IC/88/313

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ABSTRACT

The resistively and capacitive shunted junction model is used to investigate hysteresis development in superconducting Josephson junctions. Two empirical formulas that relate the hysteresis width and the quasi-particle diffusion length in terms of the junctions electrical parameters, temperature and frequency are obtained. The obtained formulas provide a simple tool to investigate the full potentials of the hysteresis phenomena.

MIRAMARE - TRIESTE
September 1988

* To be submitted for publication.

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1. INTRODUCTION

Superconducting Josephson junctions are of great interest because of their potential use in the field of superconducting electronics. Knowledge of the current-voltage characteristics of Josephson junctions is thus important for the development of such a field. One of the main current-voltage phenomenon that has gained wide interest (experimentally and theoretically) is the hysteresis phenomena [1-6]. In this paper we study the development of hysteresis in Josephson junctions in terms of the junctions electrical parameters, temperature, and the applied (driving) ac current. To accomplish this we used the resistively and capacitive shunted junction (RSJ) model in which the driving ac current is adjusted to simulate a real life hysteresis experiment. Our results show that when the junctions damping parameter $G > 0.18$ hysteresis does not develop for any temperature or applied ac frequency. On the other hand, for each $G < 0.18$ hysteresis will start to develop once the applied ac frequency approaches a unique critical frequency. The hysteresis width then continues to increase on increasing the frequency. On approaching the Josephson plasma freqeuncy ω_j, the hysteresis width approaches a unique saturation limit. We have derived a formula for the ratio $I_c(T)/I_d$ (I_d is the critical current at which the junction switches back from the resistive state to the superconducting state) as a function of the junctions resistance R, capacitance C, damping parameter G, applied frequency and $I_c(T)/I_{ds}$, where I_{ds} is the saturation value of I_d at high frequencies. The formula is valid for $G \nless 1$, and the entire operable frequency range. We have also been able to relate the quasiparticle diffusion length Λ given in Ref.4 to our formula. Finally, we would like to mention that although the RSJ model gave excellent predictions that agree with the experimental results, it fails to account for hysteresis development for applied frequencies equal to or greater than ω_j (see below). The structure of our paper is as follows. Following the introduction we present the RSJ model and the computer results. The results are then discussed and analyzed followed by the presentation of the $I_c(T)/I_d$ and Λ formulas. Finally, we give a closing conclusion.

2. MODEL AND RESULTS

The resistively and capacitive shunted junction (RSJ) model [1-2] leads to the following dynamical equation of motion:

$$\ddot{\phi} + G\dot{\phi} + \sin\phi = A \sin\Omega t \quad . \qquad (1)$$

The parameters that appear in the above equation are as follows: $G = \sqrt{\hbar/2eI_c(T)R^2c}$ is the damping parameter, $I_c(T)$ is the temperature dependent critical current, R is the junctions resistance, C its capacitance, and ϕ is the difference in phase across the junction of the tunnelling supercurrent. All times are measured in reduced units ($\omega_j t$), where $\omega_j = \sqrt{2e\, I_c(T)/\hbar C}$ is the Josephson plasma frequency. The right-hand side of Eq.(1) represents the driving (applied) ac current, where A is the amplitude normalized with respect to $I_c(T)$, $\Omega = \omega/\omega_j$ is the normalized frequency, and ω is the applied frequency. In a real life hysteresis experiment there are two cases to be considered.The first is the case of increasing the current on moving from the superconducting state to the resistive state up till a given current $I_m > I_c(T)$. The second case corresponds to decreasing the current to return back to the superconducting state. Bearing this in mind,we now replace the driving ac current by the following two expressions:

$$A \sin\Omega t = \begin{cases} \alpha t|\sin\Omega t| & \text{case 1} \quad (2a) \\ \alpha(t_m - t)|\sin\Omega t| & \text{case 2} \quad (2b) \end{cases} ,$$

where α is the normalized slope of the applied sawtooth function, and t_m is the normalized time that corresponds to the current range $0 \leq I \leq I_m$. In both expressions we used a rectified sinsoidal signal to avoid a negative current level since we are only interested in the positive path. Eq.(1) (also known as the equation of the damped driven pendulum), despite its relatively simple appearance cannot be solved analytically in order to obtain a closed form solution unless certain approximations are made [7 and 8]. Now since we are interested in the behaviour of Eq.(1) as is, we are then forced to solve it numerically. This was done using the fourth order Runge-Kutta method. To assure a constant integration step over the entire applied frequency range ($0.001 \leqslant \Omega \leqslant 0.5$), the integration steps per each half rectified cycle varied from 20,000 steps for the low limit frequency up till 50 steps for $\Omega = 0.5$. In all our computer work we took $\alpha = 10^{-4}$ to assure a slow rising signal. In Fig.1 $\langle\dot\phi\rangle$ (averaged over half a rectified cycle) is plotted versus the corresponding normalized current level i for four different G values and a constant $\Omega = 0.05$. It is to be noted that the variation in G for a single junction is accomplished by varying its temperature., Thus each curve represents the same junction with different temperature $I_c(T)$, ω_j and ω settings. Another way of viewing the different curves is to assume that each curve represents a different junction of the same material. Thus the junction that has the maximum $(I_c(T)R^2C)$ product will have the lowest G value. Now in order to avoid a complex situation we will theoretically assume that all junctions have the same $I_c(T)$ and C. Thus the junction with the highest R will correspond to the curve of the lowest G and vice versa. In Ref.5 it has been shown that as the damping parameter G decreases, the phase derivative $\dot\phi$ increases leading to an increase in the

voltage jump at $i = 1$, this we see in Fig.1. On the other hand, as G increases the voltage jump decreases until it eventually becomes undetectable. The second feature observed in Fig.1 is the increase in the hysteresis width as G decreases for a constant $\Omega = 0.05$. This is expected since as the junction becomes less damped, it becomes less able to exchange energy resulting in a larger hysteresis area. In Fig.2 we plot $\langle\dot\phi\rangle$ versus i for $G = 1/\sqrt{50}$ and different normalized frequencies. As we see for frequencies $\Omega < 0.03$ hysteresis does not develop. At $\Omega \sim 0.03$ hysteresis starts to develop (not shown for purposes of clarity). On increasing the normalized frequency the hysteresis width increases. As the applied normalized frequency approaches unity and beyond, the width reaches its saturation limit. This behaviour has been experimentally observed, (cf.Ref.4). From Figs.1 and 2 we may conclude that the damping parameter (or temperature) affects only the upward voltage jump at $i = 1$ and the frequency range over which hysteresis exists, while the hysteresis width is a function of frequency only.

As seen in Fig.2 we did not go beyond $\Omega = 0.5$. We found out that as Ω approaches unity, the RSJ model becomes unpredictable and fails to explain hysteresis. This may be due to the fact that within this frequency limit the voltage dependence of the critical current (resulting from the energy gap of the superconductor) must be taken into account [9]. In order to understand the relation between hysteresis and the different parameters involved in our work, we plotted $I_c(T)/I_d$ versus Ω for two G values ($G = 1/\sqrt{50}$ and $= 1/\sqrt{200}$) (see Fig.3). It is seen that as G decreases, the normalized frequency range over which hysteresis exists increases. Our results show that as G approaches zero hysteresis will be observed over nearly the entire Ω range. While as G aproaches 0.18 and beyond hysteresis does not exist at all. To obtain a formula to relate the ratio $I_c(t)/I_d$ to Ω, the curve fitting procedure was used. We found that the best formula that can fit both curves is

$$I_c(T)/I_d = 1 + a\exp(-b/\Omega) , \quad (3)$$

where for $G = 1/\sqrt{50}$ we obtained the values $a \simeq 2.44$ and $b \simeq 0.083$, while for $G = 1/\sqrt{200}$, $a \simeq 10.8$ and $b \simeq 0.019$. In order to express the parameters a and b by an understandable (meaningful) physical expression, we propose the following expression for the parameters a:

$$a = \{I_c(T)/I_{ds}\} - 1 , \quad (4)$$

where I_{ds} is the maximum critical down current (saturation limit). For the exponent (b/Ω) we begun by expressing it as

$$\frac{b}{\Omega} = k \quad \tau/RC , \quad (5)$$

where τ is the period of the applied frequency. To do this we assumed that our junction has a critical current 10^{-3} mA, and a capacitance $C = 10^{-9}$ f. Next for each given G the corresponding resistance R was calculated. Then using expression (5) the factor k was calculated for each G. On plotting k versus G we found that k may be best expressed as

$$k = -1/(4\pi \log G) \quad . \tag{6}$$

On changing the temperature G varies (instead of R), and we would still get the above relation since ω_j in this case is a variable while in the first it was a constant. Thus expression (3) can now be expressed as

$$I_c/(T)/I_d = 1 + \left\{\frac{I_c(T)}{I_{ds}} - 1\right\} \exp\left(\frac{1}{4\pi \log G} \frac{\tau}{Rc}\right) \quad . \tag{7}$$

Although hysteresis is observed for $G \leq 0.18$, formula (7) is valid for $G \leq 1$. For $G > 1$, the exponent becomes positive and formula (7) is no longer valid. In Fig.3 formula (7) was plotted (solid dots for $\Omega \leq 0.5$ and a broken line afterwards) against the computer curves from which we observe a good fit between both.

In Ref.4 it was proposed that the ratio $I_c(T)/I_d$ can be expressed as

$$\frac{I_c(T)}{I_d} = 2\Lambda/(\sqrt{3}\pi\xi(T)) \quad , \tag{8}$$

where Λ is the quasiparticle diffusion length,

$$\Lambda = \Lambda_0 \exp(-t/\tau_d) \quad , \tag{9}$$

where $\Lambda_0 = \sqrt{\ell v_F \tau_s/3}$ is the Pippard diffusion length, ℓ is the mean free path, v_F Fermi velocity, τ_s is the quasiparticle inelastic scattering time, τ_d is the relaxation time of the non-equilibrium state, and t is the recovery time. From (7) and (8) we get,

$$\Lambda = \frac{\sqrt{3}\pi\xi(T)}{2} \left\{1 + \left(\frac{I_c(T)}{I_{ds}} - 1\right) \exp\left[\frac{1}{4\pi \log G} \frac{\tau}{RC}\right]\right\} \quad . \tag{10}$$

Thus we now have an expression that relates the Λ length to the junctions parameters, temperature and frequency. At the high frequency saturation limit formula (10) reduces to

$$\Lambda = \frac{\sqrt{3}\pi\xi(T)}{2} \frac{I_c(T)}{I_{ds}} = \Lambda_0 = \sqrt{\ell v_F \tau_s/3} \quad , \tag{11}$$

from which the time τ_s can be calculated for any $G \leq 0.18$, and its corresponding temperature T. By its corresponding temperature T, we mean that expression (11) is not valid for all temperatures (as it may appear at first glance), but only at those temperatures for which $G \leq 0.18$. This is because for $G > 0.18$ (temperature increases) Eq.(10) reduces to (the case of no hysteresis)

$$\Lambda = \frac{\sqrt{3}\pi\xi(T)}{2} \quad . \tag{12}$$

In other words, we can say that expression (11) is valid for the temperature range $0 \leq T \leq T\,(G \simeq 0.18)$, while expression (12) is valid for $T(G \simeq 0.18) \leq T \leq T_c$.

3. CONCLUSION

We have obtained two empirical formulas for the ratio $I_c(T)/I_d$ and the quasiparticle diffusion length Λ in terms of the junctions electrical parameters, temperature, and applied ac frequency. All parameters involved can be measured prior to performing the hysteresis experiment except the current I_{ds}. The current I_{ds} is to be obtained experimentally by using at least one data point $I_c(T)/I_d$. Once T_{ds} is determined, we then have two formulas that can predict the full scale of the junctions hysteretic behaviour without the use of extensive computer work. Our work shows that for junctions with a damping parameter greater than 0.18 hysteresis does not develop. This last statement defines the operable temperature range in order to observe hysteresis experimentally. Finally, we would like to mention that while the RSJ model fails to account for hysteresis behaviour for $\Omega \geq 1$, our formulas do not. The only restriction on our formulas is that G must be less than or equal to unity. This we do not see as a handicap since hysteresis vanishes for $G > 0.18$.

ACKNOWLEDGMENTS

One of the authors (L.N.S.) would like to thank Professor Abdus Salam, The International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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FIGURE CAPTIONS

Fig.1 Hysteresis curves for four different G values at a given constant $\Omega = 0.05$.

Fig.2 The dependence of hysteresis development on the applied frequency for a given $G = 1/\sqrt{50}$.

Fig.3 $I_c(T)/I_d$ versus Ω for two different G settings.

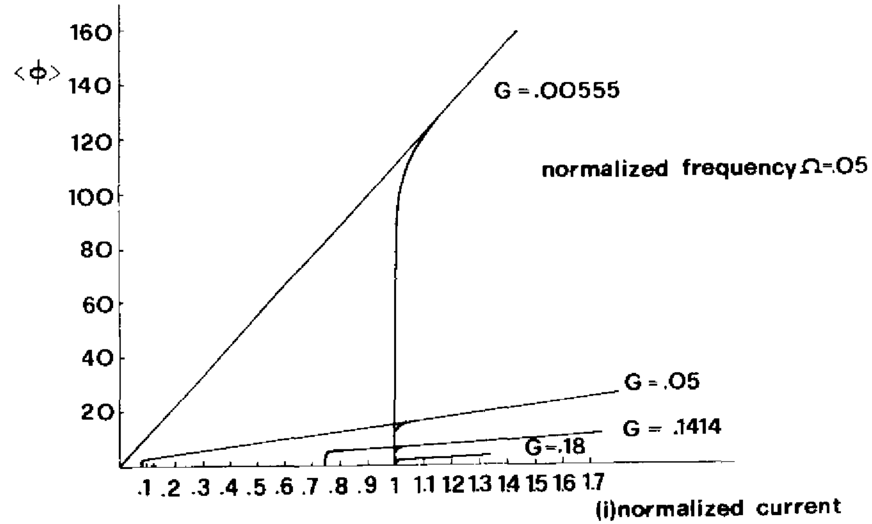
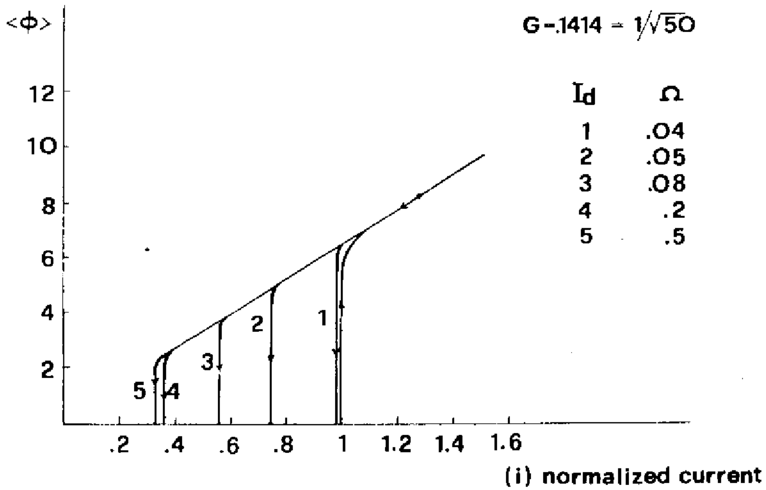
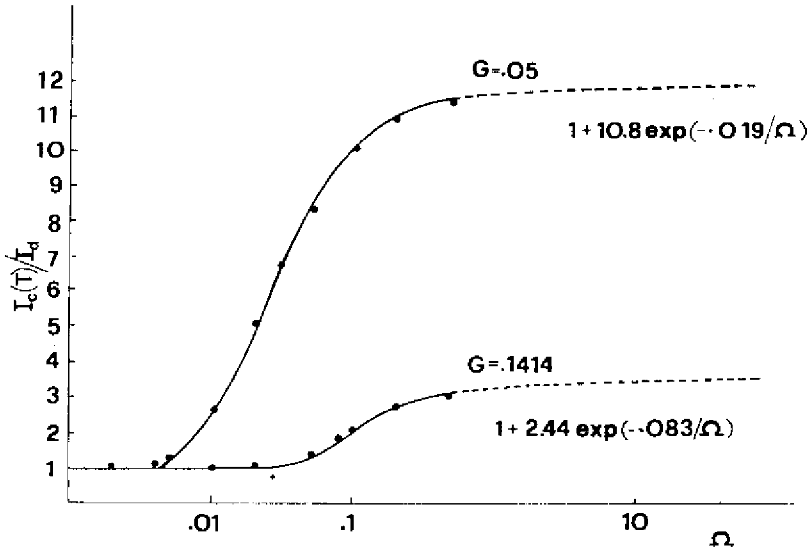


Fig.1



<Φ>
G=.1414 = 1/√50
Id Ω
1 .04
2 .05
3 .08
4 .2
5 .5
(i) normalized current

Fig.2



G=.05
1+10.8 exp(-.019/Ω)
G=.1414
1+2.44 exp(-.083/Ω)
Ic(T)/Id
Ω

Fig.3

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