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OPTIMIZATION OF MULTIWIRE COIL ENDS HAVING 45 DEGREE BENDS*

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Multiwire[1] is the name of a proprietary process for affixing small diameter wires to a flat substrate using digitally controlled machinery. It is currently being used to wind trim coils for the SSC dipoles on a flexible substrate which is wrapped around the beam tube. It is proposed[2] for making multipole coils for the Corrector, a regular arc magnet in each half-cell of the Relativistic Heavy Ion Collider (RHIC).

The current Multiwire process does not permit a change in direction of the wire other than 45 degree. The present paper answers the question of whether the 45 degree bends in the flattened coil can be located along straight lines in such a way as to eliminate or reduce higher harmonics in the ends. The more general question of bends located along curves is not addressed.

Single-layer coils typically consist of a band of wires with constant spacing in the two-dimensional part of the coil or straight-section. Two such bands, with returns at each end, complete one pole of a 2m pole magnet. If when wrapped to form a circular cylinder of radius r, each band of a pole subtends an angle of $\pi/3m$ radian at the axis of the cylinder, with $\pi r/3m$ of gap between the two bands of the pole, the coil will have zero third harmonic. For constructional reasons, the winding of a pole sometimes cannot have the full angular width of π/m radian. If the turn-less width of edge subtends an angle δ ($= \theta_i$ in Figure 1), then the band angular width must be $\pi/3m - 2\delta$ for elimination of the third harmonic. The allowed harmonics n satisfy $n/m = k$, $k = 1,3,5,\cdots$ the fundamental is $k = 1$ and the third harmonic is $k = 3$. For a dipole ($m = 1$), the third harmonic has $2n = 6$ poles, in a quadrupole ($m = 2$) the third harmonic has $2n = 12$ poles, etc.[3] The nomenclature for n and m as given here is for consistency with reference 3; harmonics "b_i" more commonly used correspond to $i = n-1$.

Figure 1 shows half of a developed (flattened) end. The axes are z in the direction of the magnet axis and s; $z = 0$ is the end of the two dimensional part of the magnet. The band of conductors has width $a = s_i - s_f$ before bending at the first short-dashed line, width b after the first bend and width $c = z_i - z_0$ after the second bend. It is found that

$$b/a = \sin(\pi/4)(1 + \tan(\alpha)) \tag{1}$$

$$c/a = (1 + \tan(\alpha))/(1 + \cot(\beta)) \tag{2}$$

Equation (2) shows that if $\beta = \pi/2 - \alpha$, $c = a$, i.e., the spacing where the wires cross the pole ($s = 0$) is the same as on the side. If $\alpha = 22.5°$ and $\beta = 67.5°$, $a = b = c$, that is, the end maintains constant wire spacing everywhere. Such an end for which $s_1 = s_2 = 0$ is half of a regular octagon.

In all that follows, it will be assumed that the point A lies on the $z = 0$ axis, i.e., that $s_1 = s_f$.

The harmonics can be computed using eqn (12) of Ref. 3; for constant radius that equation becomes

$$q_n = \frac{4mQ_n}{n}\int_0^{\pi/2m} N(\theta_0)d\theta_0 \int_{C_{\theta_0}} \cos(n\theta)dz \tag{3}$$

where q_n is 1/n of the n^{th} Taylor expansion coefficient of the integral of B_y, $Q_n = (\mu_0 I/(2\pi))(r^n/R^{2n} + 1/r^n)$ and R is the iron radius. For a 2m pole magnet, the relation between s and θ is $s = (\pi/(2m) - \theta)r$, where r is the winding radius. Note that α and β of Fig. 1 are independent of m. For a winding with constant wire spacing, $N(\theta_0) = r/d$, where d is the spacing; if the band of wires begins at $\theta_0 = \theta_i$ and ends at $\theta_0 = \theta_f$ as in Fig 1, eqn (3) becomes

$$q_n = -\frac{4mQ_n r}{nd}\int_{\theta_i}^{\theta_f} d\theta_0 \int_{C_{\theta_0}} \cos(n\theta)dz \tag{4}$$

The latter part of this double integral (with respect to z) has 3 parts $0 \le z \le z_1$, $z_1 \le z \le z_2$ and $z = z_2$, where z_1 and z_2 lie on the first and second lines of bends as shown in Fig. 1 for a typical wire beginning at θ_0. On the first part, $\theta = \theta_0$ and

$$z_1 = r(\theta_f - \theta_0)\tan\alpha \tag{5}$$

The second line of bends intersects the s axis at s_2 or θ_2, and has the equation

$$z = r(\theta_2 - \theta)\tan\beta \tag{6}$$

and on the middle segment, the typical wire has the equation

$$z = r(\theta - \theta_0(1 + \tan\alpha) + \theta_f \tan\alpha) \tag{7}$$

Combining (6) and (7) with the elimination of θ gives

$$z_2 = r\tan(\beta)(\theta_2 - \theta_0 + (\theta_f - \theta_0)\tan\alpha)/(1 + \tan\beta) \tag{8}$$

Equation (4) then becomes

$$q_n = -\frac{4mQ_n r}{nd}\int_{\theta_i}^{\theta_f}\left(\int_{z_1}^{z_2}\cos\left\{n\left[\frac{z}{r} + \theta_0(1 - \tan\alpha) - \theta_f\tan\alpha\right]\right\}dz + \int_0^{z_1}\cos(n\theta_0)dz\right)d\theta_0 \tag{9}$$

where z_1 and z_2 are given by eqs (5) and (8), resp. Note that the third segment of the typical wire is independent of z and does not contribute to the integral. Equation (9) can be evaluated in closed form; the result is

$$q_n = \frac{4mQ_n r^2}{n^3 d}\Big\{(\cos(n\theta_f) - \cos(n\theta_i))(\tan\alpha - 1) + \left(\frac{1 + \tan\beta}{1 + \tan\alpha}\right)\left[\cos\left(\frac{n(\theta_f + \theta_2\tan\beta)}{1 + \tan\beta}\right) - \cos\left(\frac{n[\theta_i + \theta_2\tan\beta - (\theta_f - \theta_i)\tan\alpha]}{1 + \tan\beta}\right)\right] + (\theta_f - \theta_i)\sin(n\theta_i)\tan\alpha\Big\} \tag{10}$$

There are three independent parameters in eqn (10): α, β, and θ_2. There are, however, constraints on them. Firstly, b and c of eqs (1) and (2) must be greater than or equal to a. Secondly, the distance between bends must be greater than about 3 d. It is intended to use 15 mil (bare) wire in the RHIC corrector, for which d = 24 mil. The distance between points A and B of Figure 1 is given by

$$\overline{AB} = r(\theta_2 - \theta_f)\sin(\beta)/\sin(3\pi/4 - \beta) \tag{11}$$

and the distance from the z axis to B is given by

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$$s_B = r[\pi/2m - (\theta_2 \tan(\beta) + \theta_f)/(1 + \tan\beta)] \qquad (12)$$

The constraints are $\bar{A}\bar{B} \geq 3$ d and $s_B \geq 1.5$ d. Since $\theta_f \equiv \pi/3m$, these constraints result in an end shape which changes with m, if the constraint is in force. Using these numbers, eqs. (11) and (12) can be rewritten

$$\theta_2 \geq \theta_f + \frac{3d\sin(3\pi/4 - \beta)}{r\sin\beta} \qquad (13)$$

$$\theta_2 \leq \left(\frac{\pi}{2m} - \frac{1.5d}{r} - \frac{\theta_f}{1 + \tan\beta}\right)(1 + \cot\beta) \qquad (14)$$

It is found by a numerical survey that infinitely many roots of $q_n = 0$ for the third harmonic (k = 3) exist. The same is not true for k = 3 and k = 5 simultaneously, and it is found that q_n for k = 5 is minimized by a configuration such that $s_B = 1.5$ d, i.e., the equality in eqn (14) holds, and such that a = c.

For comparison, the harmonics of the 2-D part of the winding can be computed using eqn (4), which for $\theta = \theta_0$ and z = 1, becomes

$$q_n = -\frac{4mQ_n r}{n^2 d}\left[\sin(n\theta_i) - \sin(n\theta_i)\right] \qquad (15)$$

If q_n of eqn (10) for the k^{th} harmonic is designated A_k and q_n of eqn (15) is designated C_k, then the effective length of the end (in meters) is $L_e = A_1/C_1$. The physical length of an end, L_p, is the maximum value of z_2 of eqn (8), obtained when $\theta_0 = \theta_i$.

Ends satisfying the equality in eqn (14) ($s_B = 1.5$ d) have the greatest physical length, and ends satisfying the equality in eqn (13) ($\bar{A}\bar{B} = 3$ d), with a = c have the shortest physical length. All such ends (for a given m) have the same physical length, regardless of α. It was mentioned that the long ends with a = c give the lowest A_5; how good are the short ends? To discuss this, use as a measure of "goodness" the magnitude of the unwanted harmonics, viz. k = 3 and 5 at a reference radius. A typical reference radius is about 2/3 the coil radius; in the RHIC Corrector, the reference radius is $x_0 = 25$ mm. The magnitude is $M_k = n\, q_n x_0^{n-1}$. Two relative measures of quality suggest themselves: the first is the integrated ratio $RI_k = 2\, M_k(\text{end})/(L_S M_1(S))$ where $M_1(S)$ is the magnitude of the fundamental in the straight-section which has length L_S. The second is $RM_k = [M_k(\text{end})/L_e]/M_1(S)$ or the magnitude of the unwanted harmonic per unit effective length in the end divided by the magnitude of the fundamental in the straight-section. This second ratio is an indication of the size of the "bump" in the unwanted harmonics at the end. Table 1 gives these two ratios (times 10^4) for two short ends, α = 22.5 and α = 45 degree, and for optimized ends. For these calculations, θ_i was assumed to be zero, and the optimized ends were close to long ends: $\theta_2 = 100/m$ degree which made $s_B = 1.96$ d in the decapole (m = 5). The radii used were r = 51.69, 46.76 and 42.21 mm, respectively for m = 2,4 and 5; $\theta_f = \pi/3$ m and $L_S = 0.5 - 2\, L_e$.

Table 1: Field Magnitude Ratios times 10^4

type	m	α	L_e, mm	L_p, mm	RI_3	RI_5	RM_3	RM_5
short	2	45.0	16.2	28.4	12.8	0.0	176.2	0.4
short	4	45.0	8.0	13.5	0.4	0.0	22.8	0.0
short	5	45.0	6.1	10.1	0.2	0.0	16.8	0.0
short	2	22.5	15.2	28.4	1.4	-.5	22.1	-8.0
short	4	22.5	7.4	13.5	-.1	0.0	-3.3	-.1
short	5	22.5	5.6	10.1	-.1	0.0	-6.3	-.1
opt.	2	39.55	22.8	37.0	0.0	-.2	0.0	-2.5
opt.	4	39.55	10.3	16.7	0.0	0.0	0.0	0.0
opt.	5	39.55	7.5	12.1	0.0	0.0	0.0	0.0

The short end with α = 22.5 (equal wire spacing everywhere) is probably acceptable, although there is a sizable bump in the 12 pole term of the quadrupole, RM_3, the optimized end is preferred.

The 2-d part of the RHIC Corrector dipole winding is not a single-layer 60 degree winding. It has 6 layers in 3, approximately equal pairs, with angles θ_i and radii as given in Table 2. The angular widths are chosen to minimize the harmonics with n = 3, 5 and 7 in the straight-section.

Table 2:

Layer No.	1	2	3	4	5	6
r, mm	60.41	61.10	61.79	62.47	63.16	63.84
θ_i	78.75	78.43	62.87	62.74	33.85	33.49

Optimization of the ends is most conveniently done by adjusting the straight-section lengths of two of the three double layers, similar to what is presently done with cable-wound dipoles. Since the Corrector dipoles do not occupy a large fraction of the ring, it is sufficient to minimize only the first unwanted harmonic, k = n = 3, but there is no reason not to optimize both. The three pairs of straight-sections will have incremental lengths at one end ℓ_i, i = 1,2,3, one of which will be zero, chosen so that the other two will be greater than zero. The set of linear equations to be solved for the ℓ_i is

$$[U]\,\vec{\ell} = -\vec{E} \qquad (16)$$

where [U] is one of the three 2 x 2 subsets of a 2 x 3 matrix, an element of which is $U_{ik} = \sum M_k$ where the sum is of the 2 values of M_k in the i^{th} double layer and M_k is calculated using q_n given by eqn (15). This M_k will hereafter be termed MS_k. Likewise, $E_k = \sum M_k$, where the sum is over the 6 layers of M_k is calculated using q_n given by eqn (10); this M_k will be termed ME_k. In principle, each of the three double layers could have its own end configuration, but adequate designs can be obtained using the same configuration for all three pairs; By the "same configuration" is meant short end ($\bar{A}\bar{B} = 3$ d), long end ($s_B = 1.5$d) or θ_2 equal to a fixed value. The setting up and solving of eqn (16) is done in a computer program "AUTOEND" which gives the two ℓ_i for each of the three solutions, and in addition calculates L_e and L_p for that solution which has both ℓ_i greater than or equal to zero. The effective length is now given by

$$L_e = \left[\sum_{i=1}^{6} (\ell_i\, MS_{1,i} + ME_{1,i})\right] \Big/ \left[\sum_{i=1}^{6} MS_{1,i}\right] \qquad (17)$$

The physical length L_p is now the maximum value of $z_2(\theta_0 = \theta_i) + \ell_i$ for the six layers.

Figure 2 is the output generated by AUTOEND for four configurations; each configuration has five lines of print. The first of the five lines gives m, α and a parameter T2 which controls how θ_2 is generated: T2 = − 1 generates short ends, T2 = 0 generates long ends and T2 > 0 is equal to a fixed value of θ_2. The next three lines give the calculated straight-section lengths ℓ_i, i = 1,2 or 3 which make the integrated harmonics k = 3 and 5 equal to zero (in each line, the missing ℓ_i is held at zero). The final line is the effective length and the physical length in meters. The surprising thing is that the listed value of α, selected (by cut and try) to minimize the positive lengths, results in a second layer also having zero length. Of the four cases, the shortest physical length results from the "long end" case, i.e., $s_B = 1.5$ d in all six layers. Figure 3 shows the developed inner layer of each pair of the three double layers for this case, which has an effective length of 62.7 mm and physical length of 99.4 mm.

References

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[3] F.E. Mills and G.H. Morgan, "A Flux Theorem for the Design of Magnet Coil Ends", Particle Accelerators 5, pp. 227 – 235 (1973).

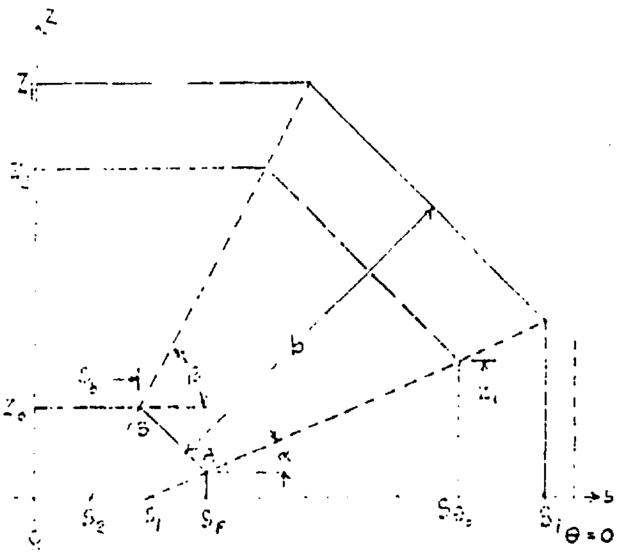

FIG 1

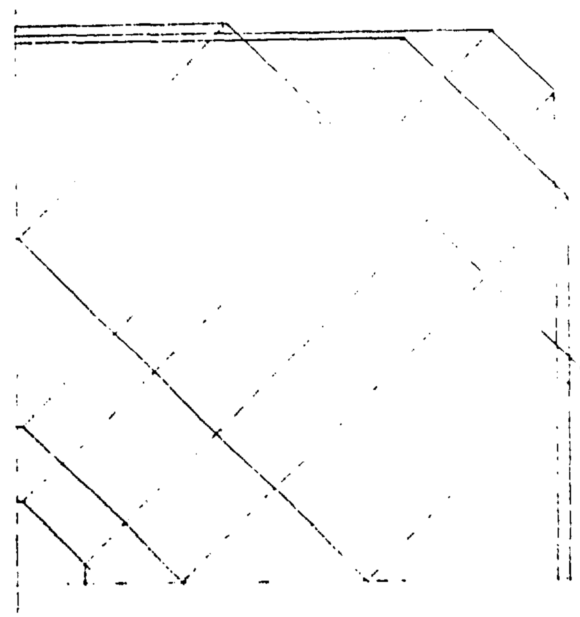

FIGURE 3


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M,A,T2 =  1    32.7145   -1.0000
L(1), L(2) =   0.06995   0.00000
L(1), L(3) =   0.06995   0.00000
L(2), L(3) = -0.07396 -0.06972
EFF LENGTH, PHYS LENGTH = 0.06532 0.15488
M,A,T2 =  1    44.9500    0.0000
L(1), L(2) =   0.00350   0.00000
L(1), L(3) =   0.00350   0.00000
L(2), L(3) = -0.00370 -0.00349
EFF LENGTH, PHYS LENGTH = 0.06271 0.09937
M,A,T2 =  1    43.7718   98.0000
L(1), L(2) =   0.06050   0.00000
L(1), L(3) =   0.06050   0.00000
L(2), L(3) = -0.06397 -0.06030
EFF LENGTH, PHYS LENGTH = 0.07965 0.15479
M,A,T2 =  1    42.2531   90.0000
L(1), L(2) =   0.07261   0.00000
L(1), L(3) =   0.07261   0.00000
L(2), L(3) = -0.07677 -0.07237
EFF LENGTH, PHYS LENGTH = 0.08105 0.16271
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FIGURE 2