Abstract: The systematics of the experimental data on the density of neutron resonances and low-lying levels is considered taking into account of the vibrational increase of the level density together with the shell and superconducting effects.

The density of excited level is the most important characteristic of the statistical description of different processes related to the decay of a compound nucleus. The widely used Fermi gas model /1,2/ does not allow to describe consistently the existed experimental data on the statistical properties of nuclei because its relations do not take into account significant shell inhomogeneities in the spectrum of single-particle levels, the correlation effects of the superconducting type and the coherent effects of a collective nature. The rigorous microscopic methods of analysis of these effects prove to be very time consuming and that strongly restricts their practical application/3,4/. It is therefore important to look for a description of the level density which will take into account to the necessary extent the main ideas of the theory about the structure of highly excited nuclei, while remaining sufficiently simple and convenient for practical usage. For heavy nuclei with $A \gtrsim 150$ where the rotational effects play essential role, the consistent phenomenological description of the nuclear level densities was considered in Ref./5/. In present work we want to expand this approach on more light nuclei in which the collective increase of level density is associated with the vibrational excitations.

Let us discuss shortly the main components of the consistent description of the nuclear level density. The influence of the pairing effects of the superconducting type on the nuclear properties can be characterised by the correlation function $\Delta_0$ which directly defines the even-odd difference of nuclear masses and the gap $2\Delta_0$ in the spectrum of quasi-particle excitation of even-even nuclei. The correlation function is connected the critical temperature $t_{cr} = 0,567 \Delta_0$ of the phase transition from the superconducting state to the normal state and
with the critical excitation energy

$$U_{cr} = 0.472 \alpha_{cr} \Delta^2 - n \Delta$$

(1)

where $n = 0, 1$ and $2$ for even-even, odd and odd-odd nuclei. Above $U_{cr}$ the level density and other statistical characteristics of a nucleus can be described by the relations of the Fermi-gas model with the effective excitation energy

$$U^* = U - 0.152 \alpha_{cr} \Delta^2 + n \Delta$$

(2)

Below the phase transition point the relations for the thermodynamical functions are more complicated, but the simple parametrisation of these functions suitable for the practical calculations is given in Ref. /5,6/. In these works there are also the discussions of the differences of the level densities in superfluid nuclear model and the Fermi gas model.

The shell inhomogeneities in the single-particle spectrum lead to a certain dependence of the level density parameter $a(U)$ on the excitation energy. However the shell effects become weaker with the increase of the excitation energy and at high energies the level density parameter will be defined by the asymptotic value

$$\tilde{a} = \alpha A + \beta A^{2/3}$$

(3)

For phenomenological description of the energy dependence of this parameter we can use the relation

$$a(U, Z, A) = \begin{cases} \tilde{a} \left\{ 1 + \frac{\delta \xi_0 (Z, A)}{\xi_0} \frac{f(U^*)}{U^*} \right\} & \text{for } U > U_{cr} \\ \alpha_{cr} \left( \frac{U_{cr}}{Z, A} \right) & \text{for } U < U_{cr} \end{cases}$$

(4)

where $\delta \xi_0$ is the shell correction in the nuclear binding energies /7/ and $f(U) = 1 - \exp(-\gamma U)$ is a "universal" function defining the energy behaviour of the shell effects. We used the same values of parameters as in Ref. /5/ (in units of MeV$^{-1}$): $\alpha = 0.730$, $\beta = 0.1147$ and $\gamma = 0.40 /A^{1/3}$.

When the collective effects are taken into account the level density of quasiparticle excitations must be multiplied by the coefficient of the vibrational increase of the level density

$$\kappa_{vib} = \exp (\delta S - \delta U / t)$$

(5)

where $\delta S$ and $\delta U$ are changes of the entropy and the excitation energy arising as the result of addition of the collective modes in the heated nucleus with the temperature $t$. These functions are defined by the relations:

$$\delta S = \sum_i (2 \lambda_i + 1) \left[ (1 + \bar{n}_i) \ln (1 + \bar{n}_i) - \bar{n}_i \ln \bar{n}_i \right], \delta U = \sum_i (2 \lambda_i + 1) \omega_i \bar{n}_i$$

(6)

where $\omega_i$ are energies of the vibrational excitations, $\lambda_i$ are degrees of degeneracy of them and $\bar{n}_i$ are mean occupation numbers. If we use the relations of an ideal Bose-
gas for occupation numbers then \( \text{Exp. (5) has} \) \( \alpha \) meaning of the adiabatic addition of the vibrational excitations to all possible quasi-particle excitations of the nucleus /9/.

For non-ideal Bose-excitations the occupation numbers can be defined as following:

\[
\overline{n}_i = (2\pi)^{-1} \int_{-\infty}^{\infty} d\omega \gamma_i(\omega) \left( e^{\omega/\epsilon_i} - 1 \right)^{-1} \left[ \omega_i + \epsilon_i - \omega \right]^2 - \frac{\epsilon_i^2}{\omega_i + \gamma_i} \right]^{-1}
\]

where \( \epsilon_i \) is energy shift and \( \gamma_i \) is damping width of the vibrational excitations. These values depend both on the energy \( \omega_i \) and the integrate variable \( \omega \), so for the definition of them it is necessary to solve correctly the complicated many body task of interacting Bose and Fermi-excitations. We did not analyse such task but only took the simplest approximation for the occupation numbers

\[
\overline{n}_i = e^{-\gamma_i/2\omega_i} \left( e^{\omega_i/\epsilon_i} - 1 \right)^{-1}.
\]

This approximation gives the relations of an ideal Bose-gas for \( \gamma_i \to 0 \) and provides necessary decrease of the coefficient \( K_{\text{vibr}} \) for increasing \( \gamma_i \).

We can expect that the damping of the vibrational excitations in nuclei is similar to the damping of the zero-sound in the Fermi-liquid theory which is described by the relation

\[
\gamma_i = C \left( \omega_i^2 + 4\omega_i^2 + \epsilon_i^2 \right).
\]

From the observed spreading widths of the giant isoscalar quadrupolar resonance it is possible to obtain a rough estimation of the constant \( C = 0.05 \text{ A}^{1/3} \text{ MeV}^{-1} \). The temperature dependence both of the occupation numbers and the coefficients which obtained from the considered relations (5-8) are shown on the Fig.1. The behaviour of the same functions in the adiabatic limit is shown also.

In calculations of the level densities we used the experimental values of the energies \( \omega_2^{+} \) of the first \( 2^{+} \) - levels of even-even nuclei /11/ and the simple interpolations of these energies for nearest odd and odd-odd nuclei. For the octupolar excitations, which are influenced on the level densities essentially weaker than the quadrupols, we used the averaged description of the observed energies \( \omega_3^{-} = 50 \text{ A}^{-1/3} \text{ MeV} \). The experimental values of the shell corrections \( \delta \xi \) were taken from Ref. /7/ and the correlation functions were accepted as \( \Delta_\omega = 12 \text{ A}^{-1/2} \text{ MeV} \). From optimal description of the experimental data on the neutron resonance densities /12/ the value of coefficient \( C = 0.075 \text{ A}^{1/3} \text{ MeV}^{-1} \) was obtained which characterized phenomenologically the effective decrease of the vibrational enhancement of the level density at highly excited nuclei.
The temperature dependence of the occupation numbers and the coefficients of vibrational increase of the level density of the nucleus $^{52}$Cr under adiabatical approximation (desh-dot curves) and taking into account the damping of vibrational excitations (solid curves).

The optimal parameters do not guarantee of course an exact agreement with the experimental data for any nucleus. However such coincidence is needed often for the calculations of the neutron spectra and the excitation functions of different nuclear reactions. In the framework of described approach we determined also the set of individual parameters $\tilde{\alpha}$ and $\delta_{\text{eff}}$ which provides the description of the neutron resonance density /12/ and the number of low-lying levels /11/ for each nucleus. These parameters are shown on Fig.2. The individual parameters display the fluctuations which are correlated with the shell structure of nuclei. These fluctuations reflect first and foremost the simplifications connected with the replacement of the realistic correlation functions for protons and neutrons by the averaged value $\Delta \omega$. For the magic numbers of protons or neutrons the values of the correlation functions must be essentially smaller than for the nonmagic numbers and just this effect is displayed in the fluctuations of individual parameters on Fig.2.

At first glance it might seem that the considered systematics of the level density parameters are not distinguished strongly from the systematics based on the relations of the back-shift Fermi gas model /2/. But it is not correct conclusion. The Fer-
The effective shifts of excitation energies $n_0 + \delta_{\text{eff}}$ and the asymptotical values of the level density parameters $\tilde{\alpha}/A$ for the even-even (x), odd (o) and odd-odd (*) nuclei. Solid curves are the averaged values parameter $\tilde{\alpha}/A$ and $n_0 = 12n/\sqrt{A}$ MeV.

The Fermi-gas model parameters can be considerably distorted by the shell, superconductive and collective effects. In our approach the obtained parameters have the strict physical meaning and they are in good agreement with the results of pure theoretical calculations of the statistical characteristics of excited nuclei /4/.

The necessity of using more rigorous, but inescapably more complicated than the Fermi-gas, models for analysis and description of the nuclear level densities seems almost obvious today. The
complications of the analysis are justified by the consistency of resulting parameters characterizing the diverse experimental information on the statistical properties of nuclei. We hope that the above suggested approach can be fruitful for the practical calculations of the level densities in a wide range of excitation energies and mass numbers.

References