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Bistability in a laser with injected signal

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ABSTRACT : A unified description of bistability is given in free running lasers, optical bistable devices, ring lasers and lasers with an injected signal (LIS). A general review of laser instabilities is also presented in the frame of the theory of elementary catastrophes, emphasizing the appearance of higher order catastrophes in the case of a LIS suggesting thus a possibility to devise from first principles the whole hierarchy of laser instabilities. Experimental results on the bistability in the polarization of LIS are also discussed.

1. Introduction

The optical bistability refers to the coexistence of two stable steady states at the output of an optical device. Optical bistability is of great interest due to both the fundamental physics involved in its study as the result of a nonlinear interaction of the electromagnetic field with matter (atomic systems) and its potential use for building smaller, faster and parallel working optical logic devices.

This phenomenon was a useful contributor to the general theory of synergetics and extending the stability analysis has added impetus to the already active research on chaotic behaviour in physical, chemical and biological systems. It is very interesting that, despite the very large number of processes which lead to optical bistability, there is a general physical model which can describe the dynamics of this phenomenon -the generalization of the Haken laser model introduced by Bonifacio and Lugiato. For this reason, free-running lasers, passive optical bistable devices, ring lasers and lasers with an injected signal can all be described in an unified way by using small variations of the same model. The optical devices are ideally suitable for modelling much more complex phenomena, as for instance the Lorenz instability from hydrodynamics, cooperation in dissipative systems and so on.

On the other hand, it now appears that optical bistable devices are likely to operate in picosecond (to femtosecond) and microwatt range and devices a few hundred microns in size have already been tested. Their use in future logical components is much discussed at annual meetings dedicated to optical bistability, ultrafast technology and optical computing.

Considering the importance of the active bistable devices, in this lecture we wish to give a broad treatment of bistability in a laser with an injected signal (LIS). This simple configuration, previously used for heterodyning and frequency stabilization, shows the power of the most complicated optical bistable systems. It is easy to be integrated and matched to various laser structures. It can maintain a constant logic level and yield two complementary outputs. All these properties recommend it as a most serious candidate for future optical logic circuits operating at very high speed.

The plan of our lecture is as follows:

- An unified description of bistability in lasers and optical devices: free-running lasers, optical bistable devices, ring lasers and lasers with an injected signal; review of instabilities.
- The study of bistability in the laser with injected signal: transformation of the rate equation in the cusp-type equation; control parameters for bistable operation; absorptive and dispersive LIS; similarity of LIS and ring laser.
- Polarization bistability in a LIS: field equations in terms of Jones vectors; coupling the field components using the hierarchy in the cusp lines; representation of the bistable solutions; experimental verification.
- Conclusions - some reflections on the optical logic, use of bistable LIS in future logical components and use of LIS for modelling complex instabilities.

2. An unified description of bistability in lasers and optical devices.

The laser and various optical devices are almost ideal for the study of nonlinear dynamical problems due to the fact that the resonance condition in the field-matter interaction in the cavity, forces a drastic selection of the number of active degrees of freedom. For this reason, free-running lasers, optical bistable devices (OBD), ring lasers (RL) and lasers with an injected signal (LIS) can all be described in an unified way by using small variations of the same single-mode model. As the steady-state solutions of this class of models exhibit all bistable characteristics, the unified approach must show the special conditions imposed to lasers and optical devices for the description by single mode equations, the form and range of the control parameters to obtain bistable operation and stability of the solutions.

2.1. The single-mode (homogeneously broadened*) laser model

This model was formulated by Haken and his school /1,2/. Starting from the semiclassical (Lamb) laser theory, it was possible to obtain a description in terms of a few collective variables - the complex slowly-varying field amplitude, E (related to the boson ensemble), the complex macroscopic polarization, P (related to the ordered induced atomic dipoles), and the saturated population difference (inversion), D (related to the finite ensemble of two-level atoms). The dynamics of the field can be described by the Maxwell equation for E which, in the slowly-varying amplitude approximation and normalization and with proper boundary conditions, takes the form:

$$\dot{E} = -K(E - AP) \qquad (2.1a)$$

The two-level atoms resonantly coupled with the field are described by normalized Bloch-like equations (in a spin 1/2-representation, like for spin precession in presence of a magnetic field):

$$\dot{P} = -\gamma_{\perp}(P - ED) \qquad (2.1b)$$

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* Homogeneous broadening means that all the atoms in the active medium have the same transition frequency. An approximative equivalent situation appears in inhomogeneous broadened single-mode lasers close to the threshold

and $$\dot{D} = - \gamma_{\parallel} (D + EP - 1) \tag{2.1c}$$

In eqns. (1), there are three damping constants:

K - the field damping constant (the halfwidth of the cavity line);

$\gamma_{\perp}$ - atomic damping rate related to spontaneous emission (halfwidth of the gain line)

$\gamma_{\parallel}$ - the atomic damping rate related to collisions among atoms,

and a pump parameter, A, related to the gain of the laser medium.

This model can be shown to be equivalent to the now famous one due to Lorenz in fluid dynamics, the first model on which chaotic behaviour has been studied /3/. The laser has always the trivial solution $E = P = 0$, $D = 1$, which becomes unstable for $A > 1$. Beyond the laser threshold, $A_{thr} = 1$, we find two stationary solutions

$$E = \pm (A - 1)^{\frac{1}{2}}$$
$$P = \pm (A - 1)^{\frac{1}{2}}/A \tag{2.2}$$
$$D = 1/A$$

If two conditions are met together, namely

$$K > \gamma_{\perp} + \gamma_{\parallel} \tag{2.3a}$$

and $$A > A_c = 1 + (K+\gamma_{\perp}+\gamma_{\parallel})(K+\gamma_{\perp})/\gamma_{\perp}(K-\gamma_{\perp}-\gamma_{\parallel}), \tag{2.3b}$$

the stationary solutions become unstable. A_c is called the second laser threshold. For $A > A_c$, the laser output is not stationary even if the laser is pumped continuously; it oscillates spontaneously in time, with a behaviour called self-pulsing (undamped and often chaotic oscillations). The instability conditions (3a) and (3b) require laser operation with a bad (lossy) cavity and high above the threshold ($A_c \geq 9$), which makes them difficult to be observed (with few, not very clear, exceptions, like e.g. *FIR lasers* and hard excitation).

Actually, Haken eqns.(2.1) are similar to the Lorenz equations up to the scale of the damping rates, which in the *last case have the following orders of magnitude:*

$$K \sim 1 \div 10; \ \gamma_{\perp} \sim 1; \ \gamma_{\parallel} \sim 8/3; \ \ A \sim 1$$

According to the above mentioned three damping rates, Arecchi et al. /4/ introduced the following laser classification:

Class A (e.g. He-Ne, Ar, dye): $\gamma_\perp \gtrsim \gamma_\parallel >> K$

The equations for P and D can be solved at equilibrium (adiabatic elimination) and only the nonlinear field equation describes the laser. The number of degrees of freedom, N = 1, means that we have a fixed point attractor, hence coherent emission.

Class B (e.g. ruby, Nd, CO_2): $\gamma_\perp >> K \gtrsim \gamma_\parallel$

Only P can be adiabatically eliminated and the laser dynamics is governed by the equations for field and inversion. N = 2 allows also for period oscillations or bistability (a fixed point and a limit cycle).

Class C (e.g. FIR lasers): $\gamma_\perp \simeq \gamma_\parallel \simeq K$

The complete set of eqns. (1) allows also for Lorenz-like chaos (N = 3). In order to increase the number of degrees of freedom to $N \geq 3$, one can inject an external signal in a three-mirror configuration or a ring configuration, or modulate the internal losses with a feedback signal provided by the output intensity.

The scale of the damping rates of a visible transition in a diluted gas (such as in He-Ne, Ar^+ lasers) is:

$$K \sim 3.10^6\, s^{-1} \text{ (for a cavity 1 m long with 1\% losses)}$$

$$\gamma_\perp \sim \gamma_\parallel \sim 10^8\, s^{-1}$$

$$A \sim 3.10^{-3}$$

which explains why it is not possible to see Lorenz -type instabilities in these lasers.

2.2. Optical bistable cavity (OBC)

Supposing homogenous broadening and single mode operation in a (Fabry-Perot) cavity filled with a nonlinear (passive) optical material as in Fig. 1, the dynamics of the device can be described by the following model introduced by Bonifacio & Lugiato* /5/

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* Starting from Maxwell-Bloch eqns., in the mean-field limit (i.e., considering the field concentrated along the propagation axis and very small losses: $T \to 0$; $\alpha L \to 0$, but $\alpha L/2T = C$ = constant, see Fig. 1).

$$\dot{E} = -K[E(1+i\theta) + 2CP - E_1] \qquad (2.6a)$$

$$\dot{P} = -\gamma_{\perp}[P(1+i\Delta) - ED] \qquad (2.6b)$$

$$\dot{D} = -\gamma_{\parallel}[D + (1/2)(E^*P + EP^*) - 1], \qquad (2.6c)$$

where the new parameters are:

ω_c - the frequency of the cavity mode;

ω_1 - the frequency of the input (laser) field E_1;

ω_a - the frequency of the atomic transition from the cavity;

$\theta = (\omega_c - \omega_1)/K$ - the cavity detuning parameter;

$\Delta = (\omega_a - \omega_1)/\gamma_{\perp}$ - the atomic detuning parameter;

$C = \alpha L/2T$ - the (bistability) control parameter.

The system of five equations (6 a-c, E and P complex functions) is a generalization of the single-mode laser model (Haken). Assuming

$$E_1, \theta, \Delta = 0, \qquad 2C = -A \qquad (\text{and } E, P \in \mathbb{R}) \qquad (2.7)$$

the Bonifacio-Lugiato model goes over into the Haken model.

The stationary solution of eqns. (6), obtained when $\dot{E} = \dot{P} = \dot{D} = 0$ can be written in the form

$$|E_1|^2 = |E|^2\left[\left(1 + \frac{2C}{1+\Delta^2+|E|^2}\right)^2 + \left(\theta - \frac{2C\Delta}{1+\Delta^2+|E|^2}\right)^2\right] \qquad (2.8)$$

where eqns. (6 b,c) have been used to eliminate the variables P and D. This standard bistability function is shown in Fig. 2 for various values of the control parameter, C. When $C > C_{min}(\Delta, \theta)$, the stationary curve of OBC transmission is S-shaped, yielding bistable operation. In the resonant case

$$\theta = \Delta = 0, \qquad C_{min} = 4 \qquad (2.9)$$

The transient behaviour of OBC can be analysed by studying the stability of the stationary solution (8) against small perturbation at various values of the control parameters. It was shown /6, 7/ that, by the control of OBC, one can force a segment of positive slope of the stationary curve to become unstable, leading to oscillation in the transmitted intensity.

For resonance, $\theta = \Delta = 0$, no positive-slope instability exists.

Out of resonance, the instability appears in either the absence or presence of bistability. When $\theta = -\delta$ (pulling condition) and $C > 150$, the oscillations are almost sinusoidal, with a frequency of the order of the cavity damping constant, K (i.e. we have a device which converts c.w. into pulsed radiation, simultaneously acting as an optical clock).

Unc.r appropriate conditions, Ikeda /8/ s.owed that the spontaneous oscillations yielded by the instability are not periodic but chaotic. Recently, Orozco et al. /9/ produced a first experimental proof of a Lorenz-type instability in homogeneously broadened systems in good semi-quantitative agreement with the theory. In order to better observe period doubling and chaos, it is necessary to design an experiment with uniform transverse field.

2.3. Laser with an injected signal (LIS)

Lugiato et al. /10/, Arecchi et al. /11/ and Scholtz et al. /12/ showed very recently that the well known device /13/ called LIS and presented in Fig.3 is an exact active counterpart of OBC. In the single-mode operation, LIS is governed by a set of equations identical to eqs. (6), apart from the replacement of the bistability parameter (2C) with the pump (-A).

Reconsidering the stationary (equilibrium) solution from eq.(8), one can write in this case:

$$E_1 = |E| \left[\left(1 - \frac{A}{1+\Delta^2+|E|^2}\right)^2 + \left(\theta + \frac{A\Delta}{1+\Delta^2+|E|^2}\right)^2 \right]^{1/2} \qquad (2.10)$$

and see again the bistable characteristics (Fig.4).

The linear stability analysis shows that the operating point corresponding to $E_1 = 0$ (no injection) is stable and for some values of the control parameters, the entire segment from origin to point A (not only the position with negative slope) is unstable against self-pulsing /10/. We emphasize that this result is in agreement not only with OBC but also with the theory of the ring laser, which will be presented in the following section. The instability of LIS can be explained by the competition between the internal and injected oscillations ($\omega_L \neq \omega_1$). Thus, when the injected signal E_1 is very small, the competition can produce some type of oscillations and for an appropriate choice of parameters, a chaotic output. When E_1 is increased enough, it "locks" the laser for an operation at ω_1 (injection locking).

It is important to stress again on the fact that injecting an external signal into a single-mode laser provides an additional degree of freedom and this makes possible to readily and conveniently observe theoretically and experimentally, bistability and instabilities, otherwise very difficult to be tackled in free-running lasers or CBD.

2.4. The ring laser (RL)

This device was the horse on which many scientists looking for laser instabilities rode since they discovered the difficulties to find them in a free-running laser. A possible scheme of RL is shown in Fig.5. Bonifacio and Lugiato, solving the Maxwell-Bloch equations in the mean-field limit and with corresponding boundary conditions, found for it the stationary bistable solution /14/.

$$E_1 = E + 2\,CE/[1 + |E|^2\,] \qquad (2.11)$$

Here, a general mean-field limit is used:

$$|E_1|^2 \to 0; \quad |E|^2 \to 0; \quad |E_1|^2/T = \text{const.}; \quad |E|^2/T = \text{const.}$$

$$(2.12)$$

$$\theta \; = -[\,k_o(L + \ell\,) + 2\varphi_R\,]\ \text{mod.}\ 2\pi \to 0; \;\; \theta/T = \text{const.}$$

As will be shown in Section 3, RL is very similar with LIS, after all it implies a double injection into the laser cavity.

A more complex configuration, the bidirectional ring laser was taken into consideration by Arecchi et al. /15/. The two counterpropagating beams cannot work at the same time, because they must compete for the same amount of population inversion. They are also slightly detuned between each other and with respect to the line center due to the intrinsic asymmetries in the cavity losses ($K_1 \neq K_2$). Through the grating induced in the population inversion by the interference of the two waves, an exchange of energy from one field to the other appears by backscattering, very similar to LIS (here the injection comes from the counterpropagating mode). Denoting:

x, y - the two complex fields;

z - the (real) spatially uniform component of population inversion;

w - the complex amplitude of the grating induced in the inversion;

δ - cavity detuning;

z_0 - pump parameter,

with suitable normalization and eliminating adiabatically the polarization, Arecchi wrote a system of seven equations

$$\begin{aligned} \dot{x} &= (zx + w^* y) / (1 + i\delta) - x \\ \dot{y} &= (zy + wx) / (1 + i\delta) - (K_2/K_1)\, y \\ (K_1/\gamma_\parallel)\dot{z} &= z - z_0 + [\, z(|x|^2 + |y|^2) + w^* x^* y + wxy^* \,] / (1 + \delta^2) \\ (K_1/\gamma_\parallel)\dot{w} &= -w - [\, zx^* y + w(|x|^2 + |y|^2)] / (1 + \delta^2) \end{aligned} \tag{2.13}$$

which can display a wealth of unstable behaviours.

2.5 Instabilities in lasers and optical systems

In the previous analysis, we have worked mainly on the stationary solution of the Maxwell-Bloch equations. A general solution of these equations must be of the form:

$$\vec{\mathcal{A}}(z,t) = \vec{\mathcal{A}}_{st}(z) + \Sigma_\lambda \ \vec{A}_\lambda(z)\, e^{\lambda t} \tag{2.15}$$

where $\vec{\mathcal{A}} \equiv (E, E^*, P, P^*, D)$, $\vec{\mathcal{A}}_{st}$ means the stationary solution, $\vec{A}_\lambda$ is the spatially dependent mode corresponding to eigenvalue λ . The linear stability analysis of (linearized) Maxwell-Bloch equations for feedback systems implies to solve some matrix equations as in the following:

$$[A_\lambda(L)] = [M(\lambda)]\ [A_\lambda(0)]$$

$$[A_\lambda(0)] = [M_0(\lambda,\theta)]\ [A_\lambda(L)] \qquad 2.16)$$

so that the characteristic equations can be written as:

$$\mathrm{Det}\ [M_0(\lambda,\theta)][M(\lambda)] = 0 \qquad (2.17)$$

The analysis of the instabilities was done by different authors for absorptive and dispersive systems under some special conditinns /16/

(a) - mean-field multimode limit: $\gamma\tau_r$=const; $k/\gamma = T/\gamma\tau_r \to 0$ (2.18)

(b) - mean-field monomode limit: $\gamma\tau_r \to 0$; $k/\delta = T/\gamma\tau_r$=const. (2.19)

(c) - outside mean-field limit,

($\tau_r = (L + \ell)/c$ is the cavity round trip time).

We shall review the principal results gained from this stability analysis.

<u>Case a: Absorptive systems</u> /17-19/- Re $\lambda_{1,n}$ may be positive for $n \neq 0$ along a part of the upper branch of the bistability curve, leading to oscillations; no chaos.

<u>Dispersive systems</u> /20/- *P eliminated adiabatically;* Re $\lambda_{1,n}$ *may be positive* for $n \neq 0$ along the upper branch of the bistability curve; no oscillations were reported.

<u>Case b: Dispersive systems</u> /21-22/- instability along some part of the upper branch; oscillations, period-doubling to chaos.

<u>Dispersive systems, the dispersive limit</u>* /23/- *instability along part of the* upper branch leading to period-doubling and chaos (Ikeda instability).

<u>Case c: Absorptive systems</u> /24/ - for $\Delta = 0$, $|\theta| < \frac{\pi}{2}$ (in particular $\theta = 0$, tuned cavity), the bistability occurs always with stable upper and lower branches; for $|\theta| \in [\frac{\pi}{2}, \pi)$, *there is no bistability, but periodic oscillations between* saturated and unsaturated states with period $2\tau_r$; no chaos.

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* The dispersive limit means that the incident laser field is tuned far from the resonance, so that the nonlinear absorption can be neglected. *This implies the* use of the low intensities for overcoming the significant power broadening compared to this detuning.

Dispersive systems /8/- P, D adiabatically eliminated; instability along some part of the upper branch of the bistability curve, leading to periodic oscillations, period-doubling and chaos.

Dispersive systems, the dispersive limit /25/- instability in the dispersive bistability associated to periodic oscillations, period-doubling and chaos, which were experimentally observed in hybrid /26/ and all - optical /27, 28/ systems (Table 1).

3. The study of bistability in the laser with an injected signal (LIS). Similarity of LIS and RL

It is well known that the injection of an external signal into a laser allows obtaining a stable output even if the laser itself is unstable. The situation is somewhat reminiscent to that of a system of spins above the Curie temperature put in an external magnetic field.

On the other hand, as emphasized in the previous Section, the injected signal can provide the additional degrees of freedom which, by suitable manipulation, make possible the observation of the wealth of instabilities including deterministic chaos.

We have considered the particular case of a laser coupled to a bistable cavity and used as injected signal the intensity I_B in this cavity. We were thus able to observe a bistable behaviour of the laser. It is interesting to note that a careful analysis of the equation shows in a very simple and elegant way, by means of catastrophe theory, that bistability is indeed to be expected in this case. We can show in this way that the problem of a LIS can be reduced to the general problem of bistability, as discussed for instance in the works of Lugiato and Arecchi /10,11,14,15/.

3.1. LIS with a nonlinear external cavity

We start with the equation for the light intensity, I_B in an external cavity filled with a nonlinear absorbing optical material (Fig.3)

$$\dot{I}_B = - \quad I_B(1 + I_B/I_{SB})^{-1} + cL_B^{-1}\ln R \, . \, I_B + cL_B^{-1}(TI_L I_B)^{\frac{1}{2}} \tag{3.1}$$

where I_L is the injected light intensity from the coupled laser.

Here, the parameters are:

α - cavity absorption coefficient

$I_{SB}(I_{SL})$ - saturation field intensities in the cavity (laser)

c - speed of light

$L_B(L)$ - external (laser) cavity length

R(T) - intensity reflection (transmission) of the mirrors (Fig.7)

If we now denote:

$$x^2 = I_B/I_{SB}, \quad y^2 = I_L/I_{SL}$$

$$A = \alpha I_{SB};\ B = cL_B^{-1}\ \ln R\ .\ I_{SB};\ C = L_B^{-1}\ (T\ I_{SL}\ I_{SB})^{\frac{1}{2}} \cdot c \qquad (3.2)$$

then equation (3.1) can be written in the stationary case ($dI_B/dt = 0$) as:

$$-Ax^2/(1 + x^2) + Bx^2 + Cxy = 0 \qquad (3.3)$$

Since $x \neq 0$, $1 + x^2 \neq 0$ we can write it even as

$$Bx^3 + Cyx^2 + (B - A)x + Cy = 0 \qquad (3.4)$$

By a change of variables,

$$x = z - Cy/3B \qquad (3.5)$$

this equation is transformed into:

$$z^3 + [A/B - 1 - (C^2/3B^2)y^2]z + 2C^3y^3/27B^3 + C(A + 2B)/3B^2 . y = 0 \qquad (3.6)$$

This equation is equivalent with the standard one obtained by Bonifacio and Lugiato /14/ in the case of a bistable ring laser, and, moreover, it is nothing else but the cusp equation familiar in Catastrophe Theory. What we can immediately say in this way, concerning the behaviour of z is that it will exhibit bistability as soon as the coefficient of the linear term becomes negative. This settles a threshold value for the variable y which we write in the form:

$$y^2_{thr} = 3B^2/C^2\ (A/B - 1) \qquad (3.7)$$

Going back to the original quantities, we thus obtain the "critical" value for the field intensity inside the laser cavity (I_L) in order that the field behaves in a bistable way:

$$I_L \geq I_{Lthr} = I_{SL}\ L_B^{-2}\ 3(\ln R)^2 I_{SB}(\alpha L_B/c\ln R - 1) \qquad (3.8)$$

That this is indeed so is quite transparent from the aspect of the cusp surface (Fig.6). Indeed we can write eqn. (3.6) in the form

$$z^3 + Pz + Q = 0 \qquad (3.9)$$

where the two control parameters, P and Q, have the obvious expressions

$$P = A/B - 1 - (C^2/cB^2)y^2$$

$$Q = 2C^3y^3/27B^3 + C(A + 2B)/3B^2 . y$$

If we consider sections P = const. of this surface, we easily see that for P < 0 (i.e. $y > y_{thr}$) eqn.(3.9) has three real solutions, of which two correspond to stable states of the laser and one is unstable (the dotted lines in Fig.6c). As soon as the variation of the coefficient Q brings the representative point of the system (the state of the laser) in positions T or T' (also called nontransversality points), it will jump from one stable sheet to the other one, which clearly exhibits the bistable character of the laser behaviour in this case.

A further comment is here in order: one can easily show that (3.3) is an average field equation, of the type obtained by Bonifacio and Lugiato and already described:

$$y = x + 2Cx(1 + x^2)^{-1} \tag{3.10}$$

which means that the bistability of a ring laser is typically an average field effect, exactly as we would have expected from the catastrophe theory analysis. Since the behaviour exhibits a cusp singularity, the phenomenon we observe at the critical point (3.7) can be described in terms of the phenomenological Landau model of second order phase transitions which them too are of average-field type.

Moreover, it is well known that systems exhibit phase transitions in the thermodynamic limit $N \to \infty$, $V \to \infty$, $N/V = \rho$ = const., where N is the particle number, V the system volume and ρ the density. Equation (3.10) is indeed valid in the limit $\alpha L \to 0$, $T \to 0$, $\alpha . L/2T = C$, where αL is the total absorption along the laser length and T the transmissivity factor. The minimum value of the constant is $C_{min} = 4$ in the case of zero atomic and cavity detunings. We would like to stress the fact that this limit can be interpreted as an "anti thermodynamic limit" (numerator and denominator tend to zero, rather than to infinity) which makes plausible the characterisation sometimes made of the laser as a zero-dimensional system.

If, instead of a nonlinear absorbing material in the external cavity, we introduce a nonlinear dispersive optical material, the changes in the index of refraction due to the intensity of the incident light will modify the optical length of the external cavity. Finally, these changes lead to a detuning between the resonance frequency of the external cavity, ω_1, and the laser frequency, ω_L:

$$\delta = \omega_L - \omega_1/(c/2L_B) + q(I_B/I_{SB})\,(q = \text{constant}) \qquad (3.11)$$

amounting to *instability* of the LIS output. The dynamics of the external cavity will dominate the process*) and can be described /29/ by rate equations for the intensity I_B and the relative phase shifts, φ, of the fields in the two cavities:

$$\dot{I}_B = -(c/L_B)\,[(1 - R\cos\delta)I_B - \sqrt{T}\sqrt{I_B I_L}\cos\varphi\,] \qquad (3.12a)$$

$$\dot{\varphi} = (c/L_B)\,(R\sin\delta - \sqrt{T}\sqrt{I_B/I_L}\sin\varphi) \qquad (3.12b)$$

These equations are similar to those written by Lugiato et al. /10/ for a ring cavity and by Arecchi et al. /11/ for a LIS-Class B (considering $\theta = 0$). If we consider the steady-state solution of eqns. (3.12) and eliminate φ, we obtain the following relation between I_L and I_B:

$$I_L = 2I_B\,(1 - R\cos\delta)/T \qquad (3.13)$$

which, at small δ and taking into account (3.11), gives

$$I_B^3 + \frac{2\Delta\omega_L}{\bar{q}}\,I_B^2 + \frac{2(1-R)+R\Delta\omega_L^2}{R\bar{q}^2}\,I_B - \frac{T}{R\bar{q}^2}\,I_L = 0\,, \qquad (3.14)$$

which obviously is of the cusp-type (3.4).

We have denoted here:

$$\Delta\omega_L = (2L_B/c)(\omega_L - \omega_1) \text{ and } \bar{q} = q/I_{SB}.$$

By a suitable change of variables, we can straightforwardly write (3.14) in the form (3.6) which gives us thus the value of the threshold above which bistable behaviour occurs: $\Delta\omega_{Lthr}^2 = 6(1-R)/R$.

We shall not enter into any details here but refer to a forthcoming publication in which we undertake an analysis of the relevant control parameters in various cases.

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* I_L can be adiabatically eliminated and the field amplitude in the external cavity can be written as $E_B = \sqrt{I_B}\,e^{i\varphi} = \sqrt{I_B}(\cos\varphi + i\sin\varphi) = x_1 + ix_2$.

3.2. LIS with nonlinear coupling inside the laser cavity and empty external cavity

Starting from the Maxwell-Bloch equations for a single mode homogeneously broadened laser (with detuning) /30/, we can describe a LIS by the following three equations, after the adiabatic elimination of the polarization:

$$\dot{I}_L = 2K\left[I_L z(1+\delta^2)^{-1} - I_L + \sqrt{I_L I_1}\cos\varphi \right]$$

$$\dot{\varphi} = K\left[-\theta - \delta z(1+\delta^2)^{-1} - \sqrt{I_1/I_L}\sin\varphi \right] \qquad (3.15)$$

$$\dot{z} = \gamma\left[z_0 - z - zI_L(1+\delta^2)^{-1} \right],$$

where θ and δ are defined with reference to the external frequency ω_1. The stationary (equilibrium) solution of eqns. (3.15) is of the form

$$I_1 = \bar{I}_L\left\{ \left[1 - z_0/(1+\delta^2+\bar{I}_L)\right]^2 + \left[\theta + \delta z_0/(1+\delta^2+\bar{I}_L)\right]^2 \right\} \qquad (3.16a)$$

i.e. the standard bistability function, with the other variables

$$\bar{\varphi} = \arccos\left\{ \sqrt{\bar{I}_L/I_1}\left[z_0(1+\delta^2+\bar{I}_L)^{-1} - 1 \right] \right\} \qquad (3.16b)$$

and

$$\bar{z} = z_0(1+\delta^2)(1+\delta^2+\bar{I}_L)^{-1} \qquad (3.16c)$$

If the external mirror is uniformly translated with a triangular voltage:

$$\delta = \pm(2\omega_\ell v/c)/\gamma_\perp = \pm\omega_D/\gamma_\perp \qquad (3.17)$$

and $z_0 \ll \delta^2$, then eqn.(3.16a) takes the form

$$I_1 = \bar{I}_L\left\{ 1 + \left[\theta \pm \omega_D z_0/\gamma_\perp (1 + \omega_D^2\gamma_\perp^{-2} + \bar{I}_L) \right]^2 \right\} \qquad (3.18)$$

With the following notations

$$x = I_L; \qquad y = I_1$$

$$a = \theta\gamma_\perp;\quad z_\pm = a + \theta\omega_D/\gamma_\perp \pm \omega_D z_0;\quad b = 1 + \omega_D^2/\gamma^2;\quad \pm\omega_D = \omega_1 \pm \omega_\ell$$

we can write eqn. (3.18) in the form

$$(1+\theta^2)x^3 + (2b + 2z_\pm\theta/\gamma_\perp - y)x^2 + (b^2 + z_\pm^2/\gamma_\perp^2 - 2by)x - b^2y = 0 \qquad (3.19)$$

For typical values of the laser parameters we can take

$$\theta^2 \ll 1, \quad \theta^2/\gamma_\perp^2 \ll 1, \quad \omega_D^2/\gamma_\perp^2 \ll 1, \quad z_0 \simeq 1, \quad b \simeq 1 \qquad (3.20)$$

and $2\theta z_\pm/\gamma_\perp \simeq \pm 2\theta\, \omega_D/\gamma_\perp$.

Consequently, we have

$$x^3 + (2 - y \pm 2\theta\, \omega_D/\gamma_\perp)x^2 + (1 - 2y)x - y = 0 \qquad (3.19')$$

To transform eqn. (3.19') in the standard cusp form, $z^3 + Pz + Q = 0$, the coefficients P and Q have here the expressions (A and B are the coefficients of x^2 and x, respectively, and $C = -y$):

$$P = B - A^2/3$$

$$Q = 2A^3/27 - BA/3 + C.$$

We shall consider the critical case, $P = 0$ and look for the threshold value of y.

From the condition $A^2 = 3B$ we get (by systematically neglecting terms with $\theta^2/\gamma_\perp^2$)

$$y_{thr} = \mp \theta\omega_D/\gamma_\perp \qquad (3.21)$$

Under the same condition

$$Q = 2A^3/27 - A^3/9 - y_{thr} = -A^3/27 - y_{thr}$$

Calculating $A^3 = A^2.A = 2(2 + 3\theta\;\; \omega_D/\gamma_\perp\;)$, we obtain

$$Q = -4/27 \mp 11/9\;\; \theta\omega_D/\gamma_\perp \lessgtr 0 \qquad (3.22)$$

which confines the behaviour to the lower /upper branch of the cusp, according to the value of $\pm\omega_D$. This formulation of LIS presents the important advantage of having the coefficients (control parameters) expressed as functions of quantities which are of a direct experimental access.

4. Polarisation bistability in a laser with injected signal

4.1. Polarisation switching between two LIS modes coupled via a nonlinear medium

We shall consider the switching of polarisation*) in the case of two single modes of a LIS laser. The coupling of the modes is realized via a nonlinear medium. We suppose that the mean field limit holds and (P, D) are adiabatically eliminated (Class A laser).

As a result, the complex fields of the two modes satisfy a system of coupled equations

$$\dot{E}_{1,2}/k = (1/\sqrt{T})\exp(i\varphi_T)E_I - E_{1,2}\left\{1 + i\varphi_{1,2} + i(1/T)(K_{1,2}L/2)\,\chi_{1,2}\right\} \qquad (4.1)$$

where: 1,2 are the two modes of frequencies $\omega_{1,2}$

$\chi_{1,2}$ are the susceptibilies

φ_T is the phase change due to mirror transmission

$K_{1,2} = \omega_{1,2}/c$

$\varphi_{1,2} = \theta/T = -([K_{1,2}(L+\ell) + 2\varphi_R]/T)(\mathrm{mod.}\ 2\pi)$

What we can observe are the cavity outputs:

$$E_{T_{1,2}} = \sqrt{T}\, e^{i\varphi_T}\, E_{1,2} \qquad (4.2)$$

Due to the linearly polarised input field of amplitude E_I and frequency ω_I, the cavity field itself is composed of two circularly polarised components, so that:

$$E(z,t) = 1/\sqrt{2}\left[E_1(t)\vec{e}_+ + E_2(t)\vec{e}_-\right]e^{-i(\omega_0 t - k_0 z)} + \mathrm{c.c.} \qquad (4.3)$$

where $\vec{e}_\pm = (\vec{x} \pm i\vec{y})/\sqrt{2}$ are unit vectors (or alternatively of two linearly polarised components).

We shall use a special type of nonlinearity found by Carmichael /16/ in the case of J = 1/2 to J = -1/2 transition and the scaled form of eqn. (4.1):

---

* Here, in polarization switching, we take into consideration the polarisation yielded by the vectorial character of the electric field, as for instance in Jones vectorial formalism.

$$\dot{\mathcal{E}}_{1,2} = \mathcal{E}_I - \mathcal{E}_{1,2}\left\{1+i\varphi + \frac{C(1-i\Delta)(1+|\mathcal{E}_{2,1}|^2)}{1+\frac{1}{2}(1+Q)(|\mathcal{E}_1|^2+|\mathcal{E}_2|^2)+Q|\mathcal{E}_1|^2|\mathcal{E}_2|^2}\right\} \qquad (4.4)$$

Here C, Δ, φ , Q are constants.

The parameter Q plays a special role. For Q = 1 (Fig. 7a), the denominator on the r.h.s. of eqn. (4.4) is decoupled, which gives two independent stationary equations:

$$Y = X_{1,2}\{[1 + C/(1+X_{1,2})]^2 + [\varphi - C\Delta/(1+X_{1,2})]^2\} \qquad (4.5)$$

where we have denoted $X_{1,2} = |\mathcal{E}_{1,2}|^2$ and $Y = |\mathcal{E}_I|^2$.

We see thus that each mode exhibits a cubic bistability - the behaviour, as a whole, being usually said to display a quadrastable character.

In particular, we can notice that there exist two stable symmetric branches (linear polarisation) with both X_1 and X_2 in either high-transmission, or low-transmission states and two asymmetric branches with X_1 in a high-transmission state and X_2 in a low-transmission state, or conversely.

We show in Fig. 7b how the loop of solution moves according to the change in Q, so that both symmetric and asymmetric branches are appearent in the projection. Point C in this figure marks the bifurcation to optical tristability /31-33/.

The most interesting phenomena appear at small values of Q. We shall take thus the limit Q $\to$ 0 in eqn. (4.4) which leads us to a very interesting result. Let us consider again the stationary case, for instance for $\dot{\mathcal{E}}_1 = 0$.

We obtain

$$\dot{\mathcal{E}}_I = \mathcal{E}_1\{1 + i\varphi + C(1-i\Delta)(1+|\mathcal{E}_2|^2)[1+1/2(|\mathcal{E}_1|^2+|\mathcal{E}_2|^2)]^{-1}\}. \qquad (4.6)$$

To simplify the discussion we denote:

$$\mathcal{E}_1 = u\,, \quad \mathcal{E}_2 = v$$

$$1 + i\varphi = \psi\,, \quad C(1-i\Delta) = M$$

Thus,

$$\mathcal{G}_I = u \;\; (\varphi + M(1 + v^2)/[1 + \tfrac{1}{2}(u^2 + v^2)\,])$$

and also

$$\mathcal{G}_I = u^3 + (1+2M)uv^2 - \mathcal{G}_I(u^2+v^2) + 2(M+\psi)u \qquad (4.7)$$

For $M = -2$, $\mathcal{G}_I = -p$, $2(M + \psi) = q$ we recognize in the r.h.s. of eqn. (4.7) the <u>elliptic umbilic</u>, another one of the Thom elementary catastrophes (/34/ and Figs. 10,11).

Now, it is very interesting that things happen in this way, for reasons we discuss in the following in a very sketchy way. A more comprehensive analysis will be published elsewhere /35/.

Let us write the r.h.s. of eqn. (4.7) in the form

$$V_{pqr}\,(u,v) = u^3 - 3uv^2 + p(u^2 + v^2) + qu + rv \qquad (4.8)$$

(our case corresponds to $r = 0$).

The catastrophe manifold of the elliptic umbilic is given /34/ by the equation

$$\partial V_{abc}(u,v)/\partial\, u = 0 = \partial V_{abc}(u,v)/\partial\, v \qquad (4.9)$$

The set of the critical points is given by

$$u^2 + v^2 = p^2/9 \qquad (4.10)$$

and is thus a double cone, which is presented in Fig. 10 where also the various critical points are indicated.

In the space of the control parameters, the bifurcation set is given by three cusp lines - as one would have of course expected from the very physics of the problem we are investigating and also exhibits a point of instability which is also mentioned in the next subsection.

It is even more interesting to note that in the list of the elementary catastrophes one can draw a subordination diagram relating catastrophes according to an ordered apparition of their relevant critical points. In this way, the cusp is subordinated to the elliptic umbilic, which in its turn is subordinated to the parabolic umbilic

$$V_{pqrs}(uv) = u^2v + v^4 + pu^2 + qv^2 + ru + sv \qquad (4.11)$$

This suggests a possibility to obtain, on a firm mathematical (topological) basis the full hierarchy of nonlinearities which a physical system is liable to demonstrate.

4.2. Experiments on polarisation bistability in LIS

We have demonstrated polarisation bistability in a LIS-clas A consisting of an (almost) single-mode, randomly polarised He-Ne laser, an empty external cavity with piezoelectric driven mirror and a dual-channel detection systems (for s- and p-polarized modes, Fig. 10) /38/. The broadening was visualized on a spectrum analyser and controlled to be close to the homogeneous case. The external mirror (M3) was an uncoated plane plate (weak injection) mounted on a piezoelectric transducer driven as shown in Fig. 10. In this setup, varying the length of the external cavity uniformly (with a sawtooth voltage applied to the PZT transducer), one can observe that the output signal switches between two different polarisation states. The experimental control parameters are: the external mirror reflectivity (I_1), the driving voltage ($\sim \omega_D$) and the polarisation of the injected signal.

The experimental results are shown in Fig. 11 for the orientation of the linear polarizer (LP) at 45°. The transitions of the output signals correspond to the transitions of the driving signal of M3. For $I_1 \sim (4 \div 15)\,10^{-3}\,I_L$, the output signals have clear rectangular shape. Increasing I_1 between $(4 \div 25)10^{-2}\,I_L$, the outputs are strongly distorted. The maximum excursion of the mirror M_3 was about (λ /8) with a frequency of $\sim$ 100 Hz.

The polarisation bistability in LIS can be phenomenologically explained with reference to Fig. 12. The single mode, randomly polarized lasers possess an intracavity birefringence /36,37/. The laser resonators containing birefringent elements have only orthogonally polarized modes. The orthogonal polarisations can be considered slightly separated in frequency ($\omega_{\ell 1} \neq \omega_{\ell 2}$) because they see different indices of refraction ($n_o \neq n_\ell$). Due to the small frequency separation, the two polarisations with the same longitudinal and transverse mode numbers are nonlinearly coupled and can switch between the two states (s and p) with high sensitivity to the initial conditions.

Such a laser in LIS configuration will be locked (slaved) at the injected signal frequency which jumps between two values. Initially, we can consider the injected signal at the frequency $\omega_{\ell 1}$, in resonence with the laser frequency (due to the random polarisation of the laser output). If $\omega_{\ell 1} < \omega_{\ell 2}$ (Fig. 12) and $\omega_{\ell 1} + \omega_D > \omega_{\ell 2}$, then the injected signal with frequency $\omega_{\ell 1} + \omega_D$ will slave E_1 (s-polarization) leading to high s-output and the injected signal with frequency $\omega_{\ell 1}$ will slave E_2 (p-polarisation) leading to high p-output. This explanation is in full agreement with the general theoretical approach of Section 4.2.

In the presence of the second mode we can see on the spectrum analyser screen that the output signals show distorted rectangular shapes (Fig. 11 b,c,d). When the laser is changed with another one with three axial modes, distorted rectangular shapes occasionally appear (Fig. 13 a, almost two modes), but most of the time complementary oscillations and irregular shapes occur at the dual output of LIS (Fig. 13 b,c).

Some experiments with semiconductor lasers are in our attention due to the potential of LIS for optical logic /38-44/. Passive OBD in array structures are also under study. Experimental results have been reported in /45/.

5. Conclusions

This lecture was guided by the idea of unification of the bistability description in free-running lasers, passive optical bistable devices, ring lasers and lasers with an injected signal. Indeed, a standard model with small variations based on Maxwell-Bloch equations can be considered as an adequate frame for studying bistable operation and the control parameters of all above mentioned laser devices. The bistability in this model is typically an average field effect exactly as is expected from the catastrophy theory. Since the behaviour exhibits a cusp singularity, the phenomenon can be described as a second order phase tr ion.

Considering the importance of the active bistable devices, the treetment of bistability in a laser with an injected signal (LIS) was largely accomplished in various situations of practical interest: LIS with nonlinearity in the laser and empty external cavity, LIS with nonlinear absorptive and dispersive external external cavity and LIS with polarisation switching. All situations finished to the same cusp equation model with important differences in the control parameter. The hierarchy of manifolds leading to the cusp was suggested s providing a useful way to predict new types of possible nonlinearities and consequently new types of optical logic devices. A LIS-class A with polarisation bistability was experimentally demonstrated.

The simple configuration of LIS posseses the power of the most complicated optical bistable systems and all necessary properties for the future optical logic circuits:

- it can maintain a constant logic level (active device);
- it is easy to be integrated in semiconductor laser structures;
- it can operate at very high speed and optically (3D) interconnected;
- it yields two complementary outputs (using polarisation switching) which define the device state;
- starting from the simple asynchronous RS (reset-set) bistable device, one can

build synchronous clocked bistable devices, master-slave (JK) devices and the whole bistable logic circuitry.

The clear and simple physics of LIS, allowing the discussion of rather complex aspects of the bistability, recommend it as a most serious candidate for future optical logical integrated circuits operating at very high speed.

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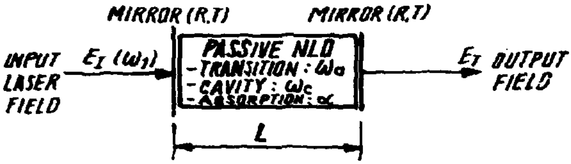
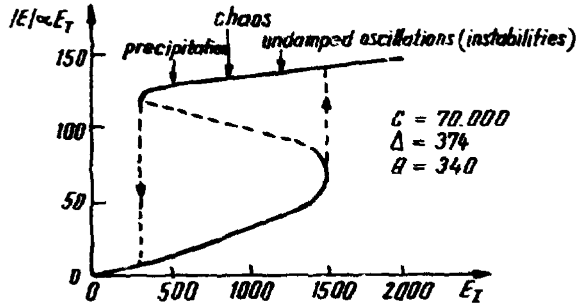


Fig. 1.



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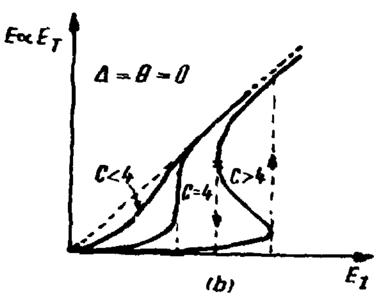

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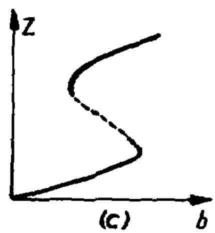
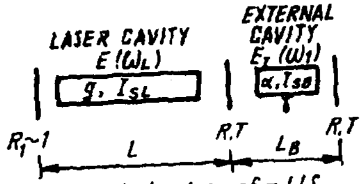


(c)

Fig. 2



General structure of a LIS

Fig.3

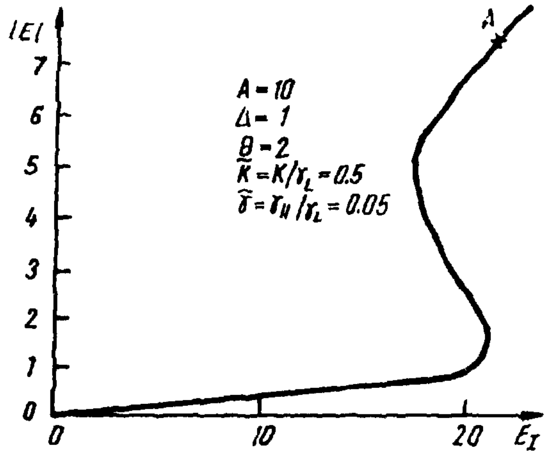


Fig.4

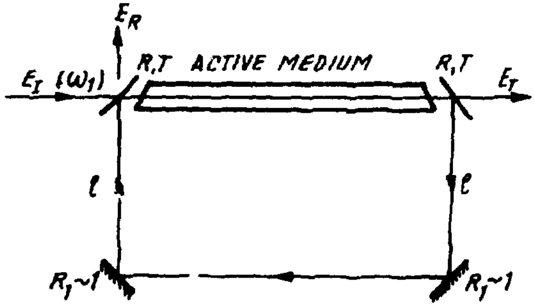
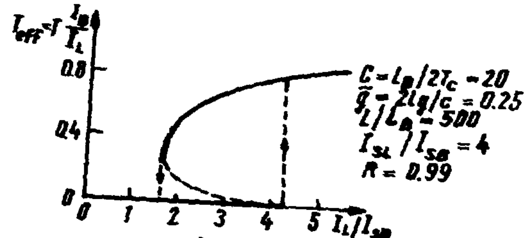
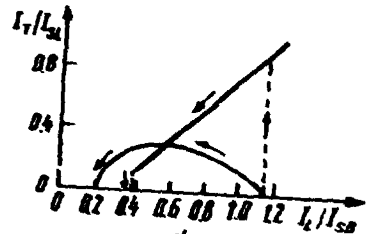


Fig.5



(a)



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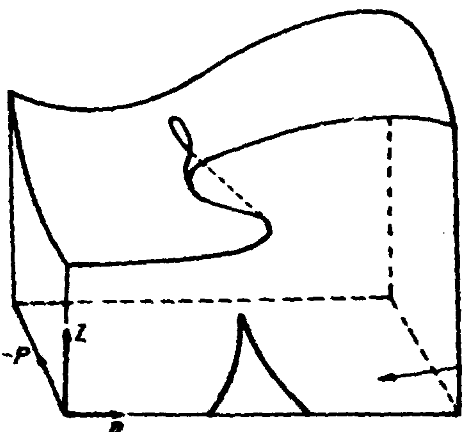
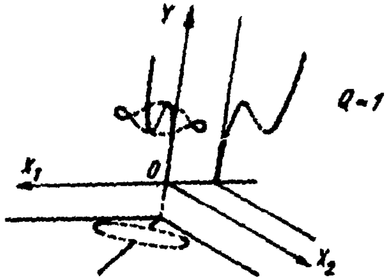


(c)

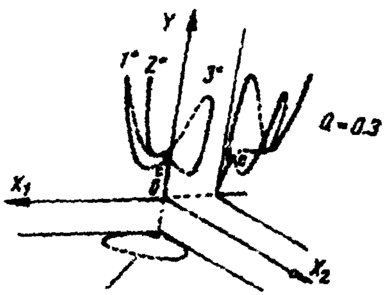
Fig.6

a) Bistability of the effective transmission in a US
b) Bistability of the transmitted intensity
c) General cusp surface (eqn (3.9)

The cusp in the plane of the control parameters $4P^3 + 27Q^2 = 0$



(a)



(b)

Fig.7

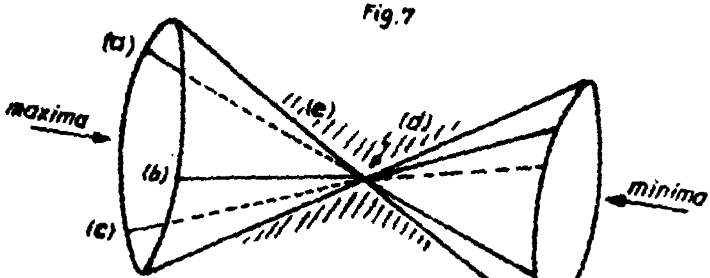


Fig.8

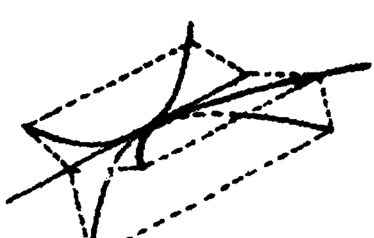
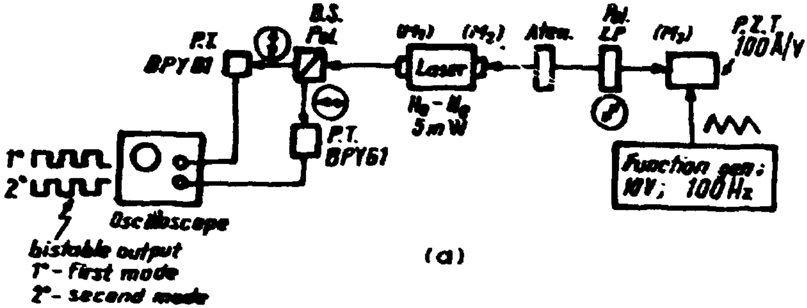
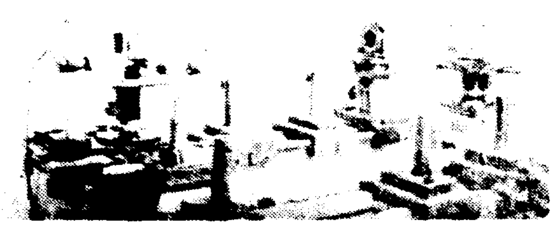


Fig.9


B.S.
Pol.
(M_2)
Atten.
LP
(M_3)
P.Z.T.
100 Å/V
P.T.
Laser
5 mW
BPY61
Oscilloscope
bistable output
1° - first mode
2° - second mode
Function gen.;
100 Hz

(a)



(b)

Fig.10

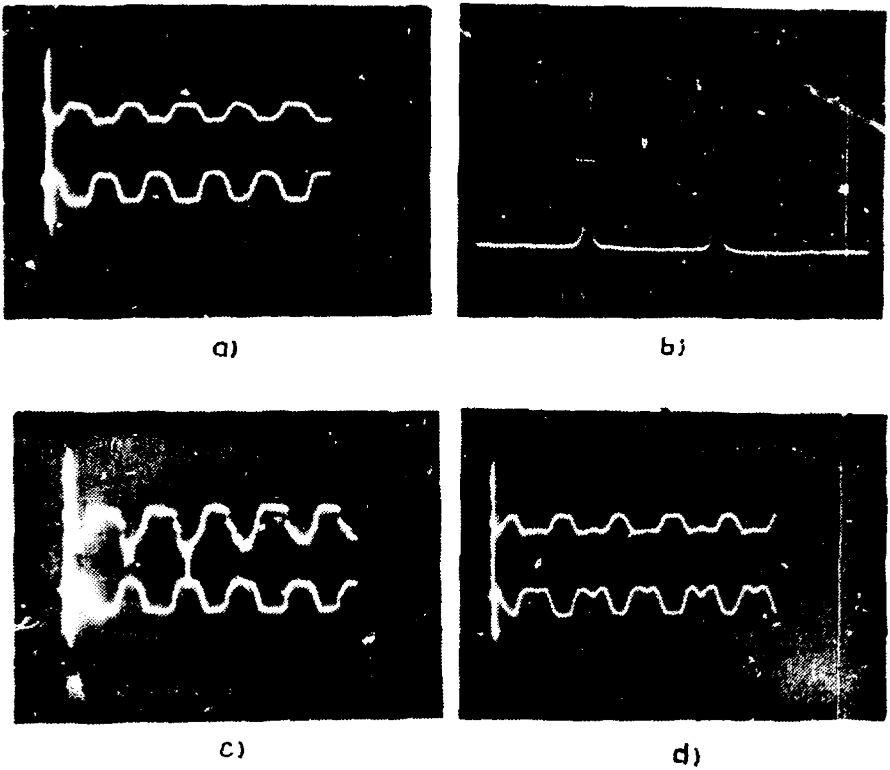

a)
b)
c)
d)

Fig. 11

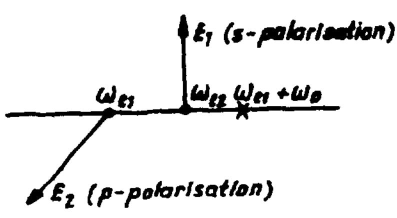
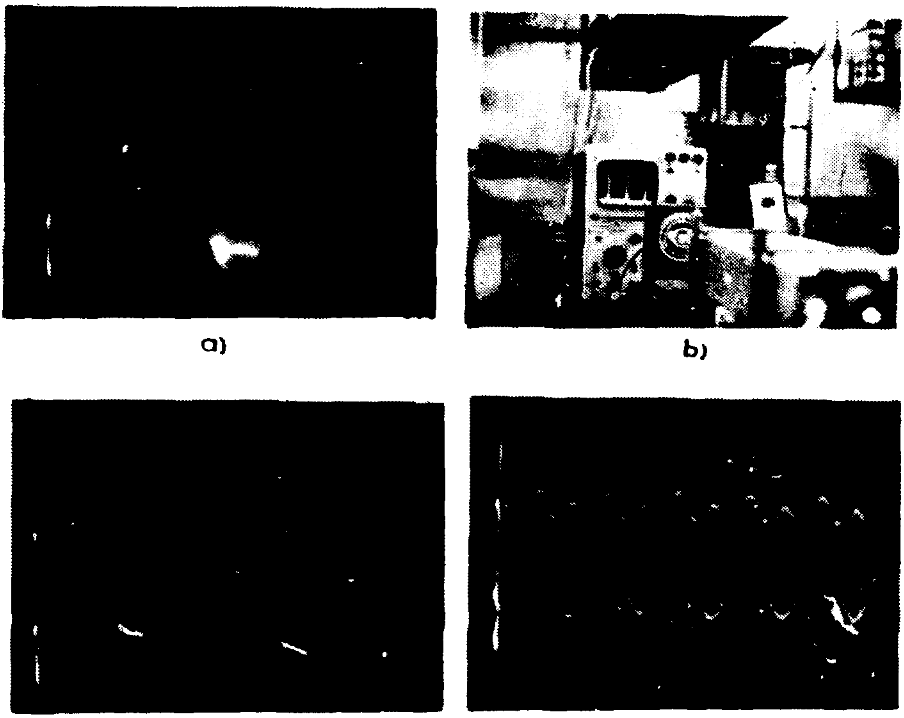

E_1 (s-polarisation)
ω_{E1}
ω_{E2}
$\omega_{E1}+\omega_0$
E_2 (p-polarisation)

Fig. 12



a) b) c) d)

Fig. 3

TABLE I. INSTABILITIES IN LASER SYSTEMS

$$\vec{A}(z,t) = \vec{A}_{st}(z) + \sum_\lambda \vec{A}_\alpha(z) e^{\lambda t}$$

(E, E^*, P, P^*, D) — $\vec{A}_{st}(z)$: stationary sol.; $\vec{A}_\alpha(z)$: spatially dependent mode corresp. to eigenval. λ

Limit \ Systems	Absorptive	Dispersive	Dispersive, At dispersive limit
MFT - Multimode $\gamma\tau_r = \text{const}; \frac{k}{\gamma} = \frac{T}{\gamma\tau_r} \to 0$	Re $\lambda_{1,n} > 0$, $n \neq 0$ on PUB in absorptive bistability -periodic and transient oscillations -precipitation on LB -no chaos *(Bonifacio and Lugiato, 1978)*	Re $\lambda_{1,n} > 0$, $n \neq 0$ on PUB in dispersive bistability and without bistability -no oscillations reported *(Lugiato, 1980)*	–
MFL - Monomode $\gamma\tau_r \to 0$; $\frac{k}{\gamma} = \frac{T}{\gamma\tau_r} = \text{const.}$	–	Instability along PUB in disp. bistability -periodic and transient oscillations -precipitations on LB -period-doubling to chaos *(Lugiato et al., 1982)*	Instability along PUB -period doubling and chaos *(Ikeda and Akimoto 1982)*
Outside MFL	$\lvert\theta\rvert < \frac{\pi}{2}$ (partic. $\theta=0$, tuned cavity) abs. bistability with stable UB and LB $\lvert\theta\rvert < (\frac{\pi}{2}, \pi)$ no bistability, -periodic osc. $(2\tau_r)$ -no chaos *(Carmichael et al., 1982)*	Instability along PUB -periodic oscillations -period-doubling and chaos *(Ikeda, 1979)*	Instability in disp. bistability -periodic oscil. -period-doubling and chaos *(Ikeda et al., 1980)* *Experim: Gibbs et al. 1981, 1982, Ikeda 1983*

MFL = mean-field limit; PUB = part of upper branch; LB = lower branch

$\tau_r = (L+l)/c$ = cavity round trip time