Fermi National Accelerator Laboratory

FERMILAB-Conf-88/134-T

Topics in B-Physics*

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September 1988

*Talk given at the IV Workshop on Recent Developments in High Energy Physics, Orthodox Academy of Crete, Chania-Crete, Greece, July 1-20, 1988

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

TOPICS IN B-PHYSICS

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Abstract

We discuss a few issues in the burgeoning field of physics of hadrons containing the b-quark. These include:

1) A simple parametrization of the Kobayashi-Maskawa matrix featuring a triangle in the complex plane,

2) a review of B_s and B_d mixing with special attention given to width-mixing and the CP-violating same-sign dilepton asymmetry,

3) a discussion of the CP-violating decay $B_d \to \psi\pi^+\pi^-$,

4) a discussion of CP-violating rate asymmetries in the two-body decays $\Lambda_b \to p\pi^-$ and $\Lambda_b \to pK^-$.

The concluding discussion concerns generalizations beyond these specific topics.

Introduction

Of the twenty-odd fundamental parameters of the standard model, four of them reside in the Kobayashi-Maskawa mixing matrix. It is obviously of high priority to not only measure them well but to provide a redundant set of measurements sufficient to overdetermine them. Only in such a way can one test that the standard-model description is indeed valid.

The decays of hadrons containing b-quarks appear to be a most promising avenue for doing this. Even CP-violating effects can be anticipated to be large in various rare decay and mixing phenomena. And although the experiments are extremely difficult, there is growing optimism that eventually the measurements will be done.[1]

The subject of b-physics is quite large.[2] In this talk, we discuss in detail only a few specialized items as enumerated in the abstract, and reserve more general remarks for the concluding section.

I. The Kobayashi-Maskawa Matrix

The amplitude for W^+ to decay into quark Q_i and antiquark $\bar{q}_j$ is proportional to gV_{ij}, where g is the weak coupling constant characterizing pure leptonic decays. The factor V_{ij} is, by definition, an element of the n x n Kobayashi-Maskawa "mixing-matrix", where n is the number of standard-model generations. It accounts for the varying strengths of non-leptonic decay processes, which depend sensitively upon the amount of flavor-violation which is present.

It is a requirement of the standard-model that the matrix V_{ij} be unitary, in order that radiative corrections to electroweak processes be finite and not lead to unacceptably large amounts of flavor-changing neutral currents. In addition to the unitarity constraint, there is an additional freedom of choice of phases of the elements of V_{ij} because physics does not change if the phases of the quark fields in the theory are redefined. That is, if

$$V_{ij} \longrightarrow e^{i(\alpha_i - \alpha_j)} V_{ij} , \tag{1.1}$$

physics does not change, even though the Lagrangian of the theory does change.[3] This reparametrization invariance is somewhat like gauge-invariance in field theories: choice of different gauges implies different Lagrangians (and sometimes different Hilbert spaces!), but the physics remains gauge-invariant.

Traditionally, two or three-generation descriptions of V_{ij} have used a generalized Euler-angle representation. This works fine, but rapidly becomes quite cumbersome for more than three generations. Here we advocate[4] a simple description which introduces no such angles, but simply fixes the 2n - 1 phases of the diagonal and upper-right next-to-diagonal elements of V_{ij} such that they are all real and positive.

That is,

$$V = \begin{pmatrix} R & R & \bullet & \bullet & \bullet \\ \bullet & R & R & \bullet & \bullet \\ \bullet & \bullet & R & R & \bullet \\ \bullet & \bullet & \bullet & R & R \\ \bullet & \bullet & \bullet & \bullet & R \end{pmatrix} \tag{1.2}$$

where the elements marked R are not only real but also positive. Independent parameters may be chosen to be the magnitudes and residual independent phases of all elements above and to the right of the diagonal.

For the 3-generation case, one finds, using unitarity plus reliable data:

$$V = \begin{array}{c} u \\ c \\ t \end{array} \overset{\begin{array}{ccc} d & s & b \end{array}}{\begin{pmatrix} .97 & .23 & V_{ub} \\ -.23-.05V^*_{ub} & .97 & .05 \pm .01 \\ V_{td} & -.05-.23V^*_{ub} & 1.00 \end{pmatrix}} \tag{1.3}$$

Unitarity implies orthogonality of the first and third rows. Neglecting 5% corrections, this simplifies to

$$V_{ub}^* + V_{td} \cong .011 \pm .002 \tag{1.4}$$

This is a triangle relation[5] in the complex plane, as shown in Fig. 1. Note that the height of the triangle is proportional to the strength of CP-violation, i.e. proportional to ϵ and ϵ'/ϵ of the neutral kaon system. The square of V_{td} is proportional to the amplitude for $B_d - \overline{B}_d$ mixing, observed to be "surprisingly" large. The squared magnitude of V_{ub}^* is proportional to $\Gamma(B \rightarrow$ nonleptonic uncharmed states). If the "surprisingly large" ARGUS measurements of $B_u^+ \rightarrow p\bar{p}\pi^+$ and $B_d \rightarrow p\bar{p}\pi^+\pi^-$ hold up (they have not been confirmed by Cornell), this implies a relatively large V_{ub}, forcing the triangle into the obtuse shape in Fig. 1. But this inference, presently favored by most theorists, is quite shaky. A right or even acute triangle is far from being ruled out.[6]

In terms of this triangle we can express some interesting features of b-physics:

1) Determination of the magnitudes of the V_{ij}, in particular V_{ub} and V_{td}, is evidently sufficient to determine whether or not the triangle is nondegenerate, i.e. whether or not there really are CP-violating phases. This simple piece of geometry has recently inspired lengthy and learned theoretical treatises.[7]

2) The CP-violating phase angles associated with various decay processes are simply related to the interior angles of the triangle; examples will be seen later.

3) It is an important experimental issue to determine whether indeed the triangle closes, i.e. whether Eqn. 1.4 is satisfied. If it does, the angles and sides should be <u>overdetermined</u>.

I believe that if the program of observation of CP-violation in b-decays is possible at all, then there is sufficient richness and variety within it that such overdetermination should be possible. The situation is in some ways comparable to the validation and determination of the Cabibbo-angle description of strange-particle decays.

Before concluding this overview of V_{ij}, it must be emphasized that the origin of the Kobayashi-Maskawa parameters is quite deep. One way of appreciating this is to set the gauge coupling constants α, α_w, α_s to zero, keeping the other 16 or so standard-model parameters fixed.[8] Upon doing this, the mass of $W^{\pm}$ goes to zero, so that processes $W^+ \rightarrow Q_i\bar{q}_j$ go into $Q_i \rightarrow q_j + W^+$. This decay exists even in the limit of vanishing gauge couplings provided the W^+ spin degree of freedom is longitudinal. This third polarization state is in fact nothing other than the Nambu-Goldstone spinless, massless boson associated with spontaneous symmetry breaking in the Higgs sector. Thus in this gaugeless limit, V_{ij} measures the Yukawa couplings of Nambu-Goldstone bosons to quarks. To go deeper into the physical origin of these couplings goes beyond questions of known gauge interactions. It is likely however that there is linkage between the origin of quark mixings and the origin of quark mass.

II. B-$\bar{B}$ Mixing

The mixing phenomenology for the neutral B system is similar to the K-$\bar{K}$ mixing phenomenology, but simpler.[9] In the rest frame of the neutral B, the Schrödinger equation reads

$$i \frac{\partial}{\partial t} \begin{pmatrix} B \\ \bar{B} \end{pmatrix} = \left[\Delta M - i \frac{\Delta\Gamma}{2} \right] \begin{pmatrix} B \\ \bar{B} \end{pmatrix} \tag{2.1}$$

where the common mass and width, i.e. the factor $\exp - i(\bar{m} - i\bar{\Gamma}/2)t$, has been removed. Unlike the situation for kaons, one has

$$|\Delta\Gamma| << \Delta m \tag{2.2}$$

This is based on the empirical measurement of Δm and strict upper bounds on $\Delta\Gamma$ (more to come later). If one neglects for now $\Delta\Gamma$, then

$$[\Delta M] \equiv \frac{1}{2} \begin{pmatrix} 0 & \Delta M \\ \Delta M^* & 0 \end{pmatrix} = \frac{\Delta m}{2} \begin{pmatrix} 0 & e^{i\alpha} \\ e^{-i\alpha} & 0 \end{pmatrix} \tag{2.3}$$

and the problem is like spin-rotation in a uniform magnetic field. The solution is

$$\begin{aligned} |B(t)\rangle &= \cos \frac{\Delta m t}{2} |B\rangle - ie^{i\alpha} \sin \frac{\Delta m t}{2} |\bar{B}\rangle \\ |\bar{B}(t)\rangle &= \cos \frac{\Delta m t}{2} |\bar{B}\rangle - ie^{-i\alpha} \sin \frac{\Delta m t}{2} |B\rangle \end{aligned} \tag{2.4}$$

Experimentally for the B_d one has

$$\frac{\Delta m}{\Gamma} \sim 0.7 \tag{2.5}$$

The origin of this is believed to be the "box-diagram" (virtual annihilation into W-pairs) as shown in Fig. 2. Roughly

$$\Delta m_{Box} \sim m_t^2 V_{td}^2 \tag{2.6}$$

The "surprisingly large" experimental value favors a relatively large V_{td} and/or top quark mass. An immediate inference is also that

$$\frac{(\Delta m)_{B_s}}{(\Delta m)_{B_d}} \approx \left| \frac{V_{ts}}{V_{td}} \right|^2 \gg 1 \tag{2.7}$$

The favored value of the ratio is somewhere between 5 and 60. This makes "complete" mixing of the B_s very likely, i.e. several mixing oscillations per mean life.

The width difference $\Delta\Gamma/\Gamma$, while small, is important for the possible observation of CP-violation in double semileptonic decay of a B-$\overline{B}$ pair. One looks at the asymmetry

$$a = \frac{N(++) - N(--)}{N(++) + N(--)} = \text{Im}\,\frac{\Delta\Gamma}{\Delta M} \;, \tag{2.8}$$

described pictorially in Fig. 3, which can be readily shown to equal $\text{Im}\,\Delta\Gamma/\Delta m$, as indicated above. The width difference, according to the Golden Rule, is fed by real intermediate states which couple to <u>both</u> B_d and $\overline{B}_d$. Most intermediate states do not have this property, because at the quark level the decay products of B_d mainly consist of $\bar{c}d(u\bar{d} + c\bar{s})$ which is not identical to the principal $\overline{B}_d$ products $c\bar{d}(\bar{u}d + \bar{c}s)$. This alone implies $|\Delta\Gamma/\Gamma| \sim 1.4\,|\Delta\Gamma/\Delta m| \ll 1$. The channels which do feed $\Delta\Gamma$ are of three categories (cf. Fig. 4):

	Intermediate State	K-M Factor	Phase Space
I.	$c\bar{c}d\bar{d}$	$(V_{cb}V^*_{cd})^2$	limited
II.	$u\bar{u}d\bar{d}$	$(V_{ub}V^*_{ud})^2$	vast
III.	$\left\{ \begin{matrix} c\bar{u}d\bar{d} \\ u\bar{c}d\bar{d} \end{matrix} \right\}$	$(V_{cb}V^*_{cd})(V_{ub}V^*_{ud})$	intermediate

Now these three classes of contributions probably destructively interfere. The reason is that were the charm quark degenerate in mass with the up quark, there would be no CP-violation effect. (This is a general principle; the extra flavor symmetry allows an SU(2) rotation in c-u space which, with our phase conventions, allows V_{bu} to be chosen real). To see this directly, we note that in that limit the phase-space factors for categories I, II, and III become identical, and

$$\Delta\Gamma \; \alpha \; (V_{cb}V^*_{cd} + V_{ub}V^*_{ud})^2 = (- V_{tb}V^*_{td})^2 = \left| \frac{\Delta\Gamma}{\Delta M} \right| \Delta M \qquad (2.9)$$

Here unitarity of the mixing-matrix has been invoked, along with the observation that the phase of $(V_{tb}V^*_{td})^2$ is the same as the phase of ΔM as calculated from the "box diagram".

In the presence of unequal c and u masses, one can still use the unitarity constraint to eliminate category III in terms of categories I and II. Structurally one has

$$\begin{aligned} a = \mathrm{Im}\left(\frac{\Delta\Gamma}{\Delta M}\right) \alpha \; & (V_{cb}V^*_{cd})^2 \left\{ (\mathrm{Ph.Sp.})_{I} - (\mathrm{Ph.Sp.})_{III} \right\} \\ & + (V_{ub}V^*_{ud})^2 \left\{ (\mathrm{Ph.Sp.})_{II} - (\mathrm{Ph.Sp.})_{III} \right\} \end{aligned} \qquad (2.10)$$

At the quark level, the value for a is no larger than 10^{-3}. Altomari, Wolfenstein, and I[10] looked at this and argued that the order-of-magnitude cancellations present at the quark level[11] need not exist at the hadron level, so that the 10^{-3} estimate is imprudently low. But it is near impossible to work the value up to 1%. A reasonable statement is that the asymmetry is almost certainly below 10^{-2}, but the theoretical uncertainties are much too large to support a claimed upper bound of 10^{-3}.

III. The Decay $B_d \rightarrow \psi\pi^+\pi^-$

Many CP-violating effects in the B system exploit the interference between CP-violating mixing and CP-violating decays, such as decays into final states in categories I - III. But these final states are not "allowed"; their branching ratios are suppressed by K-M factors $|V_{bu}/V_{bc}|^2$, $|V_{cd}/V_{cs}|^2$, etc. An "allowed" CP-invariant final state which has a very favorable signature is $B_d \rightarrow \psi K_s$. It is much discussed[12] as a prototypical CP-violation measurement. Here we choose a variant from category I of suppressed decays, namely $B_d \rightarrow \psi\pi^+\pi^-$. It is actually the <u>same</u> final state as ψK_s; the only difference being that the dipion emerges from the B decay vertex rather than downstream. This should be an experimental advantage. And while one has paid a price of a factor 20 in $|V_{cd}/V_{cs}|^2$, much of this may be compensated. In particular

$$\Gamma\,(B_D \rightarrow \psi K_s \,;\; K_s \rightarrow \pi^+\pi^-) = \frac{1}{3}\,\Gamma(B_d \rightarrow \psi K^o) \tag{3.1}$$

We guess that vector final states are more prevalent than pseudoscalars:

$$\Gamma(B_d \rightarrow \psi K^o) \overset{?}{\sim} \frac{1}{3}\,\Gamma(B_d \rightarrow \psi K^{*o}) \tag{3.2}$$

Then using

$$\frac{\Gamma(B_d \to \psi\rho^o)}{\Gamma(B_d \to \psi K_o^*} \sim \frac{1}{2} \bullet \left| \frac{V_{cd}}{V_{cs}} \right|^2 \sim \frac{1}{40} \qquad (3.3)$$

where the factor 1/2 allows for $B_d \to \psi\omega^o$ as well as $B_d \to \psi\rho^o$, we find

$$\frac{\Gamma(B_d \to \psi\rho^o \to \psi\pi^+\pi^-)}{\Gamma(B_d \to \psi K_s \to \psi\pi^+\pi^-)} \sim \frac{3 \bullet 3}{40} \gtrsim 20\% \qquad (3.4)$$

There can also be expected to be s-wave dipions (not to mention contributions from spins ≥ 2) which can add more to the branching ratio. We furthermore expect that for ψK_s the secondary kaon vertex may be occasionally lost, and that the "V2" 2-prong ψ vertex is somewhat more difficult to resolve than the V4 vertex for $\psi\pi^+\pi^-$. So I suspect the overall detection efficiency for $\psi\pi^+\pi^-$ exceeds that of ψK_s, although this assertion needs checking.

But the main reason for discussing this mode here is to point out the richness present in the final-state phenomenology. CP-violation can be seen in angular distributions and Dalitz-plot distributions in a variety of ways.[13] To examine this, we assume for simplicity we have a sample of tagged B_d, i.e. the neutral B (which of course mixes) was produced in association with a $\overline{B}_u$ or Λ_b. Then <u>for a given CP eigenstate F</u> it is easy to work out the time distribution for the decay:

$$d\Gamma_F(t) = d\Gamma_F(0)\ e^{-\Gamma t} \left[1 + (\mathrm{Im}\ \lambda_F) \sin \Delta mt \right] \qquad (3.5)$$

Here

$$\mathrm{Im}\ \lambda_F = e^{2i\alpha} \frac{T(B_d \to F)}{\overline{T}(\overline{B}_d \to F)} \qquad (3.6)$$

Again α is the phase of the mass-difference ΔM. The ratio of the decay amplitudes T depend only upon the mixing-matrix elements and is of modulus unity; therefore this is also true for λ_F:

$$|\lambda_F| = 1 \tag{3.7}$$

Note that while neither the ratio of T's nor the phase of $[\Delta M]$ is rephase-invariant, the observable λ_F is; indeed

$$\lambda_F = \frac{(V_{td}V_{tb}^*)(V_{cd}^*V_{cb})}{(\text{complex-conjugate})} \bullet (CP)_F \tag{3.8}$$

The factor $(CP)_F = \pm 1$ accounts for the CP eigenvalue of the state F in question. In terms of the triangle, Fig. 1, the phase of λ is twice the phase of V_{td}, i.e. twice the interior angle of the vertex on the right. If that angle is $\gtrsim 15^o$, $|\mathrm{Im}\ \lambda_F|$ is $\gtrsim 0.5$!

For dipion masses $\lesssim$ 1 GeV we expect the final state is a combination of only s and p waves. If s-wave dominates, then the CP eigenvalue of the final state is fixed and one is assured of a big integral asymmetry. If this is not the case, then Dalitz-plot and angular-distribution asymmetries will show the CP-violation. The structure of the decay amplitudes is

$$\begin{aligned} T(B_d) &= [s(\hat{\psi}\bullet\hat{\rho}) + ip(\hat{\psi}x\hat{\rho})\bullet\hat{k} + iP(\hat{\psi}\bullet\hat{k})] \bullet V_{cb}V_{cd}^* \\ \bar{T}(\bar{B}_d) &= [s(\hat{\psi}\bullet\hat{\rho}) - ip(\hat{\psi}x\hat{\rho})\bullet\hat{k} - iP(\hat{\psi}\bullet\hat{k})] \bullet V_{cb}^*V_{cd} \end{aligned} \tag{3.9}$$

Here the amplitudes s and p have the final-state phase-shift of p-wave π-π scattering, dominated by the ρ resonance. Evidently for the s term, the orbital angular momentum of the ψ-ρ system is zero, while for the p term it is unity. The third amplitude P describes a J=0, I=0 dipion which must be in a p-wave relative to the ψ. The phase of P is that of I=0 s-wave π-π scattering. This, as well as the previous assertion regarding the phases of the s and p amplitudes, is true under the very reasonable assumption that there is negligible final-state interaction between the ψ and the dipion.

Note that the CP eigenvalue of the s-wave $\psi\rho$ configuration is opposite to that of the two p-wave $\psi\rho$ and $\psi\sigma$ terms. This provides an extra structure in the time distribution for the decay of $B_d(t)$ and $\overline{B}_d(t)$. We have, with a suitable normalization

$$\frac{dN}{dt}\left(\overline{B}_d \rightarrow \psi\pi^+\pi^-\right) = \Gamma_{4\pi^+\pi^-} e^{-\Gamma t} \left| s\hat{\psi}\cdot\hat{\rho}\left(\cos\frac{\Delta mt}{2} + i\lambda \sin\frac{\Delta mt}{2}\right) + i\left(p\hat{\psi}x\hat{\rho}\cdot\hat{k} + P\hat{\psi}\cdot\hat{k}\right)\left(\cos\frac{\Delta mt}{2} - i\lambda \sin\frac{\Delta mt}{2}\right)\right|^2 \tag{3.10}$$

To obtain the corresponding distribution for $\overline{B}_d$, one simply replaces λ by λ^* and p,P by -p,-P:

$$\frac{dN}{dt}\left(\overline{B}_d \rightarrow \psi\pi^+\pi^-\right) = \Gamma_{4\pi^+\pi^-} e^{-\Gamma t} \left| s\hat{\psi}\cdot\hat{\rho}\left(\cos\frac{\Delta mt}{2} + i\lambda^* \sin\frac{\Delta mt}{2}\right) - i\left(p\hat{\psi}x\hat{\rho}\cdot\hat{k} + P\hat{\psi}\cdot\hat{k}\right)\left(\cos\frac{\Delta mt}{2} - i\lambda^* \sin\frac{\Delta mt}{2}\right)\right|^2 \tag{3.11}$$

We shall not explicitly expand this out for the general case. However there is very interesting information in these angular distributions, in particular for the ρ decay. For example, suppose for simplicity that $P \approx 0$, $s \approx ip$ and $\lambda = 1$, signifying maximal CP violation. Now introduce the angle ϕ between the decay planes of ψ and ρ:

$$\cos\phi = \hat{\psi} \cdot \hat{\rho}$$

$$\sin\phi = \hat{\psi} \times \hat{\rho} \cdot \hat{k} \tag{3.12}$$

Then

$$\frac{dN}{d\phi} \propto \sin^2\left(\phi - \frac{\Delta mt}{2} + \frac{\pi}{4}\right) \tag{3.13}$$

On the other hand,

$$\frac{d\bar{N}}{d\phi} \propto \sin^2\left(\phi + \frac{\Delta mt}{2} + \frac{\pi}{4}\right) \tag{3.14}$$

We see that, as a function of proper time, the decay plane of ψ rotates relative to that of ρ. The rate of rotation is $\Delta m/2\Gamma$ radians per mean lifetime. The direction of rotation (right-handed versus left) is opposite for $B_d(t)$ and $\bar{B}_d(t)$. One can hardly ask for a more graphic demonstration of CP violation.

The general case looks rather complicated. However the number of parameters which characterize all distributions is not large; probably two numbers $|s/P|$ and $|p/P|$ are all that are needed. This follows because the dependence of the functions s, p, and P on dipion mass are calculable in a resonance approximation; hence only the normalizations may be required as input. In addition there is the parameter arg λ, which is the goal of the measurement. Thus a global fit for all data should in principle be a practical procedure.

Will any of this actually be possible to do experimentally? To estimate the branching ratio for $B_d \to \psi\pi^+\pi^-$, we start with the observed branching ratio

$$\frac{\Gamma\left(B_d \to \psi X\right)}{\Gamma_{total}} = 1.1\% \tag{3.15}$$

We therefore expect

$$\frac{\Gamma\left(B_d \rightarrow \psi X_{\text{nonstrange}}\right)}{\Gamma_{\text{tot}}} \approx \theta_c^2 \times 1\% \sim 5 \times 10^{-4} \tag{3.16}$$

What remains to be estimated is the fraction of non-strange final states that end up as charged dipions. We guess, consistent with the general ideas of duality, that

$$\frac{d\Gamma}{dM^2} \approx \left(\frac{d\Gamma}{dM^2}\right)_{\text{Parton level}} \tag{3.17}$$

where M^2 is the squared mass of the hadron system $X_{\text{nonstrange}}$ recoiling against the ψ. At the parton level this is a $\bar{d}$ quark produced in the weak decay $\bar{b} \rightarrow \bar{c}c\bar{d}$ together with the spectator d in the initial B_d meson. The squared mass of the $d\bar{d}$ system is

$$M^2 \cong 2p_d \cdot p_{\bar{d}} \sim 2E_{\bar{d}} \langle E_d \rangle \left(1 - \cos \theta_{d\bar{d}}\right) \tag{3.18}$$

The energy of the $\bar{d}$ emitted in the weak decay is

$$E_{\bar{d}} \sim \left(\frac{M_B^2 - M_\psi^2}{2M_B}\right) \approx 1.5 \text{ GeV} \tag{3.19}$$

The energy (better, momentum) of the spectator d is characteristic of the bound-state wave function. We take it to be ~ 0.5 GeV. Because the orientation of the d quark momentum is evidently distributed isotropically, this leads, <u>at the parton level</u>, to a uniform distribution in M^2 from a minimum of zero to a maximum of $4E_{\bar{d}}\langle E_d \rangle \sim 3$ GeV^2. In real life the low-mass region, $M^2 < 1 \text{ GeV}^2$, will be dominated by resonance contributions, probably π, σ, ρ, and A1, as shown in Fig. 5. Duality would imply one third of the total distribution is concentrated in the mass region below 1 GeV. Taking $\pi/\rho/\sigma$ = 1/2/3 would put ~ 70% of this low mass contribution into two charged pions. The net $\psi\pi^+\pi^-$ branching ratio would then be

$$\frac{\Gamma\left(B_d \to \psi\pi^+\pi^-\right)}{\Gamma_{tot}} \approx \left(1.1\%\right) \times \frac{1}{20} \times \frac{1}{3} \times 0.7 \sim 1.2 \times 10^{-4} \qquad (3.20)$$

To this must be included the dilepton branching ratio of 7% for $\mu^+\mu^-$ or e^+e^-. Even presuming both dilepton modes can be found, this implies a net branching ratio of 1.7×10^{-5}. Assuming 1000 B_d or $\overline{B}_d$ are needed to measure well the distributions, this requires 500 x $(1.7)^{-1} \times 10^5 \sim 3 \times 10^7$ B_d produced in the course of the experiment, or roughly 10^8 $b\overline{b}$ quark pairs. But this does not include many experimental inefficiencies. For hadron-hadron collisions, these include, but are not limited to

Isolating the decay vertex (factor 2?)
Geometrical acceptance (factor 5?)
Detection efficiency (factor 3?)
Finding and tagging the spectator B (which ideally should be a B_u or Λ_b) (factor $\gtrsim$ 3?)

This leads to an extra factor in excess of 10^3, or a number of $b\overline{b}$ quarks produced into the apparatus in excess of 10^{11}. Amongst sources of hadron collisions, the SSC and only the SSC holds out promise of providing such a large number of produced b-quarks.

If one uses e^+e^- colliders, in principle the loss factors may be greatly reduced. But even if they are a factor ~ 10 instead of 10^3, this implies 10^9 $b\overline{b}$ produced to get the sample of 1000 $\psi\pi^+\pi^-$ decays.

IV. $\Lambda_b \rightarrow p\pi^-$ and $\Lambda_b \rightarrow pK^-$ as Examples of "Penguin" Processes

As we have seen, there are several distinct mechanisms for b-decays to occur, characterized by different effective weak hamiltonians. For example, we may write, for decays with initial and final states of zero net strangeness

$$H_{wk} = \xi_u H_u + \xi_c H_c + \xi_t H_t + \text{h.c.}$$

with (i = u, c, t)

$$\xi_i = V_{ib} V^*_{id}$$

and

$$H_i = \bar{b}\gamma_\mu (1-\gamma_5) q_i \; \bar{q}_i \; \gamma^\mu (1-\gamma_5) d$$

Note that the H_i are CP-invariant operators; CP violation is due to the phases of the mixing amplitudes ξ_i.

Normally, it is a good approximation to only consider one of these three terms in calculating a given decay process. However, there are circumstances when, due to strong final-state interactions, more than one contributes significantly. Because the coefficients ξ_i have distinct complex phases this can lead to CP violation, provided the (CP-invariant) decay amplitudes T_i which multiply the ξ_i are out of phase. The observable CP-violating effect is a particle-antiparticle branching ratio asymmetry, and its generic structure is

$$\frac{\Delta\Gamma}{\Gamma} \equiv \frac{\Gamma\left(B \rightarrow f\right)}{\Gamma_{tot}} - \frac{\Gamma\left(\bar{B} \rightarrow \bar{f}\right)}{\Gamma_{tot}} = \frac{2\left(\text{Im } \xi_i \xi_j^*\right) \left(\text{Im } T_i T_j^*\right)}{\left|\xi_i T_i + \xi_j T_j\right|^2} \tag{4.1}$$

Here i and j label the two contributing terms in H_{wk}.

Let us now specialize to decays containing no charm in the final state. We will be interested in particular in the decay $\Lambda_b \to p\pi^-$, although what we do is applicable to $B_u \to \rho^o\pi^-$, $B_d \to \pi^+\pi^-$, as well as quite a few other processes. Normally one considers only H_u, the source of the "tree amplitude". in estimating these processes. The two remaining contributors may be expected to be dealt with perturbatively, since the strong-interaction contribution includes $c\bar{c}$ or $t\bar{t}$ annihilation. We may write, for the strong corrections

$$\Delta T = \xi_t \sum_{\substack{\text{n containing} \\ \text{top}}} \frac{\langle f|T|n\rangle\langle n|H_t|\Lambda_b\rangle}{E_n - M} + \xi_c \sum_{\substack{\text{u containing} \\ \text{charm}}} \frac{\langle f|T|n\rangle\langle n|H_c|\Lambda_b\rangle}{E_n - M + i\epsilon}$$

$$+ \xi_u \sum_n \frac{\langle f|T|n\rangle\langle u|H_u|\Lambda_b\rangle}{E_n - M + i\epsilon} \tag{4.2}$$

where use of the perturbation theory in the last term is more dubious. But, as we shall immediately see, its short-distance contribution is crucial.

At the quark level the relevant diagrams are shown in Fig. 6. The loop, after Fierz rearrangement, still has the structure of a good old vacuum polarization loop a la QED. Aside from factors of two it will, in momentum space, take the form:

$$\sum_i \xi_i \frac{\alpha_s}{3\pi} \cdot \frac{1}{q^2} \cdot q^2 \ln \frac{\Lambda^2}{\left(4M_i^2 - q^2\right)} \tag{4.3}$$

There are several important features to this contribution, (in the jargon, the "electric penguin"):

1) The logarithmic divergence is cancelled via unitarity of the mixing matrix:

$$(\xi_u + \xi_c + \xi_t)\log \Lambda^2 = 0$$

2) This implies the top contribution can be eliminated, provided the sum-over-states $\sum_n$ in the remaining terms is cut off at a mass-scale of order the top-quark mass.

3) The short-distance contribution of the remaining charmless part is not too large; it provides a small correction to the leading tree diagram contribution.

4) The charm contribution is complex because of the possibility of real $c\bar{c}$ intermediate states in the decay. Nevertheless it is arguably[14] still a short-distance contribution, being a coarse-grained superposition of Feynman propagators with mass parameter $\gtrsim$ 3-4 GeV.

Thus the net result is that the effective short-distance interaction can be considered to be dominated by the (cut-off) charm contribution. It has the structure

$$"H"_{\text{penguin}} = \frac{G_F}{\sqrt{2}} \left(\ln \frac{M_t^2}{M_c^2} + i\pi \right) \left\{ \frac{\alpha_s}{3\pi} \xi_c \left(\bar{b}\, \frac{\lambda_i}{2} \gamma_\mu \left(1-\gamma_5\right) d \right) \left(\bar{u}\, \frac{\lambda_i}{2} \gamma^\mu u \right) + \text{h.c.} \right\} \tag{4.4}$$

Here the λ_i are QCD color matrices. Note that because of the final-state interaction phase (which does not undergo complex conjugation when external particles are replaced by antiparticles) $"H"_{\text{penguin}}$ is not hermitian. With 40 GeV < M_t < 200 GeV, this phase-factor $\pi/(\ln m_t^2/m_c^2)$ is ~ $\pi/(8.2 \pm 1.6)$, or about 15°.

Now what about the phase of the presumably dominant "tree" contribution? We might suspect it to also suffer final-state interaction phase-shifts beyond the arguably small "electric-penguin" contribution already discussed. However, we shall argue that to good approximation this amplitude is real, essentially as free of final-state interaction phases as semileptonic decays. This argument[15] is basically a defense, for this process, of the "factorization" hypothesis widely used[16] in calculations of nonleptonic B-decays and, less justifiably, of charm decays. The argument is based on the space-time evolution of the decay products. At the quark level the decay $b \rightarrow u + \pi^-$ begins as a nearly collinear configuration of $b \rightarrow u + (\bar{u}d)^-$. The color singlet $\bar{u}d$ pair recoils in the direction opposite to the u. In order that it has good overlap with the final-state pion it has low virtual mass relative to its momentum, of order 2.5 GeV. It follows that the formation time of the pion will be long because of the relativistic time-dilation. By the time the pion is formed it is several fermis away from the color fields existing in the neighborhood of the original B_d or Λ_b. And during the time the $(\bar{u}d)$ system is within those color fields, it is a small color dipole, originating from the pointlike, color-singlet weak interaction, and growing only slowly because of the long formation time. It is therefore arguable that this small dipole will not significantly interact with the spectator system. There is a calculation possible here to check this, but to my knowledge it has not been done.

In the absense of such final-state interactions, the decay amplitude becomes a product of two factors, one of which is the usual pion decay amplitude, and the other one the same (real) form factor appearing in semileptonic decays. What is implied, therefore, is a direct relationship between the nonleptonic pionic decay and semileptonic decays. An easy calculation gives[15]

$$\frac{\Gamma\left(\Lambda_b \rightarrow p\pi^-\right)}{\frac{d\Gamma}{dM^2}\left(\Lambda_b \rightarrow pe^-\nu_e\right)_{M^2=0}} = 6\pi^2 F_\pi^2 \cong 1.0 \text{ GeV}^2 \qquad (4.5)$$

This is indeed a general test of factorization; the p can be replaced by any low-mass hadron system, and the Λ_b by any b-meson B. It is quite important to test sharply this factorization hypothesis wherever possible. There may already be a way to do this in decays such as $B \to D\pi$ and $B \to D^*\pi$.

Returning to the $\Lambda_b \to p\pi^-$ decay, we may estimate the branching ratio as before. We take

$$\frac{d\Gamma}{dm^2_{e\nu}}\left(B \to pe\nu\right)\Bigg|_{m^2=0} \approx 0.2\,\frac{d\Gamma}{dm^2_{e\nu}}\left(B \to Xe\nu\right)\Bigg|_{m^2=0} \qquad (4.6)$$

Here X implies a sum over all uncharmed (nonstrange) baryonic final states. We have not studied the dilepton mass distribution but guess an average $(\text{mass})^2$ of $\sim 5\ \text{GeV}^2$. Therefore in accordance with Eqn. 4.5, we take

$$\frac{d\Gamma}{dm^2}\left(\Lambda_b \to Xe\nu\right)_{m^2=0} \overset{?}{\approx} \left(0.2\ \text{GeV}^{-2}\right)\,\Gamma\left(\Lambda_b \to Xe\nu\right) \qquad (4.7)$$

Evidently

$$\frac{\Gamma\left(\Lambda_b \to Xe\nu\right)}{\Gamma\left(\Lambda_b \to e\nu\ \text{all}\right)} = \left(\frac{V_{ub}}{V_{cb}}\right)^2 \times \left(\text{Phase Space}\right) \sim 2.5\left(\frac{V_{ub}}{V_{cb}}\right)^2 \qquad (4.8)$$

With a 15% semileptonic ($e\nu$) branching ratio, we can put all these numbers together to get

$$\Gamma\left(\Lambda_b \to p\pi^-\right) = \left(1.0\ \text{GeV}\right)^2 \cdot \left(0.2\right) \cdot \left(0.2\ \text{GeV}^{-2}\right) \cdot 2.5\left|\frac{V_{ub}}{V_{cb}}\right|^2 \cdot \left(0.15\right)$$
$$\cong \left(1.5 \times 10^{-3}\right)\left|\frac{V_{ub}}{V_{cb}}\right|^2 \qquad (4.9)$$

With $|V_{ub}/V_{cb}|^2 \sim (1\text{-}2) \times 10^{-2}$, this implies, very roughly, a branching ratio of $(1.5\text{-}3) \times 10^{-5}$. This is actually surprisingly large for a "first-forbidden" weak transition into a two-body final state. This fact provides additional support for the factorization hypothesis. It is hard to see how complicated strong-interaction mechanisms can lead to an effective "squared form factor" of 10^{-3} (after taking out the $|V_{ub}/V_{cb}|^2$ factor) at a parent mass scale of 5 GeV (Compare $e^+e^- \to p\bar{p}$, or $\psi \to p\bar{p}$, etc.). Hence it is credible that the factorized contribution in fact dominates.

Less certain is the relative magnitude of the "penguin" and "tree" contributions. We here do not try to estimate the ratio. However, for a large range of magnitudes of penguin amplitudes, the issue can be finessed when <u>comparing</u> the $\Lambda_b \to p\pi^-$ and $\Lambda_b \to pK^-$ processes. For the latter process, the tree amplitude is suppressed by a factor of Cabibbo-angle $\theta_c \sim 0.23$, while the Penguin contribution (Fig. 6) is <u>enhanced</u> by a factor θ_c^{-1}. Thus it is very likely that the process Λ_b pK^- is dominated by the Penguin contribution. We write

$$\Gamma\left(\Lambda_b \to p\pi^-\right) = \Gamma_o \left| 1 + \theta_c \left(\frac{V_{cb}}{V_{ub}}\right)\left(\frac{P}{T}\right)\right|^2$$

$$\approx \Gamma_o \left\{ 1 + 2\eta\theta_c \left|\frac{V_{cb}}{V_{ub}}\right| \left|\frac{P}{T}\right| \right\} \tag{4.10}$$

Consequently, we estimate

$$\Gamma\left(\Lambda_b \to pK^-\right) \cong \Gamma_o \left| \theta_c \left(\frac{F_K}{F_\pi}\right) + \left(\frac{V_{cb}}{V_{ub}}\right)\left(\frac{P}{T}\right) \right|^2$$

$$\cong \left|\frac{V_{cb}}{V_{ub}}\right|^2 \left|\frac{P}{T}\right|^2 \Gamma_o \left\{ 1 + 2\eta \left(\frac{F_K}{F_\pi}\right) \theta_c \cdot \left|\frac{V_{ub}}{V_{cb}}\right| \cdot \left|\frac{T}{P}\right| \right\} \tag{4.11}$$

In both equations the parameter η is the ratio of CP-violating strength to the maximum strength possible:

$$\eta = \frac{\mathrm{Im}\ V_{ub}}{|V_{ub}|} \cdot \frac{\mathrm{Im}\ P}{|P|} \overset{?}{\sim} \pm 0.2 \tag{4.12}$$

It changes sign upon replacing the particles by antiparticles.

In Table I we tabulate the decay asymmetries and decay widths for a variety of assumed Penguin-amplitude strengths (namely $|P/T| = 0.3$, 0.1, and 0.03) and values for V_{ub} (we take $V_{ub}/V_{cb} = .14$ and .07). We see that the asymmetry tends to be large either for the $p\pi^-$ mode, pK^- mode, or both. This follows because, from Eqns. (4.10) and (4.11),

$$\sqrt{a_{p\pi} \cdot a_{pK}} = 2\eta \sqrt{\frac{F_K}{F_\pi}} \cdot \theta_c \approx 0.1 \tag{4.13}$$

The mean asymmetry does not depend on the most uncertain elements (other than the question of strong-interaction phases entering η).

TABLE I

$\lvert V_{ub}/V_{cb}\rvert$	$\lvert P/T\rvert$	$\Gamma(pK^-/\Gamma(p\pi^-)$	$a(p\pi^-)$	$a(pK^-)$
.07	0.3	17	8%	3%
.07	0.1	2	13%	9%
.07	0.03	.17	4%	22%
.14	0.3	4	18%	5%
.14	0.1	0.5	6%	15%
.14	0.03	0.04	2%	20%

V. Concluding Remarks

The specific examples of $B \to \psi\pi\pi$ and $\Lambda_b \to p\pi$, pK are only special cases of a broad class of similar decay modes which can exhibit CP violation and other interesting effects in similar ways. One can find these enumerations in many places.[17] But it may well end up that the most interesting case experimentally is not on the theorists' long lists or at least not one which has received adequate scrutiny.

My own emphasis has been to scrutinize low-multiplicity, all-charged decay modes of b-hadrons, where the multiplicity count includes the cascade decays of intermediate charm or strange systems. Restriction to total $M_{ch} \lesssim 4$ still leaves ~ 60 channels, essentially all of which are in principle quite interesting for one reason or another.[18] The decay branching ratios are in some cases computable at the level of accuracy discussed for $B_d \to \psi\pi\pi$ and $\Lambda_b \to p\pi$, pK. Others (e.g. $p\bar{p}$, $\overline{pp}\pi$, etc.) are more uncertain. All can benefit from more theoretical effort expended upon them.

I find the duality techniques described above more trustworthy than direct calculation using bound state wave-functions, but that is a matter of taste. I would prefer most of all results based upon current-algebra or QCD sum rules; here the prototypical analogue is the set of estimates for strange-particle decays, e.g. $K \rightarrow \pi l \nu$, using the Ademollo-Gatto-theorem approach.[19] To my knowledge not much has been done in this direction.

As for experimental prospects of reaching the sensitivities required, it is clear that there is still a long way to go. To get to the CP-violation level of sensitivity requires at least $10^{8\pm1}$ $b\bar{b}$ pairs produced into the detection apparatus per experiment.[20] Once that level is reached, many experimental opportunities open up. This implies to me that if this is possible at all, the study of CP-violation in the B-system becomes a program, not only a discovery. This will be important because, as we already mentioned, it is overdetermination of the K-M parameters that both tests the validity of the picture and provides some estimate of the reliability of the measurements and theoretical calculations.

What facilities might provide this? Present-generation electron-positron colliders should provide between 10^6 and 10^7 $b\bar{b}$/experiment. New ideas promise another order of magnitude. The best cms energy choices appear to be either the $\Upsilon(4S)$ or the Z^o. Hadron-hadron colliders can yield from 10^9 $b\bar{b}$ (Tevatron) to over 10^{11} (SSC) per experiment, but background, rate, and triggering problems make the overall achievable detection efficiency very uncertain at present. And in every minute of fixed-target operation at Fermilab, about 10^7 $b\bar{b}$ pairs are made. While, alas, they are produced in the beam dumps, there is hope that very high-luminosity experiments looking at the simplest decay modes (eg two-body decays[21] or decays with a ψ in the final state[22]) will be possible.

No matter what the approach is, these experiments are very difficult. I guess 20±10 years as the relevant time scale to reach the CP level of sensitivity. But this physics will not go out of fashion. Even given spectacular successes with physics up to the TeV mass scale via SLC/LEP, Tevatron, and SSC, I cannot find a scenario which would diminish the importance of this b-physics. It deserves a *great deal of effort, not only by experimentalists and accelerator* builders, but also by theorists.

Acknowledgments

Much of this material has been worked out because of stimulation and interaction with many Fermilab experimental colleagues. I also would like to especially acknowledge I. Dunietz and J. Rosner and my collaborators T. Altomari and L. Wolfenstein. Thanks also go to E. Floratos, P. Dittus, and A. Verganelakis for their excellent organization of the Crete Workshop.

FIGURES

1. Unitarity triangles, with allowed regions for the vertex for various assumed values of top quark mass m_t. The base is normalized to sin θ_c (We thank H. Harari for this suggestion), and the bounds are transcribed from Ref. 6).

2. "Box-diagram" believed to be responsible for $B_d - \overline{B}_d$ mixing.

3. Mechanism for same-sign dilepton asymmetry.

4. A redrawing of the "box-diagram", Fig. 2, illustrating the presence of "real" intermediate states (shown here only at the quark level).

5. Sketch of the expected mass distribution of hadrons X produced in the decay of $B \rightarrow \psi X$.

6. Tree (a) and penguin (b) contributions to the decays $\Lambda_b \rightarrow p\pi^-$, pK^-. The subprocess enclosed by the dashed box is, to good approximation, local.

REFERENCES

1. Prospects for future experiments can be found in the Proceedings of the Workshop on High Sensitivity Beauty Physics at Fermilab, Nov. 1987, ed. A. J. Slaughter, N. Lockyer, and M. Schmidt (Fermilab, 1988), and references therein.

2. There are many reviews; see for example I. Bigi, V. Khose, N. Uraltsev, and A. Sanda, SLAC-PUB-4476; also J. Rosner, preprint EFI-88/37.

3. An extensive discussion is given by C. Jarlskog, Phys. Rev. Lett. 55, 1039(1985).

4. J.D. Bjorken and I. Dunietz, Phys. Rev. D36,7(1987), and references therein.

5. L. L. Chau and W. Y. Keung, Phys. Rev. Lett. 53, 1802(1984); C. Hamzaoui, J. Rosner, and A. Sanda; also reference 7.

6. The input data used in Fig. 1 comes from the analysis of H. Harari and Y. Nir, Phys. Lett. B195, 586(1987); Y. Nir, SLAC preprint SLAC-PUB-4368(1987).

7. C. Jarlskog, ITP preprint 8/87 (Talk presented at the 4th LEAR Workshop, Villars-sur-Ollons, Switzerland,(Sept. 1987).

8. For a discussion of this gaugeless limit, see J. Bjorken, Proceedings of the 7th Moriond Workshop, "New and Exotic Phenomena", ed., O. Fackler and J. Tran Thanh Van, Editions Frontieres (Gif-sur-Yvette, France, 1987), p. 1.

9. A useful discourse on this is provided by H. Lipkin, preprint ANL-HEP-PR- ?/88; to be written.

10. T. Altomari, L. Wolfenstein, and J. Bjorken, Phys. Rev. D37, 1860(1988).

11. J. Hagelin, Nucl. Phys. B193, 123(1981).

12. See for example B. Cox and D. Wagoner, <u>Physics of the</u>

Superconducting Supercollider, ed. R. Donaldson and J. Marx (Snowmass, Colo, 1986), p. 83.

13. The general idea goes back to S. Barshay and J. Geris, Phys. Letts. 84B, 319(1979).

14. The line of argument requires a coarse-grained energy average similar to what is used in comparing $R(e^+e^-)$ to perturbative QCD calculations; see e.g. H. Quinn and S. Weinberg, Phys. Rev. D13, 1958(1986); also R. Shankar, Phys. Rev. D15, 755(1977) and R. M. Barnett, M. Dine, and L. McLerran, Phys. Rev. D22, 594(1980).

15. I am told this is in the folklore but do not know a good reference.

16. M. Wirbel, B. Stech and M. Bauer, Zeit. Phys. C29, 637(1985); M. Bauer, B. Stech, and M. Wirbel, Zeit. Phys. C34, 103(1987).

17. For example, ref. 16.; I. Dunietz and J. Rosner, Phys. Rev. D34, 1404(1986).

18. My own compilation is unpublished, but available upon request.

19. M. Ademollo and R. Gatto, Phys. Rev. Lett 13, 264(1964).

20. There are many such estimates extant, none very reliable. A sampling can be found in K. Foley et. al, Proceedings of the Workshop on Experiments, Detectors, and Experimental Areas for the Supercollider; Berkeley, Ca., ed., R. Donaldson and M. Gilchriese, (World Scientific, 1987), p. 759; also the talks of N. Lockyer and D. Macfarlane in ref. 1.

21. D. Kaplan et. al., Fermilab proposal P-789; see the discussions by P. Garbincius and by D. Kaplan et. al. in ref. 1.

22. B. Cox et. al., Fermilab experiment E-771; see the discussions by P. Garbincius, by D. Wagoner, and by W. Selove in Ref. 1.

Fig. 1

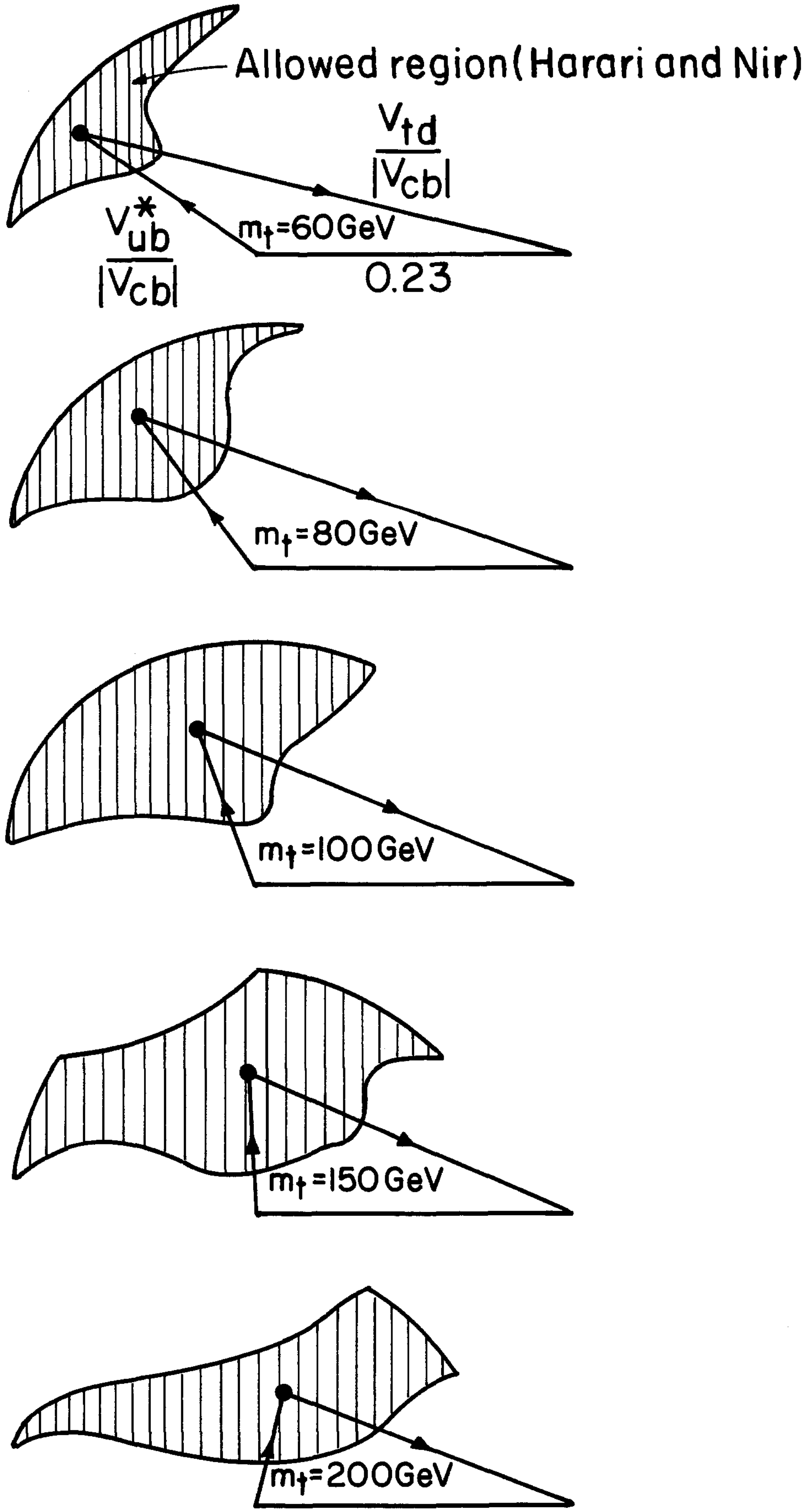
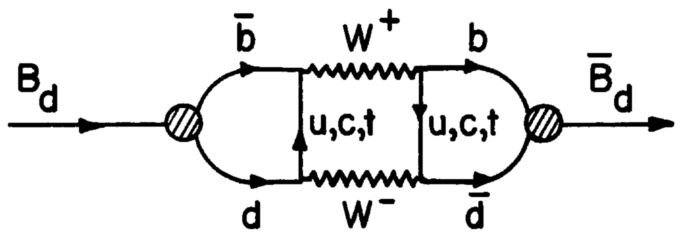
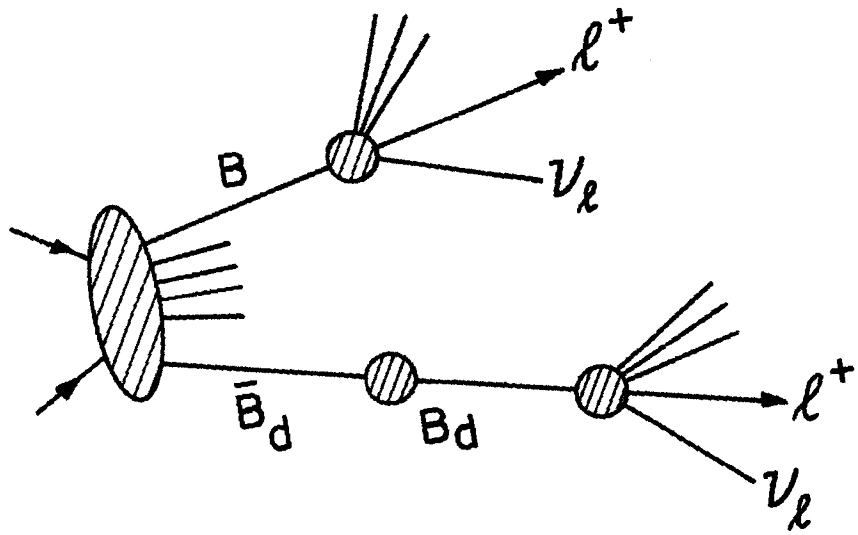

Allowed region(Harari and Nir)
$\frac{V_{td}}{|V_{cb}|}$
$\frac{V^*_{ub}}{|V_{cb}|}$
m_t=60GeV
0.23
m_t=80GeV
m_t=100GeV
m_t=150GeV
m_t=200GeV

Fig. 2





ℓ^+
B
ν_ℓ
$\bar{B}_d$
B_d
ℓ^+
ν_ℓ

Fig. 4

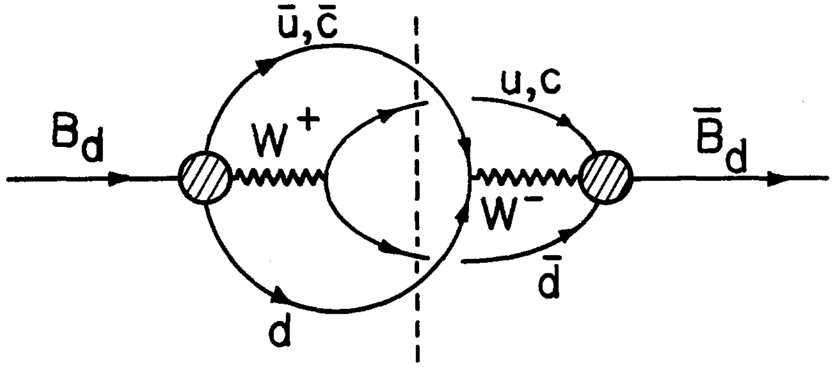
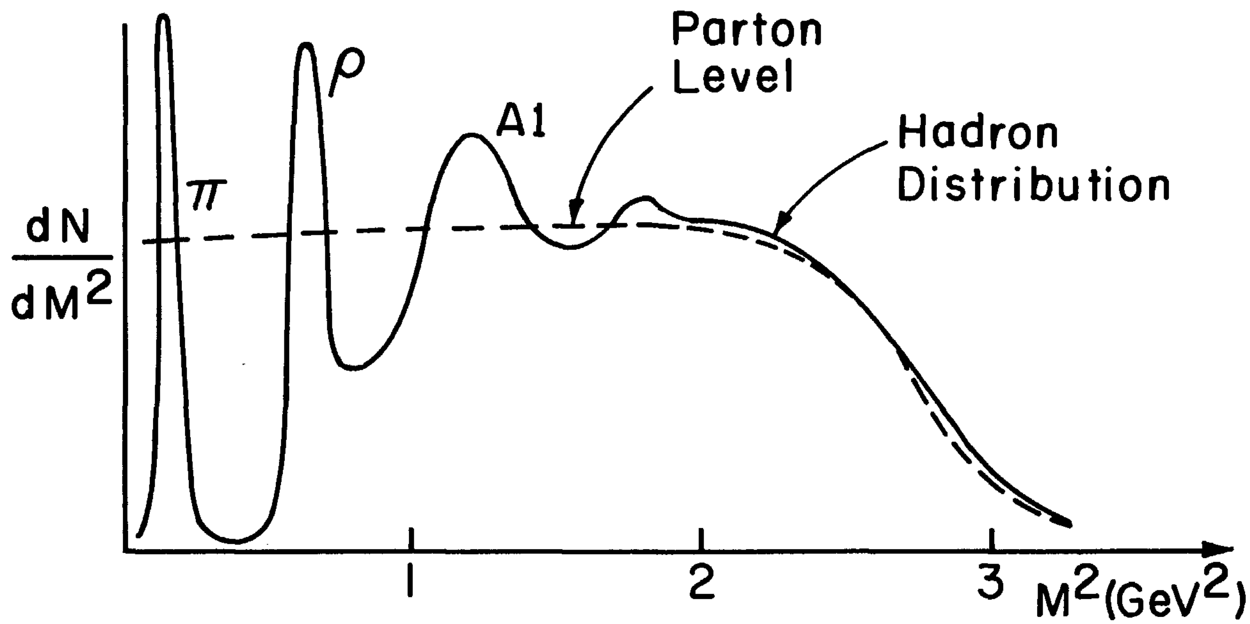
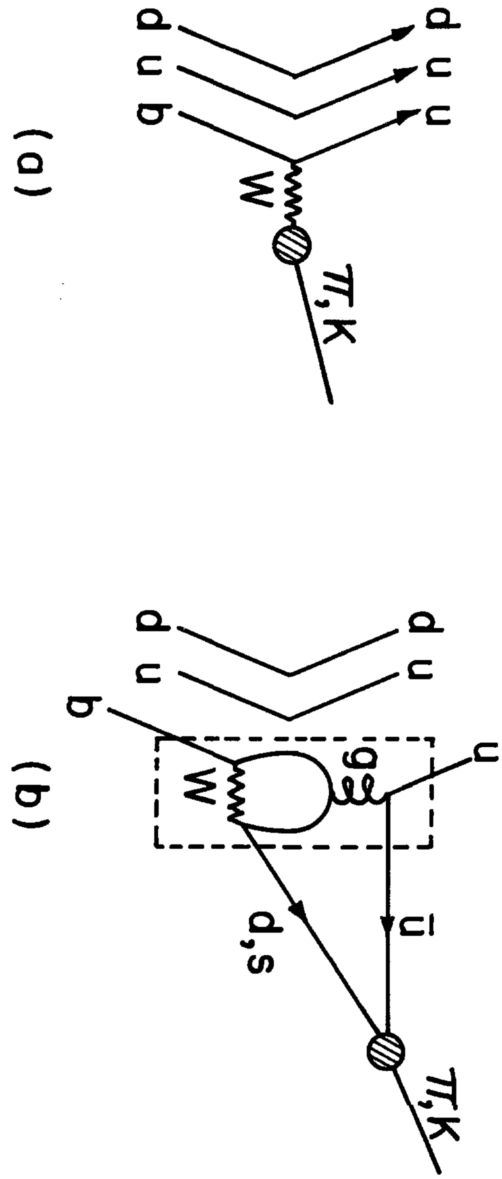


Fig. 5


Parton
Level
Hadron
Distribution
A1
π
ρ
$\frac{dN}{dM^2}$
1
2
3
$M^2(GeV^2)$


(a)
q
u
d
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W
π,K
(b)
q
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d,s
π,K