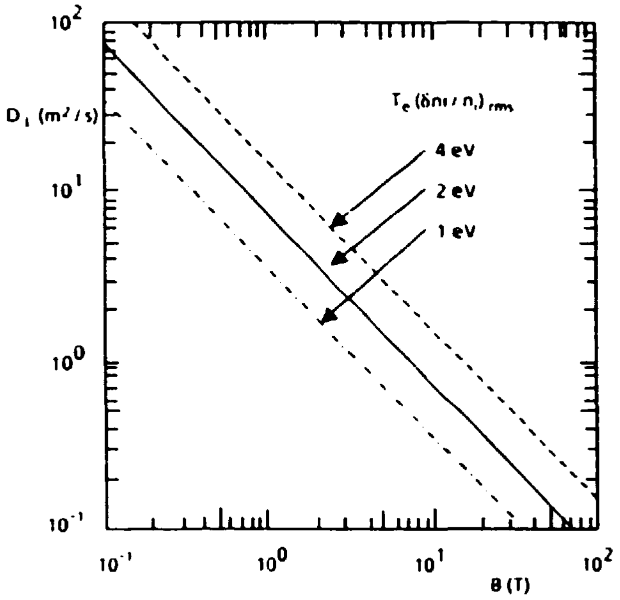

Fig. 1 - $D_\perp$ versus magnetic field for various fluctuation levels

$$t_p \approx \frac{a^2}{D_\perp} = \frac{1}{3.7}\,\frac{eB}{cT_e}\,\frac{a^2}{\left(\frac{\delta n_i}{n_i}\right)_{rms}} \tag{2}$$

In tokamaks, low frequency turbulent fluctuations are observed [3 - 6]. It may be noted that generally the electron temperature decreases and the fluctuation level increases with increasing minor radius. To the simplest approximation then, the product of electron temperature and ion fluctuation level is a constant across the magnetic field. A more detailed analysis may reveal this to be incorrect, however, the deviation probably is not large. In addition, the confinement time of particles must be obtained from radial averaging along the path a particle will travel. Hence, for clarity in the physics discussed for Figures 2 and 3, and because the difference probably will not be large, this product is treated as a constant (where more detailed information is available this need not be done). A typical product of temperature and fluctuation levels in tokamaks is in the range of $T_e\,(\delta n_i/n_i)_{rms} \approx 2$ eV and this number will be assumed to hold for all tokamaks, so that a radially averaged value of $\langle T_e\,(\delta n_i/n_i)_{rms}\rangle_r \approx 2$ eV (in these units). Hence, the considerations for Figures 2 and 3 apply to standard Ohmic discharges which are fairly MHD stable only. Auxiliary heating and pellet fueling may be expected to change the fluctuation levels, leading to different predictions, based on equation (1). The autocorrelation time of the turbulence may vary some from that used in equation (1), but as argued previously, it must be of the same order. Thus, the experimentally established

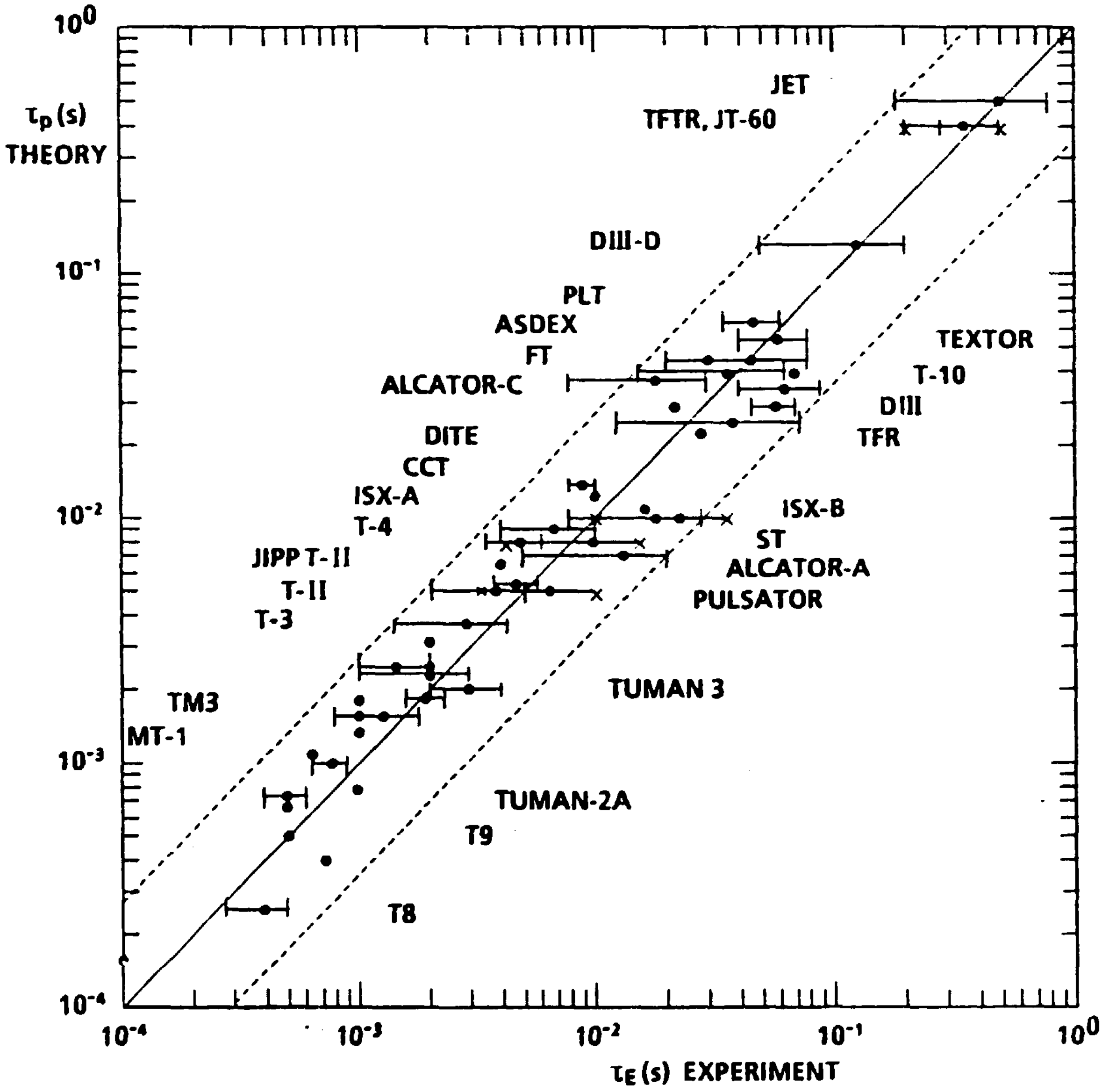

Fig. 2 - Particle confinement times predicted by equation (2) versus experimental reported energy confinement times.

numerical coefficient of equation (1) will be used. It is seen that in this theory only the minor radius, a, and toroidal magnetic field will play a direct role in the diffusion and confinement of majority species ions (the total magnetic field enters equation (1) but it is assumed here that $B_{pol} \ll B_{tor}$). Of course, it is possible that the amplitudes and autocorrelation times of the driving electric field turbulence may depend on other parameters (the knowledge of which, in combination with equations (1) and (2),

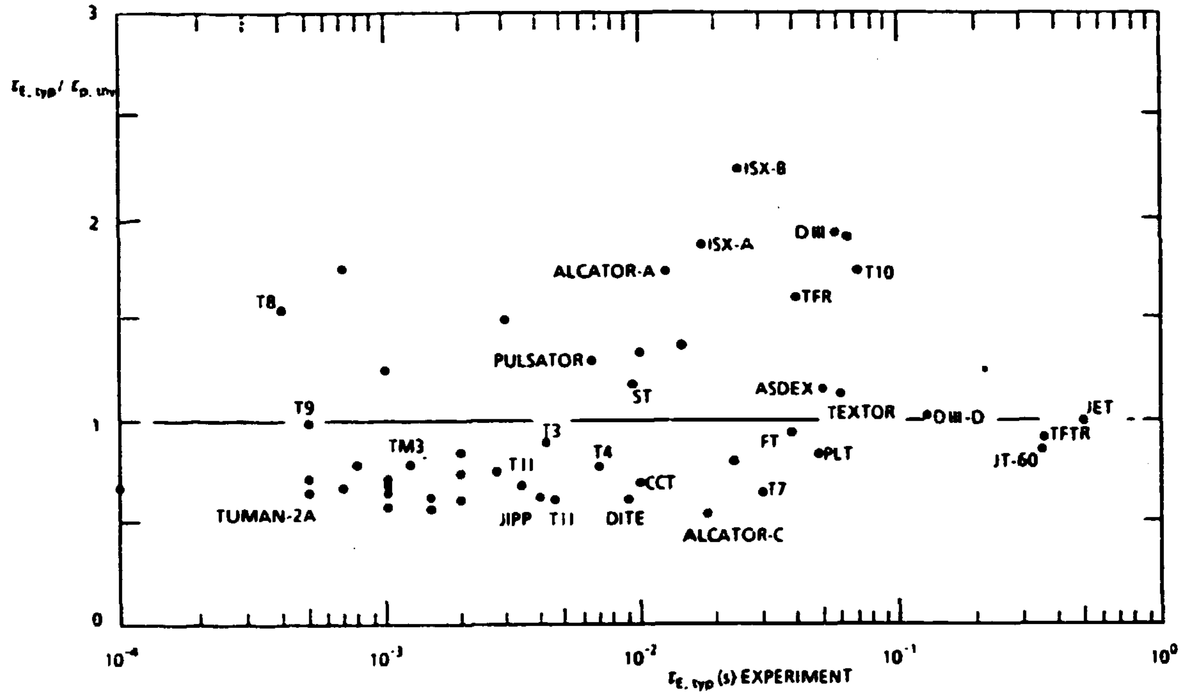

Fig. 3. Ratio of typical experimental energy confinement time to predicted particle confinement time versus typical experimental energy confinement time.

would lead to confinement scaling laws), but this is not addressed for Figures 2 and 3.

Particle and energy confinement times in tokamaks are difficult to measure and usually rely on inference from indirect measurements. Energy confinement times are perhaps more reliably measured in general and more commonly reported than particle confinement times. Since equation (2) predicts particle confinement times for a majority species thermal ion, it may also give a reasonable prediction of the energy confinement times. At the IAEA Meetings from 1961 until present, and the 1986 Nuclear Fusion Special Supplement on World Survey of Activities in Controlled Fusion Research [15], the experimental energy confinement times of tokamaks have been reported. The comparison data are taken from these collective proceedings unless otherwise noted [16]. There is commonly a range of times reported for individual tokamaks (e.g. often $\tau_E \alpha (\bar{n}_e)^x$ with x near 1/2 to 1). Figure 2 plots the reported Ohmic discharge energy confinement times with the range, if reported, represented by the horizontal lines and the dots representing the typical energy confinement time, $\tau_{E,typ}$ (i.e. the central value of the reported range), for 51 different tokamaks. Many of the devices are labelled on the plot. Some data points represent more than one device. The predictions of equation (2) are plotted versus these data, using only the published minor radius and toroidal magnetic fields for these experiments. The solid line through the data represents the line of unity correlation between τ_p (theory, equation (2)) and τ_E (experiment). The dashed lines indicate where τ_E varies by an e-fold from $\tau_{p,\,thy}$.

Figure 2 is a log-log plot, required because of the large range of data presented. For such plots, even when the statistical correlation is very strong, the human eye based judgement is often skeptical. Hence, Fig. 3 shows the ratio of $\tau_{E,typ}/\tau_p$ on a log-linear plot to emphasize the correlation in another and visually believable fashion.

Conclusions may be drawn from these plots. Only 4% (2 out of 51) of the tokamaks show any confinement time extrema outside a factor of e of the prediction of equation (2). Of significant interest is the observation that only 2% (one device) of the tokamaks exceeded the prediction of equation (2) by an e-fold for any regime of operation, and for that device it reached only a maximum factor of 3.5 above the prediction. The simple predictions of equations (1) and (2) agree within a factor of two with typical energy confinement times of 98% of the tokamaks compared. These conclusions suggest strongly that electrostatic turbulence plays a major role in tokamak confinement through the physical mechanisms elucidated in equation (1).

Additionally, unless significantly different ways to control electrostatic turbulence in tokamaks are found, the scaling predicted by these equations and assumptions is likely to hold for typical confinement times for future devices. Including the Irvine work and tokamak CSTN - II, these predictions cover a range of about 5 orders of magnitude in temperature, 5 orders of magnitude in density, 5 orders of magnitude in time, 2 orders of magnitude in magnetic field, and 3 orders magnitude in fluctuation level.

Considering the operation of single tokamaks is also in order. Observations on TFR [5] and preliminary results on FT [17] indicate that the radially averaged fluctuation level may go as $(\bar{n}_e)^{-x}$ with x in the range of 1/2 to 1. This would lead to $\tau_p \propto (\bar{n}_e)^x$ in equation (2), as is frequently observed on these devices. Hence, one may suspect that for each specific tokamak, the left hand ends of individual tokamak experimental values plotted in Figure 2 are generally for lower densities, while the right hand values are for higher densities (this trend is verified where the published data show τ_E vs $\bar{n}_e$). At the extrema in the individual device experimental τ_E values the rigid assumption of $\langle T_e\,(\delta\, n_i/n_i)_{rms}\rangle_r \approx 2$ eV may thus prove a little too low or high and actual values should be determined.

As an aside, for PDX Crowley and Mazzucato [18] have reported increased fluctuation levels with neutral beam injection in a rail limited plasma. They also report reduced fluctuation levels for H-mode operation compared to L-mode on the PDX tokamak. This shows why care must be taken in the application of equation (2) to auxiliary heated plasmas, but suggests that the physical mechanisms leading to equations (1) and (2) still apply in such regimes.

How does the prediction of equation (1) compare where detailed particle confinement times and inferred diffusion coefficients have been made in tokamaks, rather than compared to τ_E as has been done thus far? The recently published work of Rowan, et al [6] for particle confinement times provides sufficient data for a detailed comparison of these predictions with the results from the TEXT device. Reference 6 Figures 2, 3, 8, 11 (b), and 12 show TEXT measurements and inferred estimates of temperature, density, fluctuation levels, diffusion coefficient profiles and global particle confinement times, respectively. Using the measured values of electron temperature and fluctuations levels of Reference 6 Figures 2, 3, and 8 one may apply equation (1) to predict the radial dependence of $D_\perp$ for the TEXT device. Figure 4 of the present paper shows this result in comparison. The curve labeled $D_{\perp McW}$ is the prediction of equation (1) using the radial profile values given in Reference 6 Figures 2, 3, and 8 while the curve labeled $D_{\perp ROW}$ is a reproduction of the experimentally inferred diffusion coefficient of Reference 6 Figure 11 (b) (the data of Reference 6 do not allow a comparison for $r < 25.5$ cm in the graph but do allow a comparison for $r \geq 25.5$ cm, as is done). One sees that $D_{\perp McW}$ reproduces the inferred radial profile of $D_{\perp ROW}$ probably within the experimental uncertainty and that $D_{\perp McW}/D_{\perp ROW} \approx 2$ over the whole range of the graph. Given that the choice of autocorrelation time was based entirely on other work [10 - 14], this close agreement goes some way in experimentally justifying that the choice of autocorrelation time based on the experiments at Irvine is correct within order unity. An additional prediction from equation (1) may be made here. For Figure 2 a value of $\langle T_e(\delta n_i/n_i)_{rms}\rangle_r \approx 2$ eV was used for all tokamaks in that figure. Using this value for the TEXT data discussed here gives $D_\perp \approx 4 \times 10^4$ cm^2/s, which is plotted as the horizontal line in Figure 4. One sees that this is approximately the average value of $D_{\perp ROW}$ over the entire graph.

Particle confinement times also may be compared with equation (2) for some experiments. Reference 6 Figure 8 allows a direct comparison from a equation (2) of the particle confinement time for the TEXT experiment by noting the fluctuation level at the limiter edge ($r = 27$ cm). Equation (2) predicts a particle confinement time of 5.8 ms while Reference 6 Figure 12 shows a range of 4 - 8 ms. For another device, the Frascati Tokamak, De Angelis and Tonini [19] have reported particle confinement times of about 24 $\pm$ 5 ms while the prediction of equation (2) is $\tau_p = 29$ ms for FT at 60 kG.

This work has shown that electrostatic turbulence in tokamaks can play a major role in tokamak particle and energy confinement times. A simple model of turbulent

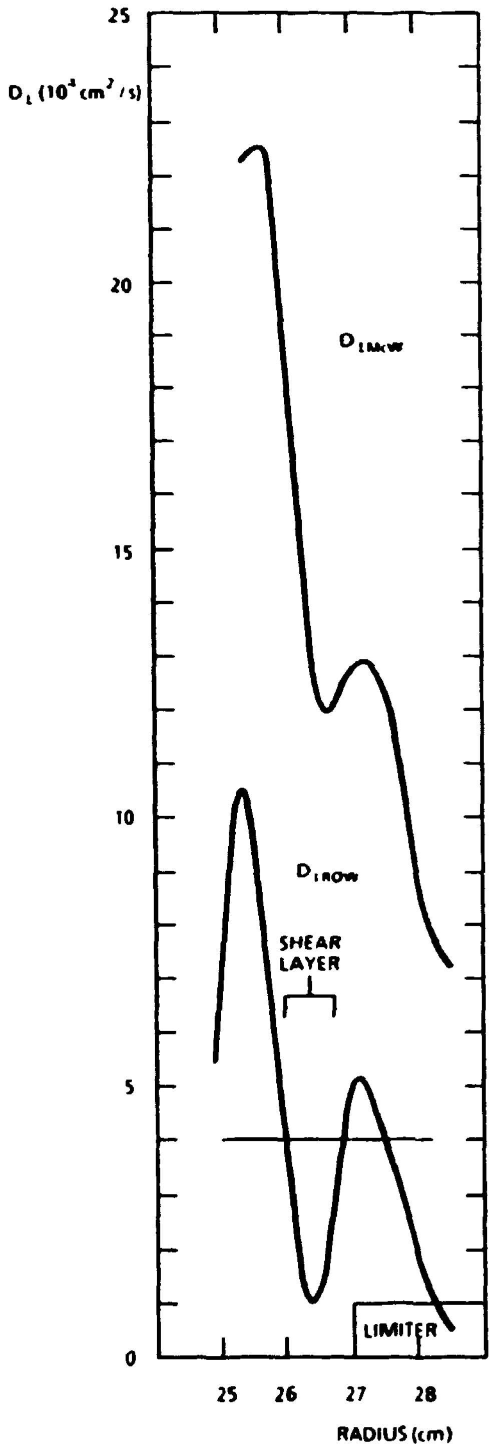

Fig. 4 - Comparison of predictions of equation (1) with TEXT results of Rowan, et al. [6].

diffusion was presented which agreed with experiment. The model predicts confinement times and diffusion coefficients covering a wide range of plasma

parameters for tokamaks operating without auxiliary heating or pellet fueling and which were fairly MHD stable. The effects of the physical mechanisms driving the cross-field transport were evident in a simple way. The causes of these turbulent electrostatic fields (which might then lead to confinement scaling laws) and how auxiliary heating or pellet fueling might effect them were not discussed.

ACKNOWLEDGEMENTS:

The basic plasma physics work leading to equation (1) was supported by the National Science Foundation Grant No. PHY-8606081.

The work directly related to this paper was supported by U.S. Department of Energy Grant No. DE-FG03-86ER53231 and Associazione EURATOM - ENEA sulla Fusione, CRE Frascati.

Illuminating discussions were held with Drs. D. Frigione, F. De Marco, G. Mazzitelli, V. Pericoli-Ridolfini, L. Pieroni, and F. Romanelli.

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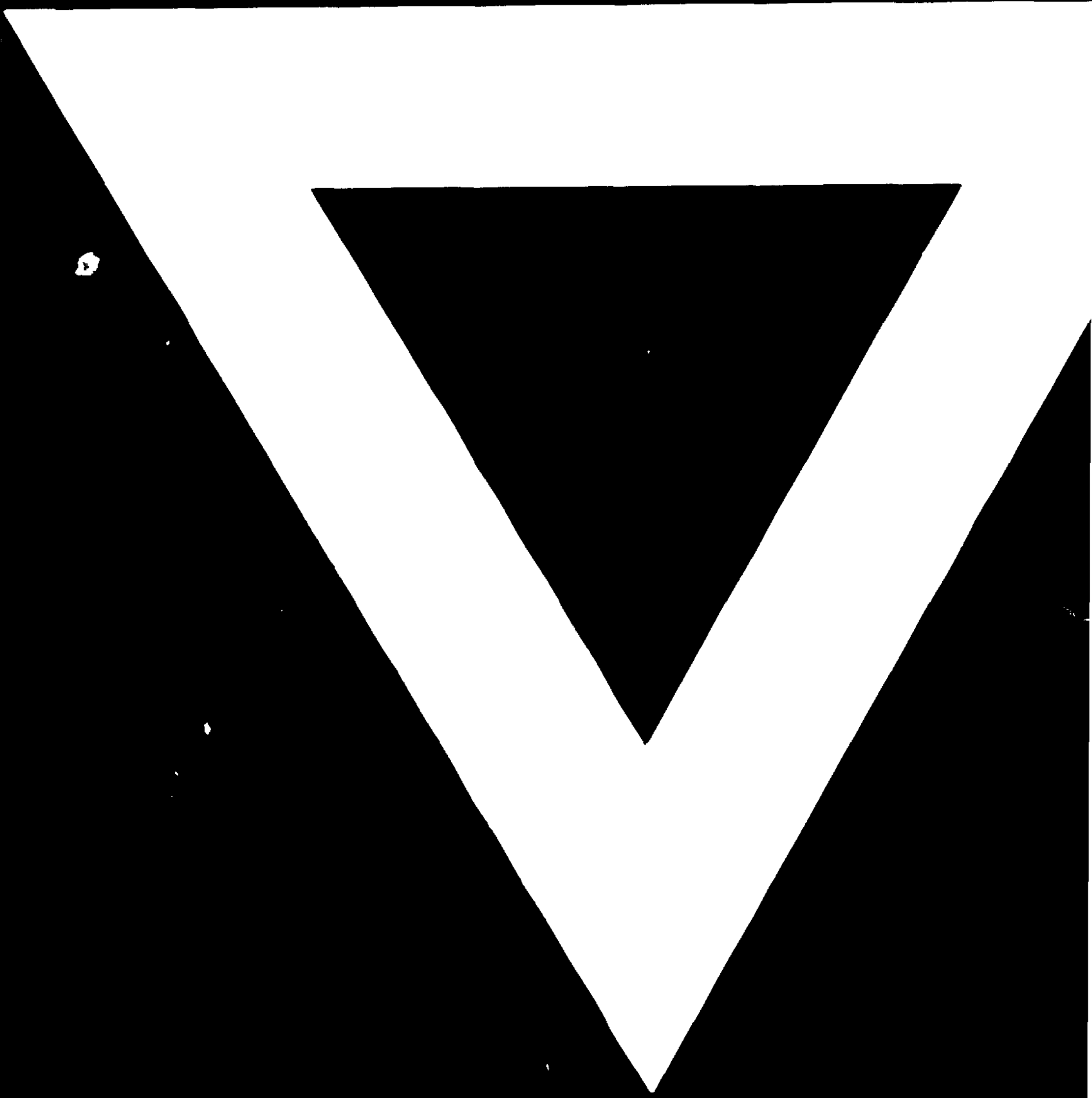
Edito dall'ENEA, Direzione Centrale Relazioni.
Viale Regina Margherita, 125 - Roma
Finito di stampare in settembre 1988

Fotoriproduzione e stampa
a cura della «Arti Grafiche S. Marcello»
Viale Regina Margherita, 176 - Roma

Questo fascicolo è stato stampato su carta riciclata



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