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SNAKE RESONANCES

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Abstract

Siberian Snakes provide a practical means of obtaining polarized proton beams in large accelerators. The effect of snakes can be understood by studying the dynamics of spin precession in an accelerator with snakes and a single spin resonance. This leads to a new class of energy independent spin depolarizing resonances, called snake resonances. In designing a large accelerator with snakes to preserve the spin polarization, there is an added constraint on the choice of the vertical betatron tune due to the snake resonances.

Introduction

Preserving the spin polarization of a beam of protons in large accelerators is a monumental task because of the large number of spin resonances that can be crossed during acceleration. The usual methods of resonance jumping and orbit correction[1-4] are too impractical. The Siberian Snake[5] was invented to eliminate all these spin resonances simultaneously. We study this by looking at an accelerator with a single spin resonance and a periodic distribution of snakes. Although the snakes make the spin resonances transparent, there is a new class of energy independent spin depolarizing resonances created called snake resonances[6,7].

In the section 2 we study the spin dynamics of an accelerator containing a periodic distribution of snakes and a single spin resonance. The results are summarized in section 3 and some consequences are discussed.

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Snake Resonances

A Siberian Snake is a collection of dipole (and/or solenoidal as well as other configurations[8,9]) magnets that are orbit transparent and rotate the spin 180° about a horizontal axis. The spin precession of the snake can be described by the following matrix:

$$S(\phi_k) = \begin{bmatrix} 0 & ie^{-i\phi_k} \\ ie^{i\phi_k} & 0 \end{bmatrix}$$

where ϕ_k is the angle of the precession axis of the k'th snake (Fig. 1).

In an ideal accelerator with N_S (assumed even) snakes that are equally spaced and no spin resonances the spin precesses through one revolution as follows:

$$U = (-1)^{N_S/2} \begin{bmatrix} e^{i\pi\nu_s} & 0 \\ 0 & e^{-i\pi\nu_s} \end{bmatrix}$$

where the spin tune ν_s is

$$\nu_s = \frac{1}{\pi} \sum_{k=0}^{N_s-1} (-1)^k \phi_k$$

independent of energy. When

$$\nu_s = \text{integer}$$

then the matrix M is a unity and would set up a resonance condition leading toward depolarization. This is true when there are no spin resonances present.

To generalize this by including spin resonances we consider the following configuration of snakes in an accelerator

$$\phi_{2k} = \frac{\pi\nu_s}{N_s}, \qquad \phi_{2k+1} = -\frac{\pi\nu_s}{N_s}.$$

Furthermore, we need the matrix that describes the precession of the spin in the arcs between snakes assuming only a single spin resonance is present. This can be found from the spin equation[2]

$$\frac{d}{d\theta}\psi(\theta) = -\frac{i}{2}\begin{bmatrix} \gamma G & -\varepsilon e^{i\kappa\theta} \\ -\varepsilon^{*} e^{-i\kappa\theta} & -\gamma G \end{bmatrix}\psi(\theta)$$

where ψ is the spinor representing the spin direction, θ is the azimuth of the particle, γ is the lorentz factor, $G = (g-2)/2$ is the anomolous magnetic moment, ε is the strength of the spin resonance and κ is the resonance position.

The spin equation can be solved exactly when there is no acceleration (i.e. γ is a constant) and for a single resonance giving

$$\psi(\theta + \frac{\pi}{N_s}) = V(\theta)\ \psi(\theta - \frac{\pi}{N_s})$$

where $2\pi/N_s$ is the spacing between snakes, θ is the azimuth at the center of the arc and

$$V(\theta) = \begin{bmatrix} Z & -W \\ W^{*} & Z^{*} \end{bmatrix}$$

with

$$Z = e^{-i\pi\kappa/N_s}\left(\cos\frac{\pi\lambda}{N_s} - i\frac{\delta}{\lambda}\sin\frac{\pi\lambda}{N_s}\right)$$

$$W = -i\frac{|\varepsilon|}{\lambda}e^{i(\rho-\kappa\theta)}\sin\frac{\pi\lambda}{N_s}$$

and $\varepsilon = |\varepsilon|e^{i\rho}$, $\delta = \gamma G - \kappa$ and $\lambda = \sqrt{\delta^2 + |\varepsilon|^2}$.

The dynamics of the spin precession in an accelerator with a single spin resonance and sna ·s can be deduced from the following matrix

$$M_k = S(\phi_{2k+1})\ V(\theta_{2k+1})\ S(\phi_{2k})\ V(\theta_{2k})$$

which is the matrix representing the spin precession between two snakes and two arcs. Expanding the matrix M_k leaves

$$M_k = \begin{bmatrix} a_k & -b_k \\ b_k^* & a_k^* \end{bmatrix}$$

where

$$a_k = -\cos^2\mu\ e^{i2\pi\nu_s/N_S} + \sin^2\mu\ e^{i(2\kappa\theta_k^s - 2\rho)}$$

$$b_k = i\sin 2\mu\ \cos(\phi_{2k} - \kappa\theta_k^s + \rho)e^{-i(\phi_{2k+1} + \beta - 2\pi\kappa/N_S)}$$

$$\mu = \arcsin(\frac{|\varepsilon|}{\lambda}\sin\frac{\pi\lambda}{N_S})$$

$$\beta = \arctan(-\frac{\delta}{\lambda}\tan\frac{\pi\lambda}{N_S})$$

and $\theta_k^s = (\theta_{2k+1} + \theta_{2k})/2$ is the azimuth of the central snake. In the limit when the spin resonance ε goes to zero the anti-diagonal element b_k in matrix M_k goes to zero as well as its trace (with half integral spin tune).

Due to the periodicity in ϕ_k, the only variation in matrix M_k from one group of two snakes and arcs to the next comes from θ_k^s. Hence, the depolarization may be enhanced by the phase variation of θ_k^s. To see this, we introduce a new matrix N_t such that

$$N_{t+1} = M_t\ N_t$$

and N_0 is the unit matrix. If the particle does not depolarize due to the spin resonance then the anti-diagonal component of N_t should remain small. Denoting:

$$N_t = \begin{bmatrix} Z_t & -W_t \\ W_t^* & Z_t^* \end{bmatrix}$$

then

$$Z_{t+1} = a_t Z_t - b_t W_t^*$$

$$W_{t+1} = a_t W_t + b_t Z_t^*.$$

To determine the effect of the phase due to θ_k^s, we must solve the above difference equation for W_t. This can be solved to an odd order in the spin resonance strength as shown by the following expansion:

$$W_t^{(n)} = W_t^{(1)} - \sum_{k_1=1}^{t-1} \sum_{k_2=1}^{k_1-1} b_{k_1} b_{k_2}^* W_{k_2}^{(n-1)} \prod_{m=k_1+1}^{t-1} a_m \prod_{m=k_2+1}^{k_1-1} a_m^*$$

where

$$W_t^{(1)} = \sum_{k=1}^{t-1} b_k \prod_{m=k+1}^{t-1} a_m \prod_{m=1}^{k-1} a_m^*.$$

This solution by itself is not very transparent. We simplify this by writing

$$b_k = b\,(\eta_+ e^{i4\pi k\kappa/N_S} + \eta_- e^{-i4\pi k\kappa/N_S})$$

where b is a real constant, η_+ and η_- are complex constants and $\theta_k^s = 4\pi k/N_S$. When $\sin\mu$ is much smaller than $\cos\mu$ then (i.e. a weak spin resonance)

$$a_k \cong -e^{i2\pi\nu_s/N_S}$$

the anti-diagonal element $W_t^{(1)}$ becomes

$$W_t^{(1)} \cong b \cos^{2t-4}\mu \; e^{i2\pi t\nu_s/N_s} \sum_{k=1}^{t-1} \left[\eta_+ e^{i4\pi k(\kappa-\nu_s)/N_s} + \eta_- e^{-i4\pi k(\kappa+\nu_s)/N_s} \right]$$

Note, if

$$\text{integer} = \frac{2}{N_s} (\kappa \pm \nu_s)$$

then $W_t^{(1)}$ cannot remain small far from the resonance where the n vector is vertical. Thus, the polarization is destroyed. This is the lowest order snake resonance.

There are many other snake resonance due to the higher order terms in W_t. The most important of these higher order snake resonances come from the higher order terms in a_k that were dropped when expanding $W_t^{(1)}$. We can see the effect of these terms by regrouping the summation as follows

$$W_{pt}^{(1)} = \sum_{k=1}^{t-1} \prod_{m=p+kp}^{t-1} a_m \prod_{m=1}^{pk-1} a_m^* \left[\sum_{n=0}^{p-1} b_{n+pk} \prod_{m=n+1}^{p-1} a_{m+pk} \prod_{m=0}^{n-1} a_{m+pk}^* \right]$$

where the second sum includes terms up to order 2p-1 in spin resonance strength. In trying several values of p in $W_{pt}^{(1)}$ we find higher order snake resonances of the form

$$\text{integer} = p \frac{2}{N_s} (\nu_s \pm m\kappa)$$

where m is an odd integer from 1 to 2p-1. Tracking studies have shown that p must be an odd integer for depolarizing the beam.

Conclusion

There are three classes of spin resonances in large proton accelerators, imperfection resonances where $\kappa=k$, and two intrinsic resonances where $\kappa=k\pm\nu$. When Siberian snakes are added to such an accelerator, the spin tune can be chosen such that one particular class of these resonances do not excite snake resonances. Then, the remaining two classes will excite a set of energy independant snake resonances. In practice, we choose $\nu_s = 1/2$, which will avoid any snake resonances that could be excited by the imperfection resonances. In this case we must carefully choose the vertical betatron tune ν in order that the two classes of intrinsic resonances are not operating on a snake resonance.

The snake resonances can be observed using a spin tracking program[10]. An accelerator containing two Siberian snakes with a spin tune of 1/2 and a single intrinsic spin resonance with $\varepsilon=1.93$ has been tracked for various values of the fractional part of the vertical tune ν. The results are shown in Fig. 2. The dips agree with the places given by the snake resonance formula. Some plots from the tracking program are shown in Fig. 3-4. Yokoya[11] has shown that these results are dependent on the strength of the spin resonance as well.

There is still plenty of work that needs to be addressed in an accelerator with Siberian snakes. For instance, there is no theory for the case involving more than one spin resonance. The spin equation for the arcs can no longer be solved analytically. Furthermore, all three classes of spin resonances are present and they may interact in ways we presently cannot forsee.

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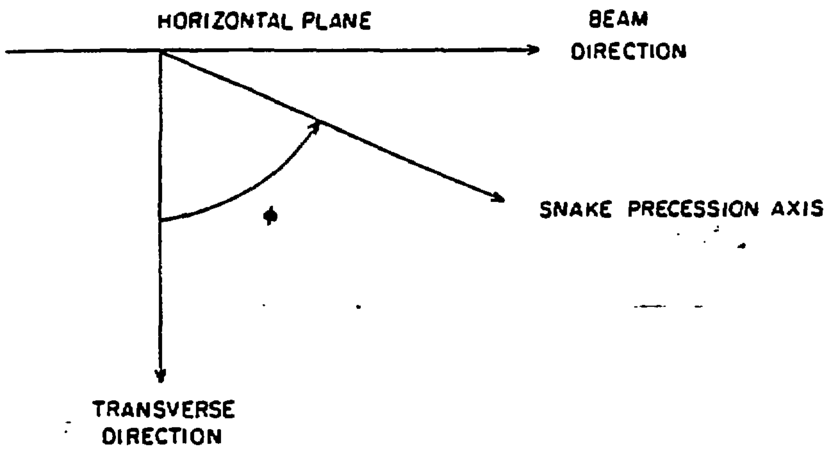

Figure 1. The snake precession axis.

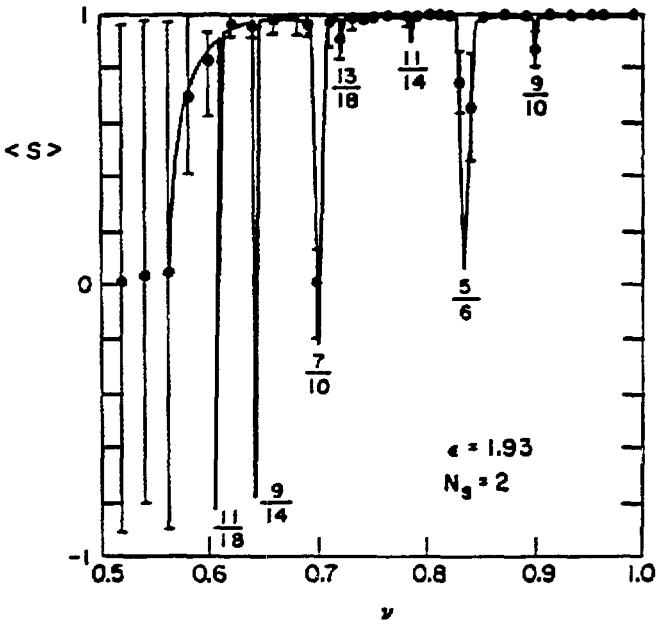

Figure 2. Polarization versus the fractional part of vertical betatron tune.

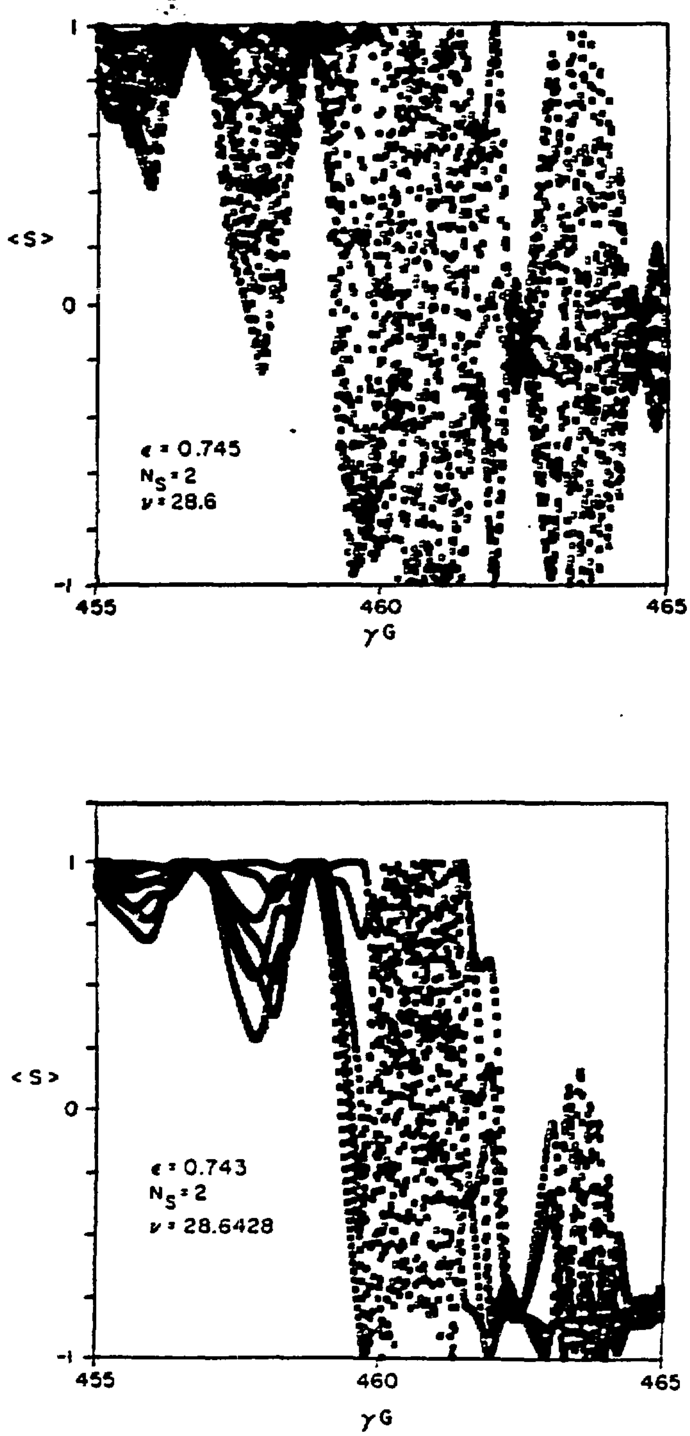

Figure 3. Tracking of the vertical component of the spin versus γG I.

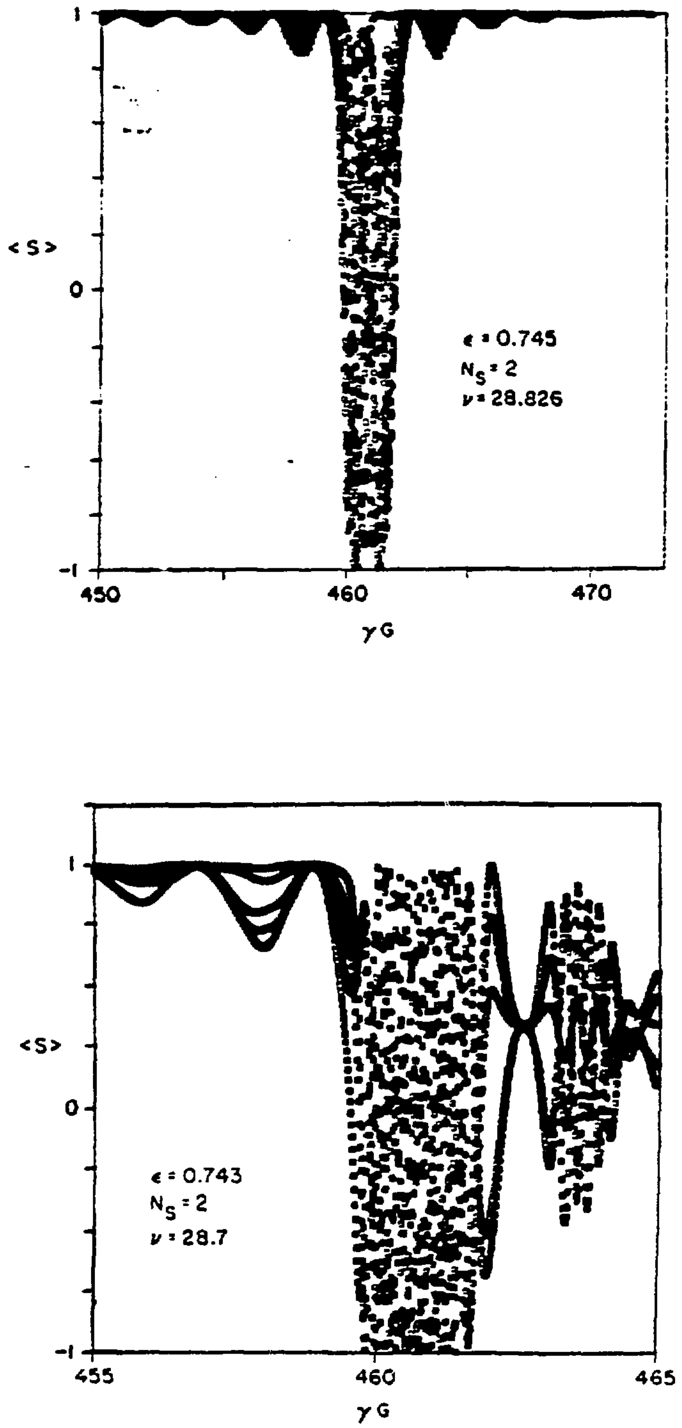

Figure 4. Tracking the vertical component of the spin versus γG II.