IC/88/141

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

FORCED AND FREE CONVECTION FLOW WITH VISCOUS DISSIPATION EFFECTS: THE METHOD OF PARAMETRIC DIFFERENTIATION

M.A. Hossain

and

O. Arbad

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

IC/88/141

International Atomic Energy Agency

and

United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

FORCED AND FREE CONVECTION FLOW WITH VISCOUS DISSIPATION EFFECTS: THE METHOD OF PARAMETRIC DIFFERENTIATION *

M.A. Hossain **

International Centre for Theoretical Physics, Trieste, Italy

and

O. Arbad

Centre for Research in Basic Science, Misurata, P.O. Box 18478, Libya.

ABSTRACT

Effect of buoyancy force in a laminar uniform forced convection flow past a semi-infinite vertical plate has been analyzed near the leading edge, taking into account the viscous dissipation. The coupled non-linear locally similar equations, which govern the flow, are solved by the method of parametric differentiation. Effects of the buoyancy force and the heat due to viscous dissipation on the flow and the temperature fields as well as on the wall shear-stress and the heat transfer at the surface of the plate are shown graphically for the values of the Prandtl number σ ranging from 10^{-1} to 1.0.

MIRAMARE – TRIESTE

July 1988

* To be submitted for publication.

** On leave of absence from: Centre for Research in Basic Science, Misurata, P.O. Box 18478, Libya.

1. INTRODUCTION

Free convection may contribute a significant effect on forced flow over solid surfaces. It may also alter the flow field and hence the heat transfer rate and wall-shear distribution. The simplest physical model is two-dimensional, mixed forced and free convection along a flat plate. Understanding of the fundamental mechanism of this interaction can help one to estimate the heat transfer rate and pumping power for more complex geometries of practical interest. Recent examples are found in the area of reactor safety, combustion flames and solar collectors, as well as building energy conservation. Extensive studies [1-10] have been conducted on mixed convection along vertical, horizontal or inclined surfaces. Very recently, Yao [11], has studied the mixed convection flow along a vertical flat plate. It has been generally recognized that $\beta(= Gr/Re^2)$ is the governing parameter for the convection flow along a vertical plate. Yao [11], has shown that the free convections limits for $Re > 1$ and $Re < 1$ are different. For $Re > 1$, the true convection becomes large when $\beta \to \infty$. In this case the value of Gr determines the required distance for the forced–convection effect to fade away. On the other hand, when $Re < 1$, free convection is the dominant mode all over the plate, and the forced–convection effect is $0(Gr^{-1/2})$. In all the above studies viscous dissipation was assumed to be negligible. Gebhart [12] and Gebhart and Mollendorf [13] have shown that the viscous dissipation contributes a considerable effect on the free convection flow fields of extreme size or extremely low temperature or in a high gravity field. Among the above authors, Gebhart [12] studied the flow generated by the plate surface temperature varying as power of x (the distance measuring along the plate surface from the leading edge) by a series expansion method and Gebhart and Mollendorf [13] derived similarity solutions for such a flow generated by the plate surface temperature varying exponentially with x.

In the present paper, we examine the effect of viscous dissipation in the combined forced and free convection flow of a viscous incompressible fluid past a vertical flat plate near the leading edge for smaller values of $\beta(= Gr/Re^2)$. The solutions of the locally similar equation are obtained by the method of parametric differentiation initiated by Rubbert [14] and Rubbert and Landhal [15]. To varify the accuracy of the present method some of the numerical values are compared with that of Schlicting [16] and Soundalgekar *et al.* [17].

2. FORMULATION OF THE PROBLEM

Here the x–axis is taken along the plate in the vertical direction and the y–axis is normal to it. The incompressible fluid is assumed to flow in the upward direction with a uniform velocity U_0. Under the usual Bousinesq approximation, the laminar boundary layer free convection flow is then governed by the following system of equations:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{2}$$

and the Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{\sigma}\,\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where u, v are respectively the velocity components in the x and y-direction, ν the kinematic co-efficient of viscosity, g the acceleration due to gravity, β the co-efficient of volume expansion, c_p the specific heat at constant pressure, $\sigma(=\mu c_p/k)$ the Prandtl number, T the temperature of the fluid in the boundary layer, T_∞ is that of the ambient fluid, and μ the viscosity of the fluid and k the thermal conductivity.

The appropriate boundary conditions for the present problem are

$$\left.\begin{array}{llll} u = 0, & v = 0, & T = T_w & \text{at} \quad y = 0 \\ u \longrightarrow U_0, & & T \longrightarrow T_0 & \text{as} \quad y \longrightarrow \infty \end{array}\right\} \tag{4}$$

Introducing the following transformations for the independent and dependent variables

$$\begin{aligned} \eta &= (U_0/\nu x)^{1/2}y, \quad u = U_0 f'(\eta) \\ v &= -\frac{1}{2}(U_0\nu/x)^{1/2}(f(\eta) - \eta f'(\eta)) \\ \theta(\eta) &= (T - T_0)/(T_w - T_0) \end{aligned}$$

into Eqs.(1) to (3), we have

$$f''' + \frac{1}{2}ff'' + \beta\theta = 0 \tag{5}$$

$$\theta'' + \frac{1}{2}\sigma f\theta' + \sigma Ec f''^2 = 0 \tag{6}$$

where

$$\left.\begin{array}{lll} \beta & = Gr/Re^2 & \\ Gr & = g\beta(T_w - T_\infty)x^3/\nu^2 & \text{(Grashof number)} \\ Re & = U_0 x/\nu, & \text{(Reynold number)} \\ Ec & = U_0^2/c_p(T_w - T_0), & \text{(Eckert number)} \end{array}\right\} \tag{7}$$

The corresponding boundary condition takes the form

$$\left.\begin{array}{ll} f(0) = f'(0) = 0, & \theta(0) = 1 \\ f'(\infty) \longrightarrow 1, & \theta(\infty) \longrightarrow 0 \end{array}\right\} \tag{8}$$

3. METHOD OF SOLUTION

It appears from the relations (7) that the Grashof number Gr and the Reynold number Re depend on x and hence Eqs.(5) and (6) are not strictly similarity equations. But for a particular x, Gr and Re can be treated as constant and hence the solutions of Eqs.(5) and (6) can be considered as locally similar solutions. Such solutions were presented by Chen and Einchorn [19] and Minkowycz and Sparrow [20]. Though for an incompressible fluid, the value of the Eckert number Ec is less than unity, its effect on the temperature and flow fluids cannot be neglected (Gebhart [12] and Gebhart and Mollendorf [13]); and therefore, to find the solutions of Eqs.(5) and (6), we first expand the functions $f(\eta)$ and $\theta(\eta)$ in powers of Ec as given below:

$$\left.\begin{array}{ll} f(eta) & = f_0(\eta) + Ecf_1(\eta) + Ec^2 f_2(\eta) + 0(Ec^3), \\ \theta(\eta) & = \theta_0(\eta) + Ec\theta_1(\eta) + Ec^2\theta_2(\eta) + 0(Ec^3), \end{array}\right\} \tag{9}$$

Substituting the expressions (9) into Eqs.(5) to (7) and retaining the terms upto $0(Ec^2)$, we get

$$f_0''' + \frac{1}{2}f_0 f_0'' + \beta\theta_0 = 0 \tag{10}$$

$$\theta_0'' + \frac{1}{2}\sigma f_0\theta_0' = 0 \tag{11}$$

$$\left.\begin{array}{lll} f(0) & = & f'(0) = 0 = 1 - \theta(0) \\ f_0'(\infty) & = & 0 = \theta_0(\infty) \end{array}\right\} \tag{12}$$

$$f_1''' + \frac{1}{2}(f_0 f_1'' + f_0'' f_1) + \beta\theta_1 = 0 \tag{13}$$

$$\theta_1'' + \frac{1}{2}\sigma(f_0\theta_1' + \theta_0' f_1 + 2f_0''^2 = 0 \tag{14}$$

$$f_1(0) = f_1'(0) = \theta_1(0) = 0$$

$$f_1'(\infty) = \theta_1(\infty) = 0 \tag{15}$$

and

$$f_2''' + \frac{1}{2}(f_0 f_2'' + f_0'' f_2 + f_1 f_1'') + \beta\theta_2 = 0 \tag{16}$$

$$\theta_2'' + \frac{1}{2}\sigma(f_0\theta_2' + \theta_0' f_2 + f_1\theta_1' + 4f_0'' f_1'' = 0 \tag{17}$$

$$f_2(0) = f_2'(0) = \theta_2(0) = 0, \quad f_2'(\infty) = \theta_2(\infty) = 0 \tag{18}$$

The set of Eqs.(10) and (12) describes a non-linear boundary value problem, whereas the sets of coupled equations (13) to (15) and (16) to (18) provide linear boundary value problems. The latter sets can easily be integrated one after another, provided the solutions of the coupled non-linear set of equations (10) to (12) are known.

Here we propose to solve these coupled equations by the method of parametric differentiation, initiated by Rubbert [15] and Rubbert and Landhal [16], taking β as the parameter. Before we introduce this method we reduce the coupled boundary value problem given by Eqs.(10) to (13) to an uncoupled boundary value problem by considering $\beta = 0$. In this consideration Eq.(10) turns to the well-known Blasius equation, which has the solution $f''(0) = 0.33201$. The corresponding solutions for Eq.(12) satisfying the boundary conditions (13) for different values of the Prandtl number σ are obtained by the method of superposition, described in detail at the time of finding the solutions of Eqs.(19) and (20). The values of $\theta'(0)$ for different values of σ, thus obtained, are entered into Table 1.

We now find the solutions of the coupled non-linear Eqs.(10) to (12), for different values of the parametes $\beta(= Gr/Re^2)$, by the method of parametric differentiation. For this study, we consider β as the parameter. In the first step, Eqs.(10) and (11) are differentiated with respect to β to get

$$G''' + \frac{1}{2}(f_0G'' + f_0''G) + \beta H + \theta_0 = 0 \tag{19}$$

$$H'' + \frac{1}{2}\sigma(f_0H' + \theta_0'G) = 0 \tag{20}$$

and the corresponding boundary conditions turn to

$$G(0) = G'(0) = G'(\infty) = H(0) = H(\infty) = 0 \tag{21}$$

where

$$\partial f_0/\partial\beta = G \qquad \text{and} \qquad \partial\theta_0/\partial\beta = H \tag{22}$$

The set of Eqs.(19) and (20) are now coupled linear ordinary differential equations which represent as the linear coupled boundary value problem with the boundary conditions (21); because, in this equation the functions f_0 and θ_0 along with their derivatives are know for β. The boundary value problem, represented by Eqs.(19) to (21), are now reduced to an initial-value problem by the following method of superposition (Na [18]). For this purpose, let G and H be expressed as

$$G = G_0 + \lambda G_1 + \mu G_2$$

and

$$H = H_0 + \lambda H_1 + \mu H_2 \tag{23}$$

respectively. Introducing the expressions (23) for G and H into Eqs.(19) to (21) we get

$$G_0''' + \frac{1}{2}(f_0G_0'' + f_0''G_0) + \beta H_0 + \theta_0 = 0 \tag{24a}$$

$$H_0'' + \frac{1}{2}\sigma(f_0H_0' + \theta_0'H_0) = 0 \tag{24b}$$

$$G_0(0) = G_0'(0) = G_0''(0) = H_0(0) = H_0'(0) = 0 \tag{25}$$

$$G_1''' + \frac{1}{2}(f_0G_1'' + f_0''G_1) + \beta H_1 = 0 \tag{26a}$$

$$H_1'' + \frac{1}{2}\sigma(f_0H_1' + \theta_0'H_1) = 0 \tag{26b}$$

$$G_1(0) = G_1'(0) = G_1''(0) = H_1(0) = 0, \quad H_1'(0) = 1, \tag{27}$$

and

$$G_2'' + \frac{1}{2}(f_0G_2'' + f_0''G_2) + \beta H_2 = 0 \tag{28a}$$

$$H_2'' + \frac{1}{2}\sigma(f_0H_2' + \theta_0'H_2) = 0 \tag{28b}$$

$$G_2(0) = G_2'(0) = H_1(0) = H_1'(0) = 0, \quad G_2''(0) = 1. \tag{29}$$

In Eqs.(24) to (29), the primes denote the differentiations with respect to η. The inital conditions are obtained in the initial value problems, represented by Eqs.(24) to (29), are based on that

$$G''(0) = \lambda \qquad \text{and} \qquad H'(0) = \mu \tag{30}$$

and the values of the parameter λ and μ may be obtained from the conditions that $G'(\infty) = 0 = H(\infty)$; which give

$$\lambda = \frac{H_0(\infty)G_2'(\infty) - H_2(\infty)G_0'(\infty)}{H_2(\infty)G_1'(\infty) - H_1(\infty)G_2'(\infty)} \tag{31}$$

and

$$\mu = \frac{H_2(\infty)G_0'(\infty) - H_0(\infty)G_1'(\infty)}{H_2(\infty)G_1'(\infty) - H_1(\infty)G_2'(\infty)} \tag{32}$$

Eqs.(24) to (29) now constitute a set of initial value problems that can be integrated without iterations to give G_0, G_1, G_2, H_0, H_1, and H_2 by any initial value solver, such as the Rungee-Kutta fourth order method. The integrations are carried out in the domain $0 \le \eta \le \eta_\infty$. In all cases, the three values of $\eta_\infty = 10, 12$ and 15 were tried and it was found that the change in the final result was insignificant. The numerical results thus obtained also give the values of the required derivatives of the functions G_0, G_1, G_2, H_0, H_1 and H_2, which are then utilized to obtain the values of the parameter λ and μ from the expressions (31) and (32). With known values of λ and μ the functions $G(\eta,\beta)$ and $H(\eta,\beta)$ are found from Eqs.(23) which are then substituted into Eq.(30) to get $f_0(\eta)$ and $\theta_0(\eta)$ at $\beta = \Delta\beta$. This is accomplished by writing Eq.(22) as

$$f_0(\eta)|_{\beta=\Delta\beta} = f_0(\eta)|_{\beta=0} + \Delta\beta\ G(\eta,\beta)|_{\Delta\beta} \tag{33}$$

and

$$\theta_0(\eta)|_{\beta=\Delta\beta} = \theta_0(\eta)_{\beta=0} + \Delta\beta\ H(\eta,\beta)|_{\Delta\beta}. \tag{34}$$

This process is repeated to calculate the solutions for $f_0(\eta)$ and $\theta_0(\eta)$ for $\beta = \Delta\beta, 2\Delta\beta, 3\Delta\beta$, etc. and for different values of the Prandtl number σ ranging from 0.01 to 1.0.

Once we knew the solutions of Eqs.(10) to (12) for different values of the parameter $\beta(= Gr/Re^2)$ and the Prandtl number σ, we can easily obtain the solutions of the subsequent sets of Eqs.(13) to (15) and (16) to (18) one after another using the same method as described above. Thus, knowing the functions $f_0, f_1, f_2, \theta_0, \theta_1$ and θ_2, we may find the velocity distribution from the following relation:

$$u/U_0 = f_0'(\eta) + Ec\ f_1'(\eta) + (Ec)^2 f_2'(\eta) + \dots \tag{35}$$

Knowing the velocity field, we now calculate the skin-friction at the surface of the plate from the relation

$$\begin{aligned}\tau_w &= \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}\\ &= \left(\rho U_0^2 | Re^{1/2}\right) f''(0).\end{aligned} \tag{36}$$

Hence, the local skin-friction co-eficient is given by

$$Re^{1/2} c_f = f''(0). \tag{36a}$$

Again, the temperature distribution may be obtained from the relation (33):

$$(T - T_0)/(T_w - T_0) = \theta_0(\eta) + Ec\theta_1(\eta) + Ec^2\theta_2(\eta) + \dots \tag{37}$$

Finally, knowing the temperature distribution, one may find the heat transfer from the plate to the fluid from the following relation:

$$\begin{aligned}q_w &= -k\left(\frac{\partial T}{\partial y}\right)_{y=0}\\ &= -\left[k(T_w - T_0) Re^{1/2} x\right]\theta'(0).\end{aligned} \tag{38}$$

In terms of the Nusselt number it becomes

$$Re^{-1/2} Nu = -\theta'(0). \tag{38a}$$

The results thus obtained from the relations (35) to (38) are presented graphically in Figs.1 to 3 for different values of the parameters.

In the present problem, the actual parameter which determines the magnitude of the buoyancy force is $\beta(= Gr/Re^2)$, which is linearly proportional to x, so near the leading edge, the value of β is smaller. With this understanding, in the present analysis, the value of β has been considered to be less than unity. Although, at this region the free-convection effect is smaller, nevertheless, it affects the flow field and the temperature field as well as the wall shear-stress and the heat transfer rate significantly, which can be seen from the present analysis.

4. RESULTS AND DISCUSSIONS

The present numerical scheme had been carried out on a *Gould*/9000 machine of the International Centre for Theoretical Physics, Trieste, Italy. As a check on the accuracy of the present numerical scheme, some results are compared with those of Soundalgekar *et al.* (1980) for $\sigma = 0.72$ in Table 2.

Figs.1a and 2a represent the velocity distributions for different values of the Prandtl number σ (ranging from 0.01 to 1.0). The Eckert number Ec (ranging from 0.0 to 0.1) and the buoyancy-force parameter $\beta(= Gr/Re^2)$ (ranging from 0.0 to 0.4). In both of the figures the dotted curves are for Ec equals 0.0, the broken are for $Ec = 0.05$ and the solids are for $Ec = 0.1$. From Fig.1 we see that an increase in the heat due to viscous dissipation as well as an increase in the heating of the plate lead to increase in the velocity field in the boundary-layer. On the other hand, from Fig.2 we observe that with the increasing values of the Prandtl number, the velocity field decreases in the boundary layer. Figs.1b and 2b represent the temperature distribution in the flow field. From Fig.1b it is clearly observed that an increase in the heating of the plate as well as in the heat due to viscous dissipation leads to a fall in the temperature distribution. From Fig.2b, we further observe a fall in the temperature distribution owing to the increase in the value of the Prandtl Number σ.

Finally, Fig.3a represents the wall shear-stress co-efficient and Fig.3b that of the heat transfer rate at the surface of the plate against the buoyancy force parameter β. From these figures it is seen that an increase in the buoyancy force leads to an increase in the wall shear-stress; whereas this leads to a fall in the heat transfer rate. On the other hand, the wall shear-stress increases and the heat transfer rate decreases owing to the increase in the heat due to viscous dissipation.

Acknowledgments

One of the authors (M.A.H.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

REFERENCES

[1] E.M. Sparrow and J.L. Gregg, "Buoyancy effects in forced-convection flow and heat-transfer",ASME J. Appl. Mech. *81*, 133-134 (1959).

[2] T. Morri, "Buoyancy effects in forced laminar convection flow over a horizontal flat plate", ASME J. Heat Transfer *83*, 479-482 (1961).

[3] A. Acrivos, "On the combined effect of forced and free convection heat transfer in laminar boundary layer flows", Chem. Engn. Scie. *21*, 505-511 (1962).

[4] E.M. Sparrow and W.J. Minkowycz, "Buoyancy effects on horizontal boundary-layer flow and heat transfer", Int. J. Heat Mass Transfer *5*, 505-511 (1962).

[5] J.H. Merkin, "The effect of buoyancy forces on the boundary-layer flow over a semi-infinite vertical flat plate in a uniform free stream", J. Fluid Mech. *35*, 439-450 (1969).

[6] J.R. Llloyd and E.M. Sparrow, "Combined forced and free convection flow on vertical surfaces", Int. J. Heat Mass Transfer *13*, 434-438 (1970).

[7] G.E. Robertson, J.H. Seinfeld and G.E. Leal, "Combined forced and free convection flow past a horizontal flat plate", AICHE Journal *19*, 998-1008 (1973).

[8] G. Wilks, "Combined forced and free convection flow on vertical surfaces", Int. J. Heat Mass Transfer *16*, 1958-1964 (1973).

[9] A. Mucoglu and T.S. Chen, "Mixed convection on inclined surfaces", ASME J. Heat Transfer *101*, 422-426 (1979).

[10] M.S. Rajn, X.R. Liu and C.K. Law, "A formulation of combined forced and free convection past horizontal and vertical surfaces", Int. J. Heat Mass Transfer *27*, 2215-2224 (1984).

[11] L.S. Yao, "Two-dimensional mixed convection along a flat plate", ASME J. Heat Transfer *109*, 440-445 (1987).

[12] B. Gebhart, " Effect of viscous dissipation in natural convection", J. Fluid Mech. *14*, 225-295 (1962).

[13] B. Gebhart and J. Mollendorf, " Viscous dissipation in external natural convection flows". J. Fluid Mech. *38*, 97-107 (1969).

[14] P.E. Rubbert, "Analysis of transonic flow by means of parametric differentiation", Ph.D. Thesis, Dept. Mech. Eng., MIT, Cambridge, Mass (1965).

[15] P.E. Rubbert and M.T. Landhal, "Solution of the transonic airfoil problem through parametric differentiation", AIAA Journal *5*, 470-479 (1967).

[16] H. Schlicting, *Boundary Layer Theory*, Chapter 12, (McGraw-Hill Book Co., New York, 1964).

[17] V.M. Soundalgekar, N.V. Vighnesam and I. Pop, "Combined free and forced convection flow past a vertical porous plate", Int. J. Energy Research *5*, 215-226 (1981).

[18] T.Y. Na, *Computational Methods in Engn. Boundary Value Problem*, Chapter 10 (Academic Press, 1979).

[19] C.C. Chen and R. Eichhorn, "Natural convection from a vertical surface to a thermally statified fluid", ASME J. Heat Transfer *98c*, 446-451 (1976).

[20] W.J. Minkowycz and E.M. Sparrow, "Local non-similar solutions for natural convection on a vertical cylinder", ASME J. Heat Transfer *86c*, 178-183 (1974).

TABLE CAPTIONS

Table 1 Values of $\theta_0'(0)$ for different value of σ.

Table 2 Values of c_f and Nu for $\sigma = 0.72$.

Table 1

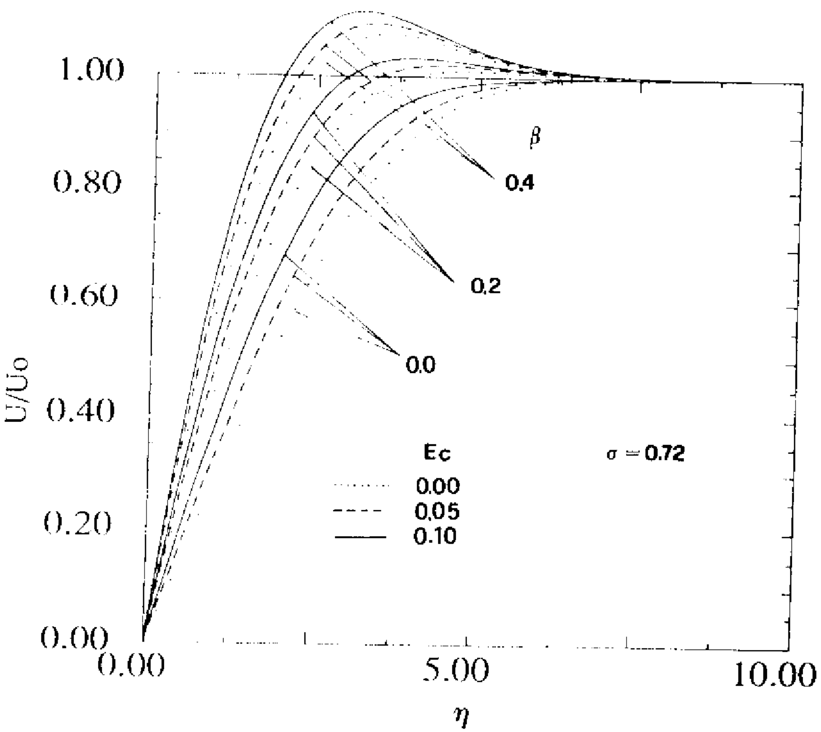
σ	0.72		1.0	
$-\theta_0'(0)$	0.2947*	0.2926	0.3320*	0.3319

* These values are due to Schlicting [16].

Table 2

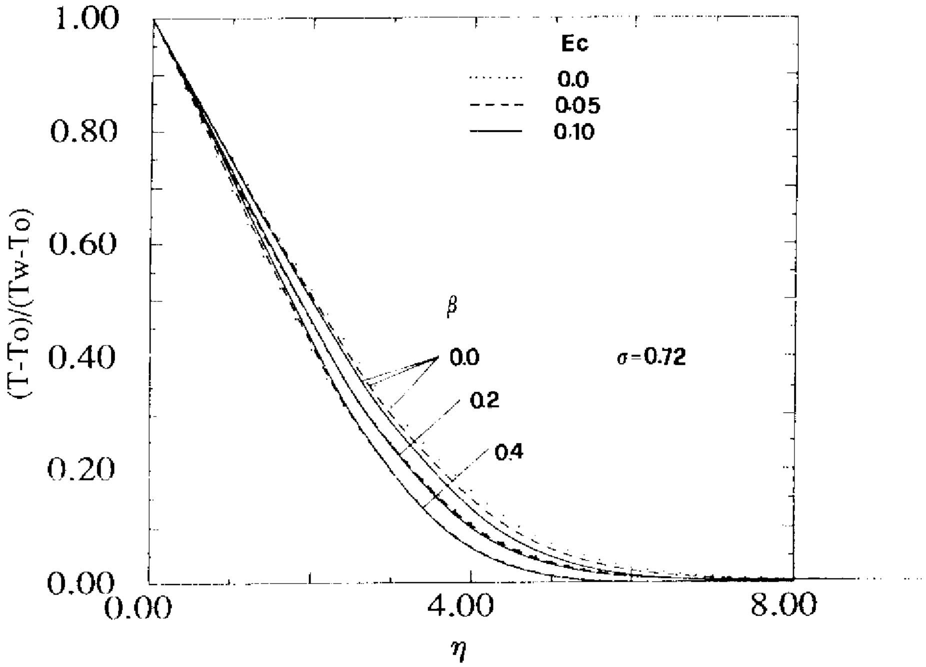
Ec	(Gr/Re^2)	$Re^{1/2}c_f$		-Nu	
0.00	0.0	0.3320	-	0.2956	-
0.01	0.0	0.3470	-	0.2966	-
0.00	0.1	0.4757	0.4719*	0.3174	0.3157*
0.01	0.1	0.4799	0.4721*	0.3169	0.3141*

* These values are from Soundalgekar *et al.* [17].



1.00
0.80
0.60
0.40
0.20
0.00
U/Uo
0.00
5.00
10.00
η
β
0.4
0.2
0.0
Ec
0.00
0.05
0.10
σ = 0.72

Fig.1a



1.00
0.80
0.60
0.40
0.20
0.00
(T-To)/(Tw-To)
0.00
4.00
8.00
η
Ec
0.0
0.05
0.10
β
0.0
0.2
0.4
σ=0.72

Fig.1b

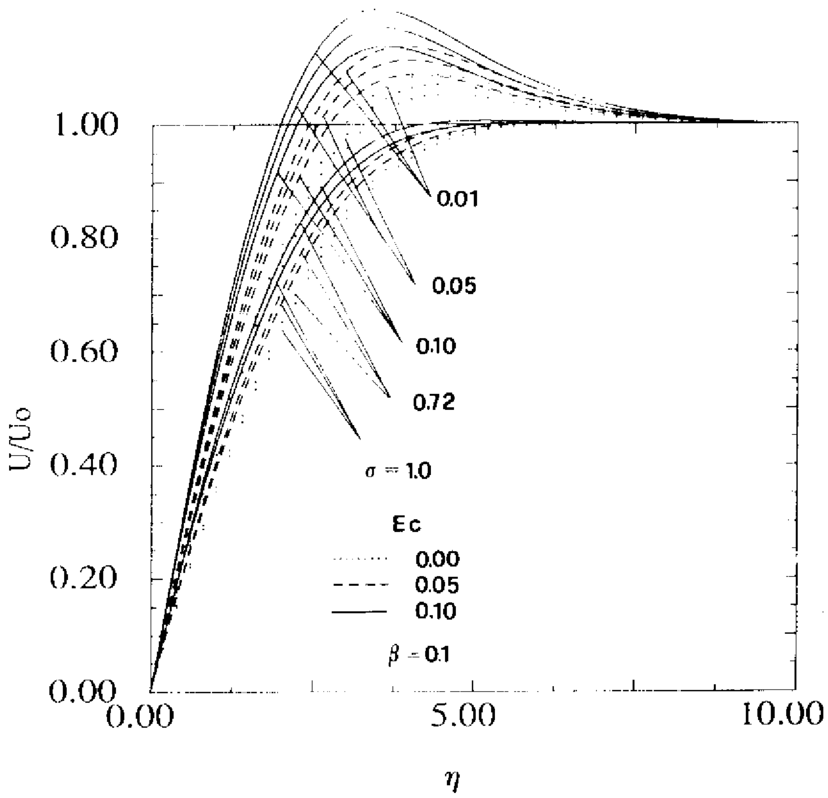

1.00
0.80
0.60
0.40
0.20
0.00
0.00
5.00
10.00
U/Uo
η
0.01
0.05
0.10
0.72
σ = 1.0
Ec
0.00
0.05
0.10
β = 0.1

Fig.2a

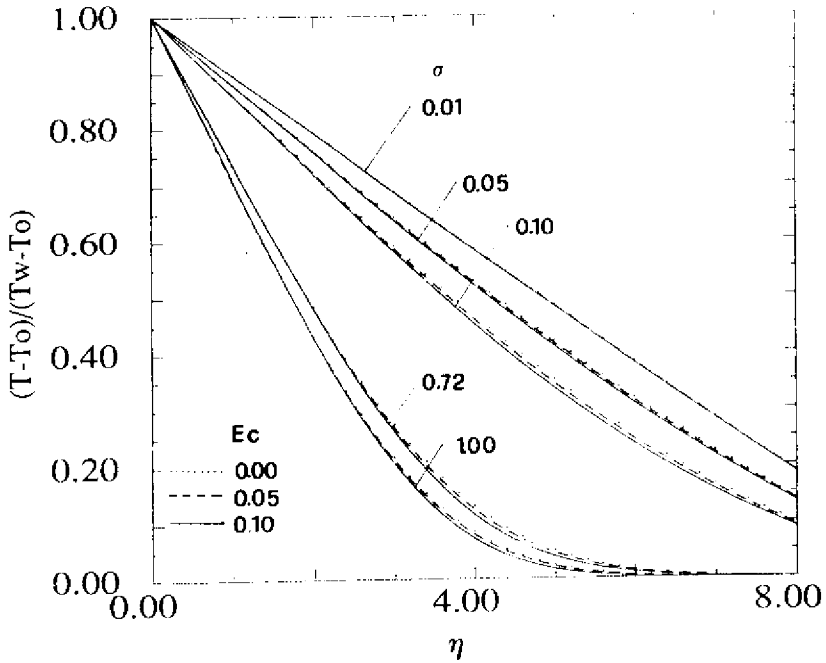
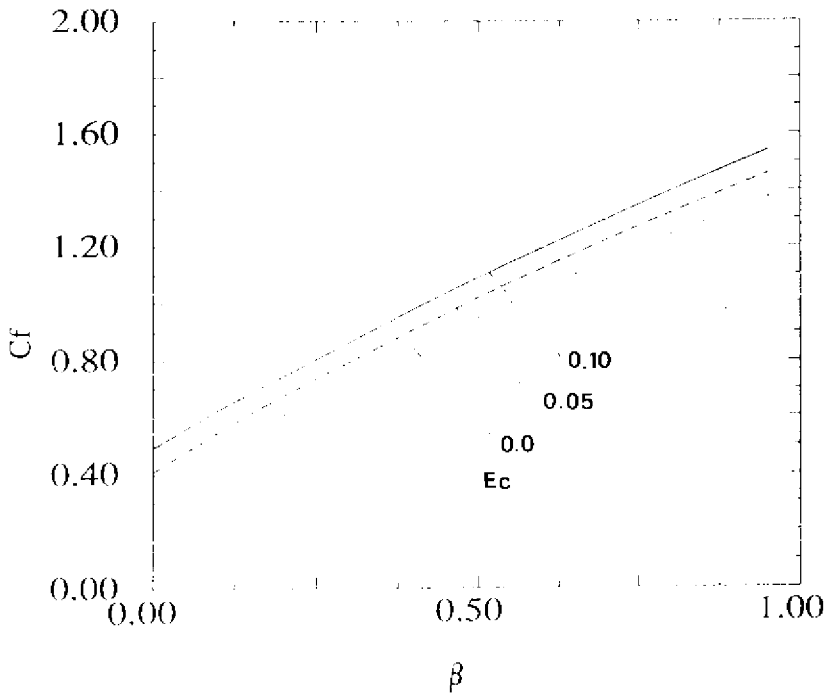
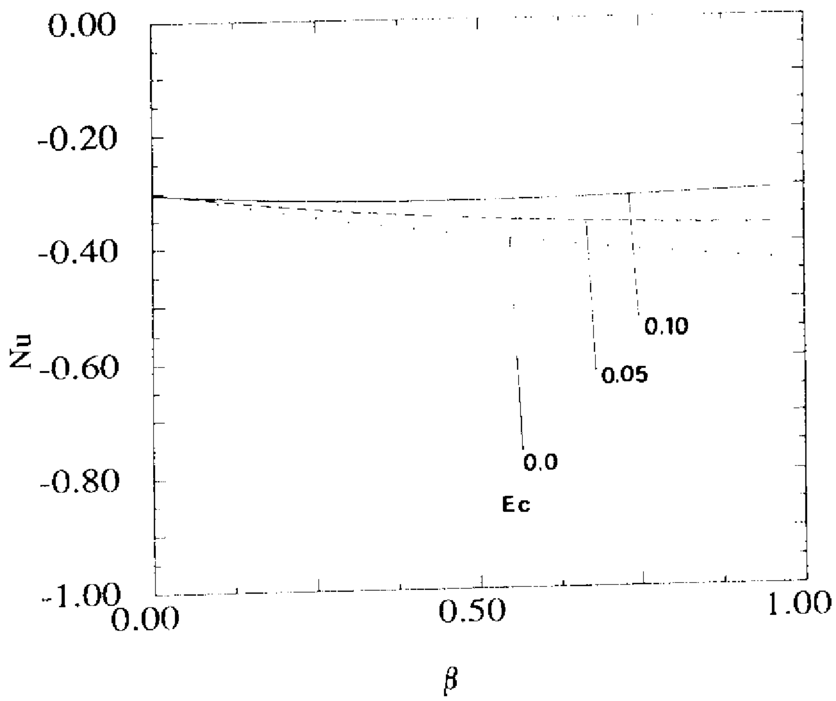

1.00
0.80
0.60
0.40
0.20
0.00
0.00
4.00
8.00
(T-To)/(Tw-To)
η
σ
0.01
0.05
0.10
0.72
1.00
Ec
0.00
0.05
0.10

Fig.2b



2.00
1.60
1.20
0.80
0.40
0.00
Cf
0.00
0.50
1.00
β
0.10
0.05
0.0
Ec

Fig.3a



0.00
-0.20
-0.40
-0.60
-0.80
-1.00
Nu
0.00
0.50
1.00
β
0.10
0.05
0.0
Ec

Fig.3b

Stampato in proprio nella tipografia
del Centro Internazionale di Fisica Teorica