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INDUCED QUANTUM CONFORMAL GRAVITY *

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ABSTRACT

Quantum gravity is considered as induced by matter degrees of freedom and related to the symmetry breakdown in the low energy region of a non-Abelian gauge theory of fundamental fields. An effective action for quantum conformal gravity is derived where both the gravitational constant and conformal kinetic term are positive. Relation with induced classical gravity is established.

MIRAMARE - TRIESTE
November 1988

* To be submitted for publication.

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1. INTRODUCTION

Twenty years ago Zeldovich [1] proposed to consider the cosmological constant as a result of quantum fluctuations and Sakharov [2] presented an argumentation in favour of induced classical gravity in general. These papers triggered studies in the search of self-consistent formulation of induced gravity in the framework of the Quantum Field theory (see review [3] and also [4]). However, there are some problems related to such a consistent formulation.

The first problem is an introduction of a dimensionful parameter. The mass of a matter field is a natural choice, but it should be extremely large. It was suggested [5] to use a regularization parameter of the Pauli-Villars scheme. However, finite auxiliary masses in this scheme have no physical meaning. The second problem is the lack of uniqueness and positive definiteness of the gravitational constant [6] for the induced classical gravity.

Another line of research considers that gravity is induced at the quantum level. Composite theories of gravity introduce vierbeins and Lorentz connections as composites of fundamental fields [7]. In non-linear sigma models [7] they are fermionic composites.

In the present paper we propose an approach to the quantum gravity as induced by quantized matter fields and related to non-invariance transformations. More specifically, our position is the following.

(i) Vierbeins and Lorentz connections are composite fields corresponding to local limits of matter field bound states. Thus, we suppose that there are fundamental fields strongly interacting in their low energy region. While fundamental fields are obviously necessary for universality of gravity, the low energy approximation reflects local properties of gravity. At present, the only self-consistent model which displays strong coupling at low energies is a QCD-type model with pre-fermions and pre-gluons.

(ii) The induced quantum gravity as a whole is a manifestation of non-invariance effects. Thus, we generalize to the case of quantum gravity the Adler's observation that the classical induced gravity is intimately related to the conformal anomaly, i.e. to the breakdown of scale invariance. An association of gravity with parameters of a non-invariance group is essential because it enables to extract gravitational variables of fundamental fields without recourse to other degrees of freedom. However, to achieve this task, it is necessary to make a reasonable assumption about non-invariance effects.

In order to demonstrate this approach we consider the case of quantum conformal gravity. We apply the four-dimensional bosonization procedure developed for QCD [8] to extract the conformal degree of freedom of prefermions in a curved space with the torsion. The effective action for quantum conformal gravity is obtained nonperturbatively by integrating conformal anomaly and separating all conformally non-invariant effects from invariant ones. We show that both problems of induced classical gravity mentioned above (introduction of dimensionful parameters and positiveness of the gravitational constant) can be resolved in a satisfactory way in the case of quantum conformal gravity and, consequently, the idea of induced gravity is still alive. Moreover, the conformal kinetic energy is also positive here.

We establish also a relation of induced quantum gravity to the induced classical one and to Einstein gravity. We also show that with the proper definition of low energy parameters (simulating the case of QCD [8]), the induced Newton constant is positive in the classical induced gravity too.

2. INDUCED QUANTUM GRAVITY AND NON-INVARIANCE GROUPS

Let us consider matter fields Φ_i on the curved space-time with the external vierbein e^a_μ. The generating functional is given by

$$Z_\Phi(e) = \int \mathcal{D}\Phi \exp i \int \mathcal{L}_m(\Phi, e) d^4x \quad , \tag{1}$$

where $\mathcal{L}_m(\Phi, e)$ is the matter Lagrangian.

The classical induced gravity (IG) is identified with external gravity after quantum fluctuations of all matter fields are taken into account. All matter degrees of freedom are integrated out. The effective action for classical IG is given by $\exp i W_{eff}(e) = Z_\Phi(e)$, see [1-6].

We consider an induced quantum gravity. It is identified with specific degrees of freedom of matter fields (to be found). The effective action for this case is given below by Eq.(4). To derive it we consider interacting fundamental fields (e.g. prequarks and pregluons) and assume that it is possible to subdivide all their degrees of freedom $\{\Phi\}$ into gravitational ones ξ and the rest X. This implies that one can perform the change of variables in $Z_\Phi(e)$ from $\{\Phi\}$ to $\{\xi, X\}$ and then derive an effective action $W_{eff}(\xi, e)$ for variables ξ ("induced quantum gravity") by integrating over variables X:

$$Z_\Phi(e) = \int \mathcal{D}X \mathcal{D}\xi \exp i W(e, \xi, X)$$

$$= Z^0(e) \int \mathcal{D}\xi \exp i W_{eff}(\xi, e) \equiv Z^0(e) Z_\xi(e) \quad , \tag{2}$$

where Z^0 does not depend on ξ.

New variables X describe composite fields which can be local, bilocal, etc. Progress can be made only if one can find an effective action $W_{eff}(\xi, e)$ in (2) without knowing variables X. Let us show that this can be done if IG variables ξ are parameters of a non-invariance transformation which change $Z_\Phi(e)$.

Suppose that we know a group $\mathcal{G}$ comprising both invariance and non-invariance transformations and $\mathcal{D}\xi\,\mathcal{D}\lambda$ is the ter Haar measure, where ν are parameters of invariance transformations. In the expression $Z_\Phi(e) = Z^0(e)\, Z_\xi(e)$ in (2) only the factor Z_ξ is not invariant under transformations from $\mathcal{G}$. It follows that in order to find an effective action $W_{eff}(\xi)$ for induced gravity it is sufficient to extract from the initial functional Z_Φ its $\mathcal{G}$-invariant part Z^0. An $\mathcal{G}$-invariant functional $Z_{inv}(e)$ can be constructed by hand by transforming fields in Z_Φ, $\Phi \to \Phi_\xi$ and averaging Z_{Φ_ξ} over a non-invariance group

$$Z^{-1}_{inv}(e) = \int \mathcal{D}\xi\, \mathcal{D}\lambda \;\; Z^{-1}_{\Phi_\xi}(e) \quad . \tag{3}$$

Note that Z_{inv} may not coincide with Z^0. It means that the effective action $W_{eff}(\xi, e)$ is defined up to $\mathcal{G}$-invariant terms

$$\exp i W_{eff}(\xi, e) = Z_\Phi(e)\, Z^{-1}_{\Phi_\xi}(e) \quad , \tag{4}$$

whereas all non-invariant effects are uniquely described by (4). Local group parameters $\xi(x)$ correspond to local limits of non-local bound state fields.

Parameters $\xi(x)$ are collective variables of matter fields, i.e. of both pregluons and prequarks. We assume that as a first approximation one can separately treat pregluonic and prequark collective variables and in this paper we discuss only prequark ones. Therefore we shall be interested in properties of the prequark functional. Note that in QCD the low mass bound states are formed exclusively out of quarks.

The variables X represent what is understood in Einstein gravity as specific matter degrees of freedom.

3. LOW ENERGY REGION

A definition of low energy region L is necessary or order to treat IG as a low energy effect of the (pre-) theory with very high mass scale. At the same time it will enable us to introduce dimensional parameters of IG. The low energy region L should replace regularization or cutoff of functional integrals Z_Φ. Usually, a regularization procedure defines an auxilliary quantity $Z_\Phi^{(\Lambda)}$ which coincides with Z_Φ (i.e. recovers all properties of Z_Φ) only in the limiting case (say $\Lambda \to \infty$), so that at nonlimiting values of Λ the quantity $Z_\Phi^{(\Lambda)}$ cannot represent Z_Φ even in a restricted sense (e.g. the Pauli-Villars regularization).

Our model for IG is pre-QCD, i.e. the QCD like theory with very high confinement scale (or Λ_{QCD}). It is assumed that strong interaction of prequarks in the low energy region L leads to formation of composite fields including a universal (gravitational) conformal field. With this choice of model we can extend to general curved space the definition of the low energy region in QCD on conformally flat space [8].

A low energy region is defined in terms of the functional integral over prefermions in the Euclidean space

$$Z_\psi^L = \int_L D\bar{\psi} D\psi \, \exp\left(- \int d^4x \, g^{1/2} \bar{\psi} \not{D} \psi \right) \qquad (5)$$

where $\not{D}$ is the complete Dirac operator

$$\not{D} = i\gamma^\alpha e_\alpha^\mu (\partial_\mu + G_\mu + \frac{1}{4} \omega_{\mu\beta\delta} \gamma^\beta \gamma^\delta) \qquad (6)$$

G_μ is a pregluon field. It will be omitted whenever it is not essential.

We choose the eigenfunction basis for the Hermitian operator $\not{D}$, $\not{D}\psi_n(x) = \lambda_n \psi_n(x)$ in the space of L_2-integrable spinors. Then the low energy region L is defined by the set of eigenvalues $L = \{\lambda_1, \ldots, \lambda_N\}$. This set L is specified by the following requirements:

a) General coordinate and local Lorentz covariance.

b) One-loop unitarity of the theory with Z_ψ restricted to L.

c) Absence of spectral flow over the border of L.

In the continuous limit L is determined by two parameters Λ and m, $\Lambda \geq m \geq 0$, in the following manner: $\Lambda + M \leq \lambda \leq \Lambda + M$. Then Z_ψ^L will meet requirements (a) - (c) if it is specified according to the finite mode regularization scheme [9], so that

$$Z_\psi^L \equiv Z_{\Lambda,M}(e_\mu^\alpha) = \det(1 - P_{\Lambda,M} + P_{\Lambda,M} \not{D} P_{\Lambda,M}) \ ,$$
$$P_{\Lambda,M} = \theta(1 - (\not{D}-M)^2/\Lambda^2) \ . \qquad (7)$$

One-loop unitarity is assured by use of the operator $\not{D}^2$ (and not $\not{D}\not{D}^+$). The parameters Λ and M are related to the spontaneous breakdown of chiral symmetry and conformal anomaly as manifested through the formation of the prequark and pregluon condensates [10].

4. INDUCED CLASSICAL GRAVITY

Now we can ask ourselves how our approach is connected with Einstein gravity. Our starting point is that non-invariance group transformations can be rewritten as transformations of background fields including metric. In the simplest case of dilation it is $g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu}$. A general form of non-invariance transformation is much more complex and as yet unknown to us. Fortunately, here we do not need a precise form of these transforms. Suppose that the matrix $\delta g_{\mu\nu}(x)/\delta\xi_i(x)$ is nonsingular. It only means that the non-invariance group is large enough to include arbitrary infinitesimal transformations of $g_{\mu\nu}(x)$. This is the correspondence principle between quantum (in terms of ξ_i) and classic (in terms of $g_{\mu\nu}$) description of gravity.

Consider small variations of background metric $g_{\mu\nu}$ in $Z_\Phi(e)$ in the form (2) where the specific matter degrees of freedom X are not integrated out. This variation

$$\frac{\delta Z_\Phi(e)}{\delta g_{\mu\nu}} = \int DX \, D\xi \, e^{iW(e,\xi,X)} \frac{\delta iW(e,\xi,X)}{\delta g_{\mu\nu}} = 0 \qquad (8)$$

vanishes due to the equation of motion $\frac{\delta W(e,\xi,X)}{\delta \xi_i} = 0$. Non-zero vacuum values of ξ_i can be absorbed in background fields. Eq.(8) means $\langle \theta_{\mu\nu}(x) \rangle_0 = 0$, where $\theta_{\mu\nu}$ is the total stress tensor of gravitation plus matter fields X. It is a stability condition of vacuum state.

Let us rewrite (8) in the form

$$\frac{\delta W(e,\xi,X)}{\delta g_{\mu\nu}} = \frac{\delta}{\delta g_\mu} W_G(e,\xi) + \frac{\delta}{\delta g_{\mu\nu}} W_X(e,\xi,X) \ ; \qquad (9)$$

where W_G depends on gravitational variables ξ_i and curved background only. The second term in (9) represents the stress tensor of matter $\theta_X^{\mu\nu}$.

Some comments concerning the cosmological constant problem are in order. Self-consistent treatment of this problem is possible only in a curved space-time. On a curved background

$$\langle \theta_\mu{}^\mu \rangle_0 = \langle \theta_\mu{}^\mu(R) \rangle + \langle \tilde{\theta}_\mu{}^\mu \rangle_0 = 0 \quad , \tag{10}$$

where all curvature independent terms are collected in $\langle \tilde{\theta}_\mu{}^\mu \rangle$. It is this term $\langle \tilde{\theta}_\mu{}^\mu \rangle$ which is proportional to cosmological constant. Thus, cosmological constant vanishes up to $\langle \theta_\mu{}^\mu(R) \rangle$ which is typically small but non zero.

Let us consider an example of prequark field and evaluate the first term in (10):

$$A_\alpha{}^\mu \equiv \frac{\delta \, \ln Z_\phi^G}{\delta e_\mu^\alpha(x)} = i \lim_{x' \to x} \mathrm{tr} \langle x' | \gamma_\alpha D^\mu \not{D}^{-1} P_\Lambda | x \rangle \quad . \tag{11}$$

Note that point shifting here is not an additional regularization but only a computational device which allows us to use geodesic waves. Calculations here are similar to those applied by Fujikawa to the chiral anomaly [11]. We obtain

$$A_\alpha{}^\mu(x) = - \frac{\Lambda^4}{(2\pi)^4} \int \frac{d\alpha \, e^{i\alpha}}{2\pi i(\alpha - i\varepsilon)} \int_\alpha^{\alpha - i\infty} d\beta \int d^4\tilde{\kappa} \; \times$$

$$\mathrm{tr} \lim_{x' \to x} \left[\gamma_\lambda \left(\frac{D_\mu}{\Lambda} + i\Delta_\mu \right) \left(\frac{\not{D}}{\Lambda} + \Delta \right) \right.$$

$$\left. \exp \left\{ - i\beta \left(\frac{\not{D}}{\Lambda} + \not{\kappa} \right)^2 + i\alpha M \left(\frac{\not{D}}{\Lambda} + \not{\kappa} \right) - i\alpha M^2 \right\} \right] ,$$

$$\tilde{\kappa} = K/\Lambda \, , \quad \Delta_\mu = \tilde{K}^\alpha D_\mu D_\alpha \, \sigma(x',x) \quad . \tag{12}$$

From (12), after construction of the Minkowski space and retaining only curvature squared terms, we get

$$A_\alpha{}^\mu = i(-g)^{1/2} \left\{ - \frac{\Lambda^4 - 6\Lambda^2 M^2 + M^4}{32\pi^2} e_\alpha{}^\mu + \frac{\Lambda^2 - M^2}{48\pi^2} \left(R_\alpha{}^\mu - \frac{1}{2} R e_\alpha{}^\mu \right) \right\} . \tag{13}$$

This equation leads to the Einstein equation with induced Newton constant

$$G_N = \frac{6\pi}{\Lambda^2 - M^2} \tag{14}$$

which is positive due to $\Lambda^2 > M^2$. The same value of induced G_N can be obtained using the conformal anomaly [12].

Note that expression (13) contains no David's anomaly [6]. Instead of it we have additional parameter M which is the same for every quantity of interest (G_N, coupling constant, condensates etc.) in contrast with the procedure of analytic continuation being arbitrary for every matrix element [6]. For instance, M can be fixed using the prefermionic condensate

$$\langle \bar{\psi}\psi \rangle = - \frac{1}{2\pi^2} \left[M\Lambda^2 - \frac{M^3}{3} \right] + \frac{7}{24\pi^2} MR \quad .$$

5. DILATION EFFECTIVE ACTION

In this section we obtain an effective action for dilation as a conformal composite degree of freedom related to the conformal part of a non-invariance group. The importance of conformal invariance breaking has been stressed by many authors [3, 13].

The functional $Z_\psi(e_\mu^\alpha)$ can be presented in the following form:

$$Z_\psi(e_\mu^\alpha) = \int D\bar{\psi}D\psi \, \exp(-\bar{\psi}\not{D}\psi)$$

$$= \int D\bar{\psi}D\psi D\omega \, \exp(-W_{eff}) Z_{inv} \tilde{Z}_\psi \quad . \tag{15}$$

Hence

$$\int D\omega \, \exp(-W_{eff}) \equiv Z_{eff} = Z_{\Lambda,M} Z_{inv}^{-1}$$

or

$$W_{eff} = - \ln(Z_{\Lambda,M}(e_\mu^\alpha)/Z_{\Lambda,M}(e^\omega e_\mu^\alpha)) \quad . \tag{16}$$

$\tilde{Z}_\psi$ reflects the presence of high-energy fermions. Both $\tilde{Z}_\psi$ and Z_{inv} contain no information about conformally non-invariant processes. Now, W_{eff} can be calculated by integration of the conformal anomaly (13).

$$W_{eff} = \int d^4x \; \omega(x) \int_0^1 d\tau \; A(e^{\omega\tau} e_\mu^\alpha) \tag{17}$$

$$A = e_\mu^\alpha \frac{\delta}{\delta \, e_\mu^\alpha} \ln Z_{\Lambda,M} = e_\mu^\alpha A_\alpha{}^\mu$$

where $A_\alpha{}^\mu$ is given by (13).

Substituting (13) into (17) we obtain in the Minkowski space

$$W_{eff}(\omega, e^{\alpha}_{\mu}) = \frac{i}{32\pi^2} \int d^4x(-g)^{1/2} \left\{ \left(\Lambda^4 + M^4 - 6\Lambda^2 M^2 \right) (1 - \Omega^4) + (\Lambda^2 - M^2) \left(\frac{R}{3} - \frac{\Omega^2 R}{3} - 2\Omega\Box\Omega \right) \right\} \quad . \tag{18}$$

Note that dilation kinetic energy in (18) is positive as well as an induced Newton constant (14). It solves a long standing problem of conformal instability in quantum gravity [14]

Let us elaborate for the sake of clarity the difference between the definition of effective conformal action in the quantum gravity given in [14] and the definition in the spirit of this paper (in the case of QCD, see [8]). Let Z_G be the quantum gravity functional [14] (the Euclidean space)

$$Z_G = \int \mathcal{D} g_{\mu\nu} \exp\left(- W(g_{\mu\nu}) \right) \quad , \tag{19}$$

where $W(g_{\mu\nu})$ is the gravitational action. Under conformal transformation $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$, $Z_G \to Z_G(\Omega)$, so that the initial Z_G is $Z_G(\Omega = 1)$. The effective conformal action is given by [14]

$$\exp(- W^H(\Omega)) = Z_G(\Omega), \qquad W^H = \int d^4x \; g^{1/2}(R\Omega^2 + 4\Omega\Box\Omega) \quad .$$

It displays a wrong sign for the kinetic term.

In our approach we represent Z_G in the form $Z_G = Z_G^{inv} Z_\Omega$, where Z_G^{inv} is invariant under conformal transformations while Z_Ω accumulates all (conformal) non-invariance effects and may be represented as

$$Z_\Omega = \int \mathcal{D}\Omega \exp(- W_{eff}(\Omega)) \quad , \tag{20}$$

where $W_{eff}(\Omega)$ is an effective action in our case. The invariant part Z_G^{inv} is obtained by integrating conformally transformed $Z_G(\Omega)$ over all conformal transformations

$$(Z_G^{inv})^{-1} = \int \mathcal{D}\Omega \; Z_G^{-1}(\Omega) \quad . \tag{21}$$

so that the effective conformal action is

$$\exp(- W_{eff}(\Omega)) = Z_G(\Omega{=}1) Z_G^{-1}(\Omega) \quad , \tag{22}$$

where $Z_G(\Omega)$ is now in denominator, thus reversing the sign of the effective action: $W^H(\Omega) = - W_{eff}(\Omega) + W^H(\Omega{=}1)$.

It can be demonstrated [15] that if we choose the group $GL(4,C)$ of linear transformations of prefermionic field then some degrees of freedom acquire positive kinetic energy while some degrees get negative kinetic terms. This subject will be discussed separately.

ACKNOWLEDGMENTS

One of the authors (Y.V.N.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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