[redacted]

AGH
IFTJ

Raport INT 191/P

ACK DECAY OF MUONIC MOLECULAR RESONANCES AND THE MEASURED VALUE OF d μ d–FORMATION RATE IN MUON–CATALYZED FUSION IN DEUTERIUM

A. Guła*, A. Adamczak** and M. Bubak***

Phys. Lett. 109A (1985) 224

[redacted]

BACK DECAY OF MUONIC MOLECULAR RESONANCES AND THE MEASURED VALUE OF $d\mu d$ - FORMATION RATE IN MUON-CATALYZED FUSION IN DEUTERIUM

ROZPAD REZONANSÓW MOLEKULARNYCH W KANAŁ WEJŚCIOWY A MIERZONA WARTOŚĆ SZYBKOŚCI TWORZENIA $d\mu d$ W MIUONOWEJ KATALIZIE SYNTEZY JĄDROWEJ

РАСПАД МЮ-МОЛЕКУЛЯРНЫХ РЕЗОНАНСОВ ПО ВХОДНОМУ КАНАЛУ И ЭКСПЕРИМЕНТАЛЬНАЯ ВЕЛИЧИНА СКОРОСТИ ОБРАЗОВАНИЯ $d\mu d$ В МЮОННОМ КАТАЛИЗЕ РЕАКЦИИ ЯДЕРНОГО СИНТЕЗА

by

A. Gula[x], A. Adamczak[xx] and M. Bubak[xxx]

Joint Institute for Nuclear Research, Dubna

Submitted to "Physics Letters" ‡

Kraków 1985

---

[x] Present address: Institute of Physics and Nuclear Techniques, AGH, al. Mickiewicza 30, 30-059, Kraków, Poland

[xx] On leave from Institute of Nuclear Physics, Kraków, Poland

[xxx] On leave from Institute of Computer Science, AGH, Kraków, Poland

‡ A modified version of JINR preprint E1-84-871, Dubna, 1984

Druk wykonano z dostarczonych oryginałów

Rozprowadza – Распространяет – Available from:
OŚRODEK INFORMACJI O ENERGII JĄDROWEJ
00-901 Warszawa, PKiN, XI p.

Wydaje:
INSTYTUT FIZYKI I TECHNIKI JĄDROWEJ AGH – KRAKÓW
30-059 Kraków, al. Mickiewicza 30

ISSN 0302-9084

Wydanie 1. Nakład 100+24 egz. Ark. wyd. 0,65, ark. druk. 1
Papier offset. A,1 kl. III, 71 g Oddano do produkcji 1985-04-16
Zamówienie nr 272/85 Druk ukończono w lipcu 1985

1987

Data złożenia maszynopisu przez autora: 1985-04-10

Wykonano w Zakładzie Graficznym AGH Kraków, al. Kawiory 49

Abstract

It is shown that the experimental values of $d\mu d$ formation rate, obtained without taking into account the decay of the μ-molecular resonance compound $[(d\mu d)^+ dee]^*$ back to the formation channel $d\mu + D_2$, are underestimated. The correction depends on the rate of this resonance back decay and the rates of processes leading to fusion in $d\mu d$.
For their current estimates the correction significantly exceeds the experimental error of the uncorrected $d\mu d$ formation rate $\lambda_m^{obs} = 2.76 \pm 0.08\ \mu s^{-1}$ reported recently. It is argued that back decay may lead to variation of λ_m^{obs} with target density which may provide useful information on the parameters of muon-catalyzed fusion.

When a negative muon enters a deuterium target it forms a muonic deuterium atom ($d\mu$) and subsequently a tightly bound $(d\mu d)^+$ system (r.m.s. $\sim 10^{-11}$cm) in which the deuterons rapidly undergo nuclear fusion [1,2]. The muon is then most likely freed ($f \gtrsim 87$ %) and becomes available to catalyze another fusion reaction.

It is now well established that $(d\mu d)^+$ formation proceeds mainly via creation of a resonance μ - molecular compound [3-5]:

$$d\mu + D_2 \longrightarrow \left[(d\mu d)^+_{J,\nu} dee\right]^*_{K,\nu} \qquad (1)$$

Such process is possible due to existence of a weakly bound ($\varepsilon_b = 1.91$ eV) rovibrational state of $d\mu d$: $J = 1$, $\nu = 1$ [2]. The energies are tuned in (1) by the vibrational ($\varepsilon_\nu \sim 10^2$meV) and rotational ($\varepsilon_K \sim 1$ meV) excitations of the compound and by the relative collision energy ($\varepsilon_T \sim 10$ meV) [2,6]. It has been commonly assumed so far, that - unless the muon decays - (1) is followed inevitably by fusion in $d\mu d$. However, as pointed out in [7], the decay of compound (1) back to the formation channel may seriously compete with transitions leading to fusion. In this note we investigate an important consequence of this observation.

Fig. 1 presents the main processes forming one muon catalyzed fusion (MCF) cycle in deuterium with inclusion of the resonance back decay (RBD) shown as a dashed line. Other important decay modes are [7] : (i) direct fusion in the ($J = 1$, $\nu = 1$; K, ν) state, (ii) de-excitation of $d\mu d$ to a lower $J = 1$, $\nu = 0$ state, (iii) vibrational de-excitation of compound (1) in collisions with D_2 molecules. In (ii) the resonance object is destroyed as $\Delta\varepsilon_b \approx 2 \cdot 10^2$ eV exceeds the ionization energy and in (iii) already a $\nu \to \nu - 1$ transition brings the compound below the elastic threshold ($\Delta\varepsilon_\nu \gg \varepsilon_T$). Although the analogous collisional K - transitions are expected to be very fast [8] we assume tentatively, as in [7], that they do not compete with RBD. In fact, the $d\mu d$-fusion data can be described by a single narrow resonance at $\varepsilon_T = \varepsilon_o \approx 50$ meV taking into account only vibrational excitation [3]. Adding the rotational excitations on top leaves the formation channel open in the ensuing

transitions, thus, the general structure of the kinetic graph in Fig. 1 is not affected.

To see how RBD influences the kinetics of MCF in deuterium let us consider the Laplace transform, $F_1(s)$, of the time distribution of fusion events ending the first cycle, $F_1(t) = dN_{f1}/dt$. Without RBD it reads [9-11]:

$$F_1(s) = \frac{s\phi(s)}{\Lambda_\mu + s} \frac{\lambda_a}{\Lambda_{d\mu} + s} \frac{\lambda_m}{\Lambda_{\mu mr} + s} H(\lambda_d, \lambda_t, \lambda_f, \lambda_{tj}'; s) \quad (2)$$

where λ_i are transition rates defined in Fig. 1, $s\cdot\phi(s)$ represents the incoming muon flux, Λ_i are sums of the transition rates over all lines leaving an i-th vertex (note: $\Lambda_{(1,1;K,\nu)} = \Lambda_{\mu mr}$) and H(s) describes the structure of the subgraph linking the μmr and F nodes (channels leading to fusion in $d\mu d$) [11].

One can easily verify, either by directly solving the first-cycle kinetic equations [10] or by applying the Mason topological formula [11], that the $d\mu \rightleftarrows \mu mr$ loop created in the kinetic graph by RBD couples $\Lambda_{d\mu}$ and $\Lambda_{\mu mr}$ in eq. (2) so that

$$(\Lambda_{d\mu} + s)(\Lambda_{\mu mr} + s) \longrightarrow (R_{d\mu} + s)(R_{\mu mr} + s) = \\ = (\Lambda_{d\mu} + s)(\Lambda'_{\mu mr} + s) - \lambda_b \lambda_m \quad (3)$$

The other Λ_i - hence also H(s) - remain unchanged and, with inclusion of RBD, $\Lambda_{\mu mr} \rightarrow \Lambda'_{\mu mr} = \Lambda_{\mu mr} + \lambda_b$. Since the time distribution of fusion events ending an arbitrary k-th cycle is given by $F_k(s) = t^{k-1}\left[F_1(s)\right]^k$, inclusion of RBD ammounts to replacing in all kinetic formulae: $\Lambda_{d\mu}, \Lambda_{\mu mr} \rightarrow R_{d\mu}, R_{\mu mr}$.

In particular, the large time behaviour of $F_k(t)$ is determined by the position of the rightmost pole in (2) [10]. This is now $(-R_{d\mu})$ since

$$R_{d\mu} = \lambda_o + \frac{1}{2}\left[\lambda_m + \lambda_F + \lambda_b - \sqrt{(\lambda_F + \lambda_b - \lambda_m)^2 + 4\lambda_b\lambda_m}\right] < \Lambda_{d\mu} \quad (4)$$

and $\Lambda_{d\mu} < \Lambda_{1 \neq d\mu}$ [2]. We put above: $\lambda_F = \lambda_e + \varphi\lambda_d^o$.

$\lambda_e = \lambda_f + \lambda_t$, $\lambda_m = \varphi \lambda_m^o$. λ_o is muon decay rate, φ is target density expressed in units of liquid hydrogen density (LHD) and index 0 denotes the rates normalized to LHD. For $\lambda_b \gg \lambda_m$ eq. (4) becomes

$$\lambda_m^{obs} = (R_{d\mu} - \lambda_o)\varphi^{-1} = \frac{\lambda_F}{\lambda_F + \lambda_b}\,\lambda_m^o \qquad (4a)$$

where λ_m^{obs} is the quantity identified so far with λ_m^o.

An immediate consequence of eq. (4) is that neglecting RBD in the determination of λ_m^o from the large time slope of $F_1(t)$ [4] leads to underestimated values of this important parameter. To estimate this effect let us remark that upon the thermal averaging [7] :

$$\lambda_b = \text{const}\; e^{\frac{\varepsilon_o}{kT}} (kT)^{\frac{3}{2}}\, \lambda_m^o(T) = \alpha(T)\,\lambda_m^o(T). \qquad (5)$$

In contrast to all previous analyses where $R_{d\mu} \to \Lambda_{d\mu} = \lambda_o + \lambda_m$ (e.g., [3,4]), λ_m^o depends now on λ_F and α, the latter being determined by the resonance energy ε_o and the rotational quantum numbers K_{D2} and K, which count the number of states contributing to relation (5). Neglecting the K-structure and using $\varepsilon_o = 53$ meV [12] one obtains [7] $\alpha(293K) \approx 300$. This value together with $\lambda_f = 870\ \mu s^{-1}$ [13], $\lambda_t = 190\ \mu s^{-1}$ [14] and $\lambda_d^o = 10\ \mu s^{-1}$ [7] applies to the result of [4] $\lambda_m^{obs} = 2.76 \pm 0.08\ \mu s^{-1}$ ($\varphi = 0.1$) gives $\lambda_m^o(293K) = 12.6 \pm 1.7\ \mu s^{-1}$. Muon transitions to impurity atoms and nonresonant $d\mu d$ production which have been neglected in writing down eqs. (4,5) may change this number by $\approx 5\ \%$ [15,16]. If, more realistically, the $K_{D2}=0 \to K=1$ transitions are assumed to dominate ($\alpha \to \alpha/3 = 100$) then $\lambda_m^o = 3.7 \pm 0.14\ \mu s^{-1}$.

To illustrate the importance of the correction for back decay let us notice that for λ_m^o to coincide with $\lambda_m^{obs} = 2.76 \pm 0.08\ \mu s^{-1}$

within two standard deviations, one should have, according to (4a): $\lambda_F/\alpha > 50\ \mu s^{-1}$, while with λ_F of [7,13,14] and $\alpha = 100$ this ratio is only $\sim 10\ \mu s^{-1}$. Thus, if λ_F is not grossly underestimated, λ_m^o is actually much higher than the existing theoretical prediction [12]. However, the results of [4,5] already challenge this estimate and indications appear that the μ-molecular formation rates may significantly differ from those reported previously [17].

More accurate data on $P_K(t)$ at small t which are sensitive to λ_F [10,16] would greatly help to resolve these ambiguities. Another interesting and relatively simple possibility is to measure λ_m^{obs} at various target densities. Fig. 2 shows λ_m^{obs} as a function of φ for several values of α, λ_e and λ_d^o with λ_m^{obs} fixed at the most accurately measured value ($2.76 \pm 0.08\ \mu s^{-1}$ at $\varphi = 0.103$) of [4]. A wide range of λ_d^o is covered to include the possibility that the rotational de-excitations also compete with RBD as conjectured in [8]. (In fact, λ_d^o for vibrational de-excitation depends strongly on the assumptions used in its determination and a wide spectrum of values can also be expected [18].)

Curves for $\lambda_d^o < 100\ \mu s^{-1}$ deviate very weakly from $\lambda_m^{obs} = \text{const}$ and are not shown. The recent data of [4] published for other densities (2.4 ± 0.6, 2.86 ± 0.11, 2.74 ± 0.14, $2.84 \pm 0.10\ \mu s^{-1}$ at $\varphi = 0.05$, 0.08, 0.09, 0.105, respectively) and the estimate of [5] ($\sim 3\ \mu s^{-1}$ at $\varphi \approx 0.3 - 0.7$) are not conclusive with the present errors. They agree with $\lambda_d^o \lesssim 100\ \mu s^{-1}$, although higher λ_d^o are not excluded if α is appropriately lowered or λ_e increased. On the other hand, the result of [3], $0.76 \pm 0.11\ \mu s^{-1}$, cannot be reconciled with these data with any choice of parameters. As pointed out in [7] λ_d^o should grow by 3-4 orders of magnitude between 400 and 2000 K and quite pronounced effects may, thus, appear at somewhat higher temperatures. Hence, back decay should be taken into account also in consideration of other factors leading to non-trivial density dependence of MCF parameters. Therefore, the measurements of λ_m^{obs} in pure deuterium over a wide range of temperatures and densities may provide very useful information.

After this work been completed we learned that L.I. Menshikov estimated the influence of RBD on the μ - molecular resonance formation rates, using a probability argument which led him to eq. (4a).

The authors are grateful to L.I. Menshikov for very helpful discussions. They also thank L.I. Ponomarev for constant encouragement and stimulating conversations and M.P. Faifman for his remarks.

References

1. F.C. Frank: Nature 160 (1947) 525,
   A.D. Sakharov: Report of the Phys. Inst. Acad. Sci. USSR, Moscow, 1948,
   Ya.B. Zeldovich: Dokl. Akad. Nauk SSSR 95 (1954) 493,
2. For introduction and a recent review see:
   L. Bracci, G. Fiorentini: Phys. Rep. 86 (1982) 170,
   L.I. Ponomarev: Atomkernenergie/Kerntechnik 43 (1983) 175,
3. V.M. Bystritsky, V.P. Dzhelepov et al.: Zh.Eksp.Teor.Fiz. 76 (1979) 460 Sov. Phys. JETP 49 (1979) 232 .
4. D.V. Balin, A.A. Vorobev et al.: Phys. Lett. 141 B (1984) 173; D.V. Balin, A.A. Vorobev et al.: Zh. Eksp. Teor. Fiz. - Pisma 40 (1984) 318,
5. S.E. Jones: Talk at Ninth International Conference on Atomic Physics, Seattle, July, 1984,
6. M.Leon: Phys. Rev. Lett. 52 (1984) 605,
7. M.Lane: Phys. Lett. 98A (1983) 337,
8. V.N. Ostrovski, V.I. Ustimov: Zh. Eksp. Teor. Fiz. 79 (1980) 1228,
9. V.M. Bystritsky, A.Guła, J.Woźniak: Atomkernenergie/Kerntechnik 45 (1984) 197,
10. M. Bubak, V.M. Bystritsky, A.Guła: Preprint JINR E1-84-164, Dubna, 1984 (to be published in Acta Physica Polonica),
11. A. Guła: Preprint JINR E1-84-301, Dubna, 1984 (to be published in Acta Physica Polonica),
12. S.I. Vinitsky, L.I. Ponomarev et al. : Zh. Eksp. Teor. Fiz. 74 (1978) 849 [Sov. Phys. JETP 47 (1978) 444] .
13. L.M. Bogdanova et al.: Phys. Lett. 115B (1982) 171.
14. Value obtained by M.P. Faifman using the algorithm of L.I. Ponomarev, M.P. Faifman: Zh. Eksp. Teor. Fiz. 71 (1976) 1689 [Sov. Phys. JETP 44 (1976) 886],

15. D.V. Balin et al. : Preprint LNPI-964, Leningrad, 1984,

16. A. Guta, M.Bubak, J. Niwicki: Report JINR E1-84-560, Dubna, 1984; preprint INT 189/P, Kraków, 1985 (to be published in Atomkernenergie/Kerntechnik),

17. J.S. Cohen, R.L. Martin : Phys. Rev. Lett. 53 (1984) 738 and M. Leon. ibid., p. 739,

18. We are grateful to L.L. Menshikov for bringing this point to our attention,

Figure captions

Fig. 1. Kinetic graph presenting the processes forming one muon-catalyzed fusion cycle in deuterium.
The nodes represent : (μ) - free muon, ($d\mu$) - deuterium μ - atom, (1,1; K, υ) $\equiv$ (μ m r) - μ-molecular resonance compound in a J = 1, ν = 1, K, υ state, (1,1; K, υ -1) - the same compound after vibrational de-excitation, (1,0) - d μ d in the J = 1, ν = 0 state. (F) is a subsidiary vertex representing fusion in dμd.

Fig. 2. $\lambda_m^{obs} = \varphi^{-1}(R_{d\mu} - \lambda_o)$ as a function of target density (in units of LHD) for different α , λ_e and λ_d^o listed in the upper - left corner. All curves correspond to $\lambda_m^{obs}(\varphi = 1.035) = 2.76\ (\pm 0.08)\ \mu s^{-1}$ of [4] . The inset (b) (bottom - right) shows in more detail the region $\varphi < 0.1$ (bottom and lower - right scales).
The numbers in brackets in the inset are the "true" values of λ_m^o.

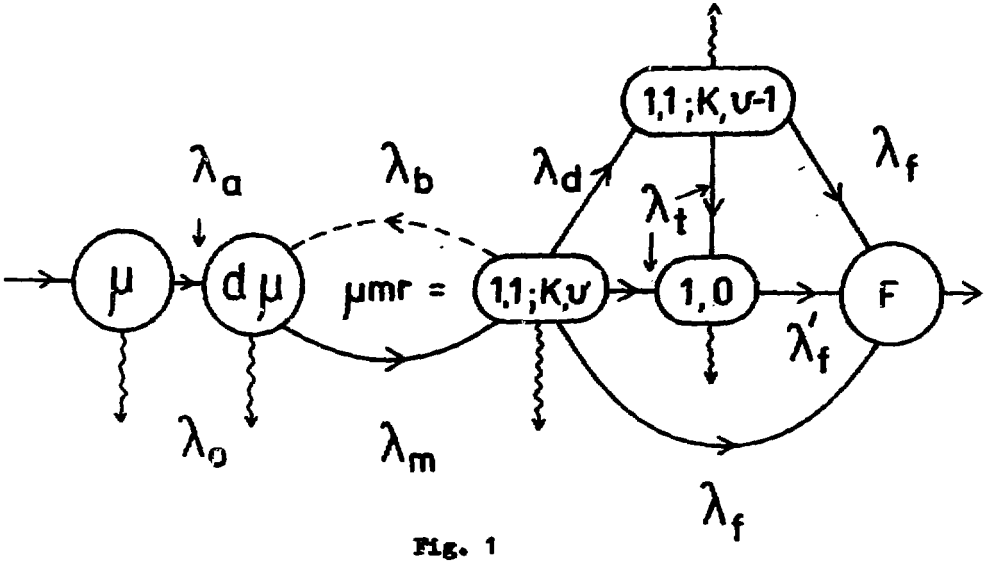

Fig. 1

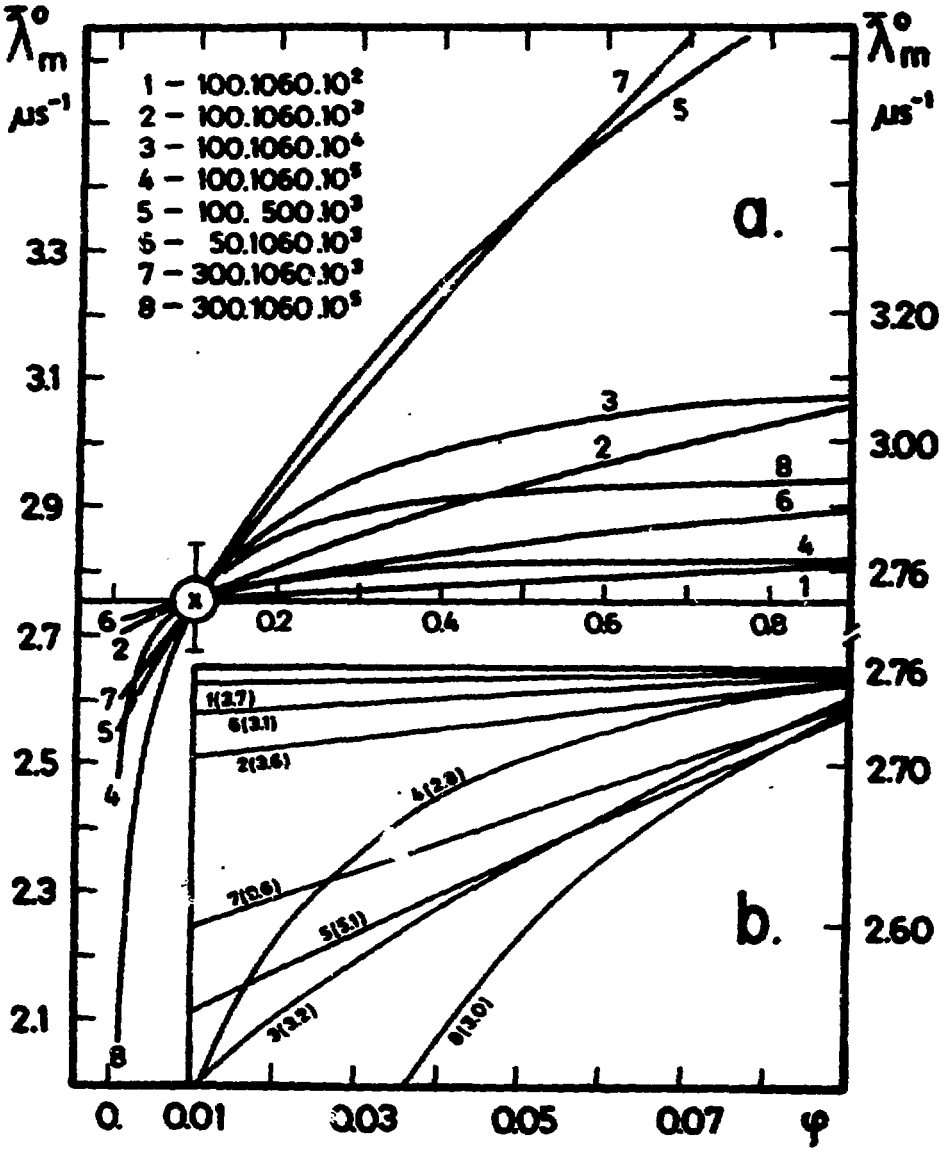
$\bar{\lambda}^{o}_{m}$
μs^{-1}
1 – 100.1060.10²
2 – 100.1060.10³
3 – 100.1060.10⁴
4 – 100.1060.10⁵
5 – 100. 500.10³
6 – 50.1060.10³
7 – 300.1060.10³
8 – 300.1060.10⁵
a.
b.
3.3
3.1
2.9
2.7
2.5
2.3
2.1
3.20
3.00
2.76
2.70
2.60
0.2
0.4
0.6
0.8
0.
0.01
0.03
0.05
0.07
φ
1(2.7)
6(2.1)
2(2.6)
4(2.9)
7(0.6)
5(2.1)
3(2.2)
8(2.0)

Fig. 2

ZAKŁAD
GRAFICZNY
AGH
KRAKÓW

ISSN 0302-903-