ABSTRACT: Magnetic surface break-up caused by overlapping of mags- netic islands, created by resonant helical windings and by plasma oscil- lations, is investigated integrating numerically the differential equations for the magnetic field lines.

1 - INTRODUCTION

Disruptive instabilities limit the tokamak operation and, in spite of the fact that they are preceded in several cases by some macroscopic modes, their causes are still unclear (Pulsator Team, 1985; Vannucci et al, 1988a). These instabilities can also be triggered by resonant magnetic field created by helical windings (Pulsator Team, 1985; Robinson, 1985). The effect of helical wind- ing perturbations for a tokamak plasma was also investigated in the brazilian small TBR-1 tokamak (Vannucci et al, 1988b). During the TBR-1 discharges, disruptive events without the total loss of the confinement, called minor disruptions, were observed and asso- ciated to the destruction of the magnetic surfaces caused by the m/n=2/1 and 3/1 magnetic islands (Vannucci et al, 1988c).

The objective of this work is to investigate the magnetic surface break-up due to the overlapping of magnetic islands, cre- ated by resonant helical windings and by plasma oscillations. The degree of chaotic distribution of the magnetic field lines outside the remaining magnetic surfaces depends upon the resonance ampli- tudes, is evaluated in this work.

A linear superposition of the equilibrium field with the resonant helical perturbations and resonances due to toroidal con- nections are considered. The differential equations for the field lines have been numerically integrated for various equilibria and perturbation strengths (Heller and Caldas, 1988). Typical parameters
of the tokamak TBR-1 are used in the numerical applications.

2 - BASIC EQUATIONS

A plasma confined in a large aspect-ratio tokamak is represented by a periodical cylinder with length $2\pi R$ and radius $a$. The MHD equilibrium is determined by the poloidal $B_{o\theta}$ and the toroidal $B_{o\phi}$ magnetic field components. The equilibrium magnetic surfaces created by these fields are characterized by the safety factor

$$q = \frac{r B_{o\phi}}{R B_{o\theta}}.$$  (1)

For the rational surface with radius $r_{m,n}$,

$$q(r_{m,n}) = \frac{m}{n}.$$  (2)

The helical perturbations on equilibrium are assumed to be created by resonant helical windings or due to saturated tearing instabilities simulated by perturbed currents at rational surfaces (Fernandes et al, 1988). In this article it is considered the linear superposition of the unperturbed equilibrium $\hat{B}_o(r)$ with two resonant perturbations with different helicities:

$$\hat{B}(r,\theta,z) = \hat{B}_o(r) + \hat{b}(r,\mu) + \hat{b}'(r,\mu')$$  (3)

where

$$\mu' = m'\theta - \frac{n'}{R} z.$$  (4)

and $r, \theta$ and $z$ are cylindrical coordinates.

The magnetic field created by electrical currents $I$ flowing in $m$ pairs of helical windings, equally spaced, with radius $b$ wounded on a circular cylinder (corresponding to a large aspect-ratio tokamak) exhibits helical symmetry. This field depends on the coordinates $r$ and $u$. The ratio $n/mR$ characterizes the winding helicity. For currents $I$ flowing in opposite direction in adjacent windings the field can be expanded in a harmonic series and near the axis ($r<<R$) approximated by a single harmonic.
The resonant poloidal magnetic field oscillations observed in low tokamak plasmas, known as Mirnov oscillations, are related to perturbations of the plasma current on the rational surfaces with \( q = m/n \), distributed poloidally and toroidally according to the measured \( m \) and \( n \) numbers of the relative phases of the Mirnov oscillations. It corresponds to helical sheet currents, following the field lines on the resonant surfaces, with amplitudes \( j_{m,n} \) obtained from the measured Mirnov oscillations. For \( r/R \ll 1 \), the magnetic field produced by such a perturbation is

\[
\hat{b}(r,\theta,z) = (-K_1 j_{m,n} e^{-r^m}) \cos \theta + (-K_1 j_{m,n} e^{-r^m}) \sin \theta + (\frac{K_1 n}{r^m}) \cos \theta
\]

for \( r/r_{m,n} < 1 \) \((-1)^m(-m)\) has to be taken in the exponent.

3. MAGNETIC SURFACE DESTRUCTION

A cylindrical symmetric system with an helical perturbation superimposed upon it possesses magnetic surfaces. Magnetic surfaces break-up occurs due to the destruction of the system symmetry. As a symmetry breaking perturbation due to magnetic islands created by different resonant helical fields grows, magnetic surfaces are destroyed (Morozov and Solovev, 1966). The degree of chaotic behaviour depends upon the strength of perturbations created by external windings currents \( I \) and by helical surface currents \( j_{m,n} \). This dependence can be indicated by the stochasticity parameter \( S \) defined as (Chirikov, 1979):

\[
S = \frac{\Delta m,n + \Delta m',n'}{2|r_{m,n} - r_{m',n'}|}
\]

where \( \Delta m,n \) is the magnetic island width.

To investigate the magnetic surface destruction, the differential equations for the field lines

\[
\frac{dr}{B_r} = \frac{r d\theta}{B_\theta} = \frac{dz}{B_z}
\]

represent the magnetic field lines.
have been numerically integrated for several equilibria and perturbation amplitudes. Intersections of the field line trajectories with a poloidal (constant $z$) plane have been determined. When a magnetic surface exists, the field line intersections obtained from a starting point lie on a closed line. However, around the magnetic islands these intersections are randomly distributed. This effect is stronger for higher perturbation amplitudes. Starting the integrations with points between the resonant surfaces but outside the islands, the fraction $\alpha$ of intersections in the Poincaré map outside an area delimited by two circles (including the intersections of the resonant surfaces with the considered plane) have been obtained as a function of the perturbation strengths and $S$. For the same starting points, it has also been computed the flux $F$ related mainly to chaotic lines going through the cylindrical surfaces associated to the area considered in the calculation of $\alpha$.

For the small helical perturbations considered in this article, the major effect of the toroidal curvature on the magnetic surface destruction of a large aspect-ratio tokamak is the appearance of $(m \pm 1)$ satellite magnetic islands on the rational surfaces with $q = (m \pm 1)/n$. The outward displacement of the magnetic surfaces proportional to $r/R$ causes the magnetic islands to overlap for points on the outside of the magnetic axis for slightly smaller perturbation strengths and has a negligible influence on the computed values of $\alpha$ and $F$. Therefore, in this work the toroidal corrections have been taken into account by multiplying $B_{0z}$ by the factor $(1 + \frac{r}{R} \cos \theta)^{-1}$. The coupling of the $m/n$ term with the $m=1/n=0$ term of $B_{0z}$ has been obtained by integrating the field lines equations with the corrected $B_{0z}$.

4. NUMERICAL RESULTS AND DISCUSSION

An example of the mentioned computations is considered in this section for an equilibrium with $(q(a)=5$ and $q(0)=1$). In the TBR-1, $R=30cm$, $a=8cm$ and $b=11cm$.

It has been considered the perturbation caused by three pairs of $m=3/n=1$ helical windings. Figure 1 shows the Poincaré map for $I=140A$. The satellite islands are due to the toroidal curvature. The intersections between the islands are randomly dis-
Figure 2 shows the fraction of intersections, outside an area including the circles with radii \( r_3,1 \) and \( r_4,1 \), as a function of \( l \) and \( S \). It can be seen that \( \alpha \) increases with \( S \) for \( S > 0.7 \) and saturates for \( S \geq 1.1 \). Figure 3 shows the increasing of the associated magnetic flux \( F \) (defined in the section 3) with \( S \) and \( l \), a significant behaviour for energy transport in tokamak (Rebu et al., 1986). The level of the fluctuations \( \sigma_\alpha \) in the computed \( \alpha \) and \( F \) values was evaluated by considering several different starting points very near each other on the Poincaré map (figure 4). As it is shown in the figures 2 and 4, \( \alpha \) and \( \sigma_\alpha \) increases with \( S \) and saturates for \( S \geq 1.1 \). This is a consequence of the random distribution of the magnetic field lines and might be used as a more realistic criterium than the Chirikov's one (\( S=1 \)), to characterize the predominance of a chaotic zone between the considered resonant surfaces. In fact, minor disruptions observed in TBR-1, associated to the magnetic surfaces destruction (Vannucci et al., 1988c), occur for \( S \geq 1 \).

*Partially supported by CNPq.

REFERENCES


Fig. 1 - Intersection of magnetic field lines with a poloidal surface for $q(a)=5$, $q(0)=1.2$, $I(1,1)=140A$.

Fig. 2 - Variation of $a$ as a function of 3/1 helical current $I$ and $S$ for $q(a)=5$ and $q(0)=1.5$. Satellite resonances were considered.

Fig. 3 - Variation of $F$ as a function of 3/1 helical current $I$ and $S$ for $q(a)=5$ and $q(0)=1$. Satellite resonances were considered.

Fig. 4 - Standard deviation $s$ of the distribution of the computed values of $a$ as a function of $S$, for $q(a)=5$. 

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