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M.A. Braun**
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

The parton picture of multiple hA and AA scattering at high energies is developed. It is shown that it leads to the standard Glauber amplitude provided the number of partons in a hadron is distributed according to Poisson's law. Within this picture collisions of more than a pair of nucleons are considered. For AA scattering a two-dimensional effective quantum field theory is constructed which allows to conveniently calculate contributions to the amplitude with a given number of loops. The AGK rules for AA scattering are established. Inclusive cross-sections for particle production in hA and AA collisions are studied both in the non-cumulative and cumulative kinematical regions.

MIRAMARE - TRIESTE
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** Permanent address: Department of High Energy and Elementary Particle Physics, Physical Faculty, Leningrad University, Leningrad, USSR.

§1 Introduction

Much attention has lately been given to interactions with nuclei at high energies, prompted by expectations to observe the quark-gluon plasma. Theoretical analysis of the scattering off nuclei involves the study of multiple scatterings of the projectile with nuclear constituents at the first step and of elementary interactions of the constituents between themselves at the second. This latter stage belongs to the realm of the standard hadron physics. It is the specific nuclear side of the problem related to multiple scattering will be the subject of this paper.

The standard picture of multiple scattering off nuclei is based on the Glauber model /1/. In this model the fast incoming hadron successively strikes several nucleons on its way through the nuclear target. It is then a simple matter to express the hA elastic amplitude via hN amplitudes at small momentum transfer and the nuclear transverse density (profile function). However it was later realized that this simple picture has little to do with what actually occurs during a collision off a nucleus at energies in the multi-Gev region. The reason is that a hadron needs some time after a collision to become once again capable of new interactions - the so called formation time, which grows linearly with the momentum of the hadron /2,3/. Once the distance corresponding to this formation time grows greater than the dimension of the nucleus, repeated interactions within the nucleus become impossible and the Glauber picture becomes wrong. In spite of this theoretical argument experimental observations show that the Glauber formula somehow remains approximately valid up to greatest energies achieved so far.

There exist several explanations of this phenomenon in literature. One of the most common is that the approximate validity of the Glauber model is a consequence of cancellations of all contributions coming from cuts in the complex cumulative transferred momenta squared, with only pole contributions remaining as a result /4, 5/. Clearly this is not an explanation at all but rather a statement of the validity of the Glauber model made in another language. The mentioned cancellations appear quite casual. A different explanation is based on the Regge model. It is assumed that multi-Regge vertices of a hadron factorise /6/. Then for hA scattering one obtains Glauber-like formulas. The difficulty with this picture are related to its asymptotic character at high energies. For finite energies problems arise such as a pattern of energy division between various interactions. In this paper we advocate still another picture of hA and AA interactions at high energies based on the well-known parton model. As usual we assume that the fast incoming particle breaks up into partons long before interactions. Each parton then only once interacts with a nucleon of the target nucleus and much later joins with other partons to form the final hadrons. Such a picture correctly describes the space-time development of the process and naturally realizes the concept of the formation time. We shall show that this picture leads to the usual Glauber formulas provided the number of partons in the hadron is distributed according to Poisson's law.

Within the parton picture of hA interactions

we then consider parton interactions with more than one nucleons at a time. Such interactions may occur if the nucleons are close to one another and form what is sometimes called nuclear correlations / 7 /. The inclusion of multinucleon interactions makes it possible to consider effects which essentially involve the bound character of nucleons in the nucleus such as cumulative particle production and nuclear structure functions at $x>1$.

The generalization of this formalism to AA scattering is considered in §§ 4,5. Although ideologically this is rather straightforward, some technical difficulties arise which have long been known to exist in the Glauber theory of AA interaction. To overcome these we introduce a formalism of an effective local 2-dimensional quantum field theory. It leads to a natural expansion of the amplitude in the number of loops and allows to analyze the bulk of the interaction, the tree part, analytically. Note that the first idea to apply an effective quantum field theory (nonlocal, 3-dimensional) to analyze AA interactions appeared in /8/. However the method used there incorrectly takes into account disconnected parts and may have some relevance to AA scattering for constant profile functions and central collisions only.

We finally study particle production. To this end the AGK-rules are established for AA scattering and multinucleon interactions. On their basis one-particle inclusive cross-sections are discussed including the production of particles in the cumulative kinematical region.

§2. hA interactions and the parton model

We consider the process shown in Fig.1: the incoming hadron breaks up into partons which then once each interact with nucleons of the target. Interactions are shown with wavy lines, which correspond to parton-nucleon amplitudes α. Spectator partons and nucleons are not shown. The number of active particles is n, the total number of partons, spectator included, is N. In the lab. system where $p_A = (M_A, 0)$ one can integrate over zero components of relative momenta of the nucleons in the target to separate out the n-fold nuclear form-factor /5/. One then obtains for the amplitude corresponding to Fig.1

$$A_{nN} = W_{nN} F_{nA} \qquad (1)$$

Here F_{nA} is the nuclear part, which in the approximation where the multinucleon nuclear density factorizes has the usual form

$$F_{nA} = C_A^n \int d^2 \ell \, e^{i q_\perp \ell} \, T^n(\ell) \qquad (2)$$

with T being the profile function

$$T(\ell) = \int dz \, \rho(\ell, z) \qquad (3)$$

and ρ being the one-nucleon nuclear density normalized to unity. W_{nN} is the high energy part of the amplitude which includes vertices for the transition of the hadron into N partons and vice versa, propagation of the partons and interaction of n of them with nucleons. For simplicity let us consider all partons to be of the same sort. It is convenient to write the expression for W_{nN} in the ordinary space in the antilab. system $p_h = (M_h, 0)$ using light-cone variables

$$i W_{nN} (2\pi)^3 \delta^3(p_h - p_h') = \left(s\sqrt{2}/(M_h (N-n)!)\right) (i\alpha)^n \times$$
$$\times \int \prod_{i=1}^{N} d^4 x_i \, d^4 x_i' \, i\bar{\chi}_N(x_i) \, i\chi_N(x_i') \prod_{i=n+1}^{N} \left(-i\Delta_c(x_i - x_i')\right) \times \qquad (4)$$
$$\prod_{i=1}^{n} d^3 z_i \left(-i\Delta_c(x_i - z_i) \, 2i\partial_{i-} \left(-i\Delta_c(z_i - x_i')\right)\right)$$

Here $\delta^3(p_h - p_h') = \delta(p_{h+} - p_{h+}')\,\delta^2(p_{h\perp} - p_{h\perp}')$, $d^3 z_i = dz_{i-} \, d^2 z_{i\perp}$, $z_{i+} = 0$.

and the derivatives are taken in z_{i-} ($i = 1, \ldots n$). We assume that the parton-nucleon amplitudes are proportional to their center-of-mass energy squared: $\alpha(s_i) = (s_i/s)\alpha = 2s_i\hat{\alpha}$, $s_i = (p_i + p_A/A)^2$, $s = (p_h + p_A/A)^2$ where p_i (p_h) is the momentum of the i-th parton (incoming hadron).

But for spectators, the vertices χ_N together with propagators would give the parton wave functions of the hadron in the hyperplane $z_{i+} = 0$:

$$\chi_N(z_i) = \langle \Psi_0 | T \prod_{i=1}^{N} \varphi(z_i) | \Psi_h \rangle_c, \quad z_{i+} = 0 \qquad (5)$$

φ is the parton field, c means the connected part. For $N > n$ we can use the identity

$$\int d^3 z \, \Delta_c(x - z) \, 2\partial_- \Delta_c(z - x') = \Delta_c(x - x') \, \mathrm{sign}(x - x')_+ \, \theta(|x_+ - x_+'| - |z|) \qquad (6)$$

to insert into the integrand of (4) an extra propagator and a derivative for each spectator parton. Note that in (4) the values of x_{i+} (x_{i+}') are positive (negative), which may be seen by transforming (4) to the lab. system. We thus obtain

$$i W_{nN} \delta^3(p_h - p_h') = \frac{s\sqrt{2}}{(N-n)!\, M_h} (i\hat{\alpha})^n \int \bar{\chi}_N(z_i) \prod_{i=1}^{N} d^3 z_i \, 2i\partial_{i-} \, \chi_N(z_i) \qquad (7)$$

The normalization condition for the parton wave function reads $\sum \rho_N = 1$ where ρ_N is the probability to find exactly N partons in the hadron:

$$\rho_N \, 2p_{h+} \delta^3(p_h - p_h') = \frac{1}{N!} \int \bar{\chi}_N(z_i) \prod_{i=1}^{N} d^3 z_i \, 2i\partial_{i-} \, \chi_N(z_i) \qquad (8)$$

The comparison of (7) and (8) gives

$$i W_{nN} = 2s (i\hat{\alpha})^n \rho_N \, N!/(N-n)! \qquad (9)$$

The total hA amplitude for a given number of active particles is obtained from (9) after summation over all $N \geq n$.

For its high-energy part W_n^{-} we get

$$iW_n = 2s(i\alpha)^n \sum_{\Lambda=n}^{\infty} \zeta_\Lambda \Lambda!/(\Lambda-n)! \qquad (10)$$

For $n=1$ we have $W_1 = \alpha\langle\Lambda\rangle$ where $\langle\Lambda\rangle = \sum \Lambda \zeta_\Lambda$. Then from (1) and (10) we conclude that $\alpha\langle\Lambda\rangle$ is the hN scattering amplitude a. Rewriting (10) in terms of $\hat{a} = a/2s$ we obtain

$$iW_n = 2s(i\hat{a})^n \xi_n \qquad (11)$$

with

$$\xi_n = \sum_{\Lambda=n}^{\infty} \zeta_\Lambda \Lambda!/(\langle\Lambda\rangle^n(\Lambda-n)!) \qquad (12)$$

The Glauber formula would follow if $\xi_n = 1$. One can easily check that this condition is satisfied if the number of partons is distributed according to Poisson's law

$$\zeta_\Lambda = \exp(-\nu)\nu^\Lambda/\Lambda!, \quad \langle\Lambda\rangle = \nu. \qquad (13)$$

§3. Multinucleon interactions

The observation of cumulative particle production unequivocally shows that the simple Glauber expression (11) cannot be the whole truth. Simultaneous interactions with more than one nucleon at a time have also to take place. A typical diagram corresponding to such a process is shown in Fig.2. Inspection of Fig.2 makes it evident that multinucleon interactions do not practically change the high-energy part W_n (11). Some of the amplitudes a have only to be replaced by a_k where k is the number of nucleons at rest with which the incoming hadron interacts simultaneously. Let n_k be the number of partonic interactions with k nucleons at a time; $\sum_k n_k = n$. Then for $\xi_n = 1$

$$iW_{\{n_k\}} = 2s\prod_k (i\hat{a}_k)^{n_k} \qquad (14)$$

In the nuclear part the nucleons interacting simultaneously prove naturally to occupy the same point in the nucleus, so that for a simultaneous interaction of k nucleons the profile function will be given by

$$F_{\{n_k\}A} = T_k(b) = \int dz\, \rho^k(b,z)(2m)^{1-k} \qquad (15)$$

Thus for a given pattern of partonic interactions characterized by a set $\{n_k\}$ the nuclear factor will be

$$F_{\{n_k\}A} = a!\, C_A^a \int d^2b \prod_k \frac{1}{n_k!} T_k^{n_k}(b) \qquad (16)$$

Here $a = \sum k n_k$ is the total number of active nucleons. To obtain the total hA amplitude we have to multiply (16) by (14) and sum over all possible sets $\{n_k\}$. We get the sum

$$\prod_k \sum_{n_k} (x_k^{n_k}/n_k!) \frac{A!}{(A-\sum k n_k)!} = \left(1+\sum_k x_k^{1/k}\right)^A - 1 \qquad (17)$$

where only integer powers of x_k are assumed to be retained on the right-hand side. With this assumption we finally get the elastic hA amplitude

$$iA = 2s\int d^2b\, e^{iq_\perp b}\left[\left(1+\sum_k (i\hat{a}_k T_k(b))^{1/k}\right)^A - 1\right] \qquad (18)$$

For large $A\to\infty$ the asymptotics can be obtained directly from (17). Approximating $A!/(A-\sum k n_k)! = \prod_k A^{k n_k}$ we get

$$iA = 2s\int d^2b\, e^{iq_\perp b}\left[\exp\left(\sum_k i\hat{a}_k A^k T_k(b)\right) - 1\right] \qquad (19)$$

If we neglect all contributions from $k>1$ we return to the standard Glauber formula.

One should keep in mind that inclusion of multinucleon interactions requires a change in the normalization of the nucleon density ρ. Indeed in the same approach the baryonic number form-factor of the nucleus is

$$B(q_\perp^2) = \int d^2b\, e^{iq_\perp b} \sum_k C_A^k\, k\, T_k(b) f_k \qquad (20)$$

where f_k is the baryonic vertex of k nucleons at the same point at zero momentum transfer, $f_1 = 1$. At $q_\perp = 0$ we get the normalization condition

$$A = \sum_k k f_k \bar{n}_k \qquad (21)$$

where

$$\bar{n}_k = C_A^k \int d^3x\, \rho^k(x) \qquad (22)$$

is the total probability for k nucleons to be at the same point. In the high-energy limit we expect the amplitudes σ_k to be proportional to k (i.e. to their center-of-mass energy squared). If they are also proportional to f_k (i.e. the baryonic charge of the target) then comparison of (19) and (21) shows that for constant ρ the hA amplitude does not change when multinucleon interactions are included. In this manner the theory knows that multinucleon interactions are nothing but the limiting case of ordinary interactions when positions of some nucleons coincide. So for total cross-sections multinucleon interactions may give non-trivial contributions only if they differ substancially from what we expect by the continuation of the ordinary pair interaction into the region of coinciding coordinates. For particle production however multinucleon interaction lead to completely new phenomena. This point will be commented further in §6.

§4. AA amplitude and the formalism of an effective quantum field theory.

The interaction of two nuclei, A' (projectile) and A (target), possesses a much more complicated geometry. Also multinucleon interactions in this process aquire some novel features absent in hA collisions. This latter point will be treated later in §5. Here we shall elaborate on the structure of the AA amplitude with only pair NN interaction taken into account.

The nuclear part of the amplitude can be separated in the same manner as for the amplitude on integrating over more components of relative momenta of nucleons in the target in the lab. system ($p_A = 0$) and in the projectile in the antilab. system ($p_{A'} = 0$). We then obtain the n-fold form-factor of the target and the n'-fold form-factor of the projectile depending correspondingly on the transferred momenta in the lab. and antilab. systems. Here n (n') is the number of active nucleons in the target (projectile). The rest of the amplitude is the high-energy part W. Diagrammatically it is a set of all graphs for the interaction of n nucleons of the target with n' nucleons of the projectile. These graphs may contain several disconnected parts. Each connected part does not depend on transferred transverse momenta. Integrations over these fix the transverse distance between the interacting nucleons equal to the impact parameter b. As a result the nuclear part corresponding to a given connected graph for the interaction of n nucleons of the target with n' nucleons of the projectile consists of the factor $\int d^2x_\perp d^2x'_\perp \, \delta^2(x_\perp - x'_\perp - b)\, T^n(x_\perp)\, T'^{n'}(x'_\perp)$.

Typical connected parts for W are shown in Fig.3. They include tree graphs (Fig.3a) as well as graphs with loops (Fig.3b). In the parton model each vertex is understood as shown in Fig.4. It is convenient to analyze W in the center-of-mass system. In this system the remaining integrations over transferred longitudinal momenta may be suitably expressed in terms of light-cone momenta $Q_{i\pm}$ transferred in partonic interactions. It is easy to see that the partonic part of the diagram belonging to the projectile depends only on Q_{i-} (as only the combination enters $p'_{i+} Q_{i-}$

$+p_{i-}' Q_{i+}$ where p_i' is a partonic momentum in the projectile and $p_{i-}' > 0$). Likewise the partonic part belonging to the target depends only on Q_{i+}. So the two integrals, in Q_{i-} and Q_{i+}, factorize and each of them can be represented in the ordinary space in the same form as (4) for hA scattering for each participating nucleon. As a result for each vertex in Fig.3 we shall get the same expression as (9). Some care has only to be taken to correctly write the amplitudes α which here refer to parton-parton interactions and may depend on transferred momenta in the loops. Upon summation over N and using the definition $1-\alpha\langle N\rangle^2$ we shall get for each vertex the same factor ε_n that entered the hA amplitude (11). For the Poisson distribution ε_n =1 and we then come to the standard Glauber formula for the AA elastic amplitude.

As is well-known the Glauber AA amplitude becomes very complicated for large A and A'. It can be substantially simplified in the formalism of an effective 2-dimensional local quantum field theory. The graphs shown in Fig.3 may be viewed upon as corresponding to a theory of two quantum fields $\varphi(y)$ and $\varphi^+(y)$ in a two-dimensional transverse space y with propagators $i\alpha(Q_\perp)$ and vertices $T(x_\perp)$ ($T'(x_\perp')$) for the target (projectile) at fixed $x_\perp$ ($x_\perp'$) (not to be confused with the coordinate y in the transverse space where act φ and φ^+). The exact correspondence is the following. Define the 2-dimensional Euclidean action

$$S = \int d^2y\, \mathcal{L}(y), \quad \mathcal{L} = \varphi^+ k \varphi + g(e^{\varphi}-1) + g'(e^{\varphi^+}-1). \qquad (23)$$

Then connected diagrams for the vacuum functional

$$\mathcal{Z}(g,g') = \int D\varphi\, D\varphi^+ \exp iS \qquad (24)$$

with $K = \alpha^{-1}$, $g = -iT(x_\perp)$, $g' = -iT'(x_\perp')$ give the correct expression for the integrand of the Glauber AA amplitude coming from a connected part of the NN interaction. As a matter of fact we shall also obtain some extra contributions from diagrams of the type shown in Fig. 5. To exclude these one should use the propagator $iK^{-1} = \ln(1+i\alpha)$. In practice one can always preserve the simpler propagator α and exlude the extra contributions directly. With these reservations the result can analytically be written as follows. Define the effective action $W(g,g')$ by

$$\mathcal{Z} = \exp i\Omega W \qquad (25)$$

where Ω is the volume $\int d^2y$. Develop W in a double series in powers of g and g':

$$W = \sum_{n,n'} W_{nn'}, \quad W_{nn'} = C_{nn'} g^n g'^{n'}. \qquad (26)$$

Then the AA amplitude with a given number n_i (n_i') of active nucleons from the target (projectile) in the i-th connected part is obtained as

$$i\mathcal{A}_{\{n_i,n_i'\}} = (A!A'!/c!(A-n)!(A'-n')!)\,2s \times$$
$$\times \int d^2b\, e^{iq_\perp b} \prod_{i=1}^{c} \int dx_{i\perp}^2 dx_{i\perp}'^2 \delta^2(x_i - x_i' - b)\, iW_{n_i n_i'}(g,g') \qquad (27)$$

with $g = -iT(x_\perp)$ and $g' = -iT'(x_\perp')$. The expression for the total AA amplitude $\mathcal{A}$ follows from (27) upon summation over all possible values of n_i, n_i' and the number of connected parts C. For small A and A' this can be done in a straightforward manner writing the sum term by term. For large A and A' this procedure becomes cumbersome.

To obtain a closed expression for $\mathcal{A}$ suitable for large A and A' we first represent $C_{nn'}$ as a double contour integral

$$C_{nn'} = \oint dz/(2\pi i z^{n+1}) \oint dz'/(2\pi i z'^{n'+1})\, W(z,z') \qquad (28)$$

The summations over n_i and n_i' facterize into

$$S_A(u_i) = \sum_{n_i} \left(A!/(A-n)!\right) \prod_{i=1}^{c} u_i^{n_i} \tag{29}$$

with $u_i = T(x_i)/z_i$ and the second analogous sum for the projectile. Using the identity

$$u^n = (u\,n!)^{-1} \int_0^\infty dt\, t^n \exp(-t/u), \quad \mathrm{Re}\, u > 0, \tag{30}$$

we introduce a facter $\prod(1/n_i!)$ in (29) whereupon the sums over n_i can be done explicitly:

$$\sum_n \left(A!/(A-n)!\right) \prod_{i=1}^{c} t_i^{n_i}/n_i! = \left(1 + \sum_{i=1}^{c} t_i\right)^A - \tag{31}$$

$$- (\text{terms with one of } n_i \text{ equal to zero}).$$

The right-hand side of (31) can be represented as a conteur integral

$$A!/(2\pi i^{A+1}) \oint d\tau\, \tau^{-A-1} \exp(i\tau) \prod_{i=1}^{c} \left(\exp(i\tau t_i) - 1\right) \tag{32}$$

So upon integrations over t_i we get

$$S_A' = A!/(2\pi i^{A+1}) \oint d\tau\, \tau^{-A-1} \exp(i\tau) \prod_{i=1}^{c} \left[(1 - i\tau u_i)^{-1} - 1\right] \tag{33}$$

Integrating over z and z' and summing over c we get finally

$$i\mathcal{A} = 2s\left(A!A'!/4\pi^2 i^{A+A'+2}\right) \int d\tau\, d\tau'\, \tau^{-A-1} \tau'^{-A'-1} \exp(i\tau + i\tau')$$

$$\times \int d^2b\, e^{iq_\perp b} \left[\exp\left(\int d^2x_\perp d^2x_\perp' \delta^2(x_\perp - x_\perp' - b)\, i h(g, g')\right) - 1\right] \tag{34}$$

with $g = \tau T(x_\perp)$ and $g' = \tau' T'(x_\perp')$.

This expression is exact and valid for any values of A and A'. For large A and A' we can use the stationary point method to find the asymptotics. Considering the internal integral over b as a smooth function of τ and τ' we find stationary points at $i\tau = A$ and $i\tau' = A'$. Therefore at $A, A' \gg 1$

$$i\mathcal{A} = 2s \int d^2b\, e^{iq_\perp b} \left[\exp\left(\int d^2x_\perp d^2x_\perp' \delta^2(x_\perp - x_\perp' - b)\, i W(g, g')\right) - 1\right] \tag{35}$$

with $g = -iAT(x_\perp)$ and $g' = -iA'T'(x_\perp')$.

The field theory formalism allows to conveniently sum the bulk of the contribution at large A and A' corresponding to tree graphs in an explicit form. The classical "equation of motion" for the action (23)

$$K\varphi = -g' \exp \varphi^+, \quad K\varphi^+ = -g \exp \varphi \tag{36}$$

allow for a nontrivial vacuum solution φ_c, φ_c^+ independent of y. The corresponding classical effective action W_0 has the form

$$W_0 = \hat{a}_0^{-1}(\varphi_c^+ \varphi_c - \varphi_c - \varphi_c^+) - g - g' \tag{37}$$

with $\hat{a}_0 = \hat{a}(q=0)$. Substitution of W_0 for W into (34) or (35) gives the AA amplitude without loops.[9, 10]

Putting further $\varphi = \varphi_c + \chi$, $\varphi^+ = \varphi_c^+ + \chi^+$ we can rewrite the action (23) in terms of χ and χ^+. Terms quadratic in χ and χ^+ will give the one-loop contribution to the effective action W_1:

$$W_1 = \frac{1}{2} i \int \frac{d^2q}{(2\pi)^2} \left[\ln\left(1 - \gamma\gamma^+ \hat{a}^2(q)\right) + \gamma\gamma^+ \hat{a}^2(q)\right] \tag{38}$$

with $\gamma = -\varphi_c^+/\hat{a}_0$, $\gamma^+ = -\varphi_c/\hat{a}_0$. Multiloop contributions will be given by vacuum diagrams for the self-interaction of χ and χ^+

$$\mathcal{L}_I = \sum_{k=3}^{\infty} \left(\gamma\chi^k + \gamma^+ \chi^{+k}\right)/k! \tag{39}$$

with propagators

$$\langle \chi\chi^+ \rangle = iK/(k^2 - \gamma\gamma^+); \tag{40}$$

$$\langle \chi\chi \rangle = -i\gamma^+/(k^2 - \gamma\gamma^+); \quad \langle \chi^+\chi^+ \rangle = -i\gamma/(k^2 - \gamma\gamma^+)$$

We expect that at large A and A' the main contribution comes from the classical part W_0. Calculations for $\hat{a}_0 g = \hat{a}_0 g' = 2.718$ (which corresponds to the overlap region for $A = A' \sim 60$) show that $W_1/W_0 \simeq 0.26\, \sigma^{el}/\sigma$ where σ (σ^{el}) is the total (elastic) NN cross-section.

For larger $\hat{a}_0 g$ this ratio becomes still smaller.

From (36) and (40) one concludes that the effective field theory becomes singular at $\varphi_0 \varphi_c^+ = 1$ (it corresponds to $a_0 g = 2.718$ for identical nuclei and appears for $A > 60$). This peculiarity of the AA amplitude was first mentioned in / /. It is evident that this singularity never appears in the amplitude itself, since in (34) we can always integrate over small τ and τ' thus avoiding the singular values of g and g'. Its meaning can only be felt when studying the asymptotics at $A, A' \to \infty$ in (35). Careful analysis shows that ambiguities associated with the singularity do not influence the asymptotical behavior of the amplitude itself. However they show themselves in the asymptotical behavior of the Glauber phase when the order of t and τ (τ') integrations in (34) is reversed. The correct prescription to handle the singularity in this latter case is not quite clear as yet.

§5. Multinucleon interactions in AA collisions

In hA collisions the detailed structure and physical origin of multinucleon interactions were irrelevant. All necessary to know reduced to the amplitudes a_K. In AA collisions, in contrast, this information is not sufficient. The reason is that in an AA collision a possibility arises for multiple interactions of a group of several nucleons at the same point in the target or projectile. In the framework of the original Glauber approach this corresponds to successive collisions of this group of nucleons, all moving at high velocity, with several nucleons of the target. To study such a process one has to know in some detail what is "a group of nucleons at the same point".

It is instructive to begin with the ordinary nuclear

theory based on a potential interaction between nucleons. Even if the potential is a purely pair one, the necessity to consider in the wave function only small relative momenta of nucleons when separating the nuclear form-factors leads to the introduction of multinucleon interactions for parts of diagrams with large relative momenta. This procedure is illustrated in Fig.6 where it is supposed that the two nucleons below interchange a large momentum Q (and correspondingly are at a distance $1/Q$). Although the interaction is a pair one, an effective 3-body interaction evidently emerges, an interaction of the projectile with the two nucleons simultaneously. To study such contributions one may split the pair potential into a long range and a short range parts and treat the short range part contributions explicitly and perturbatively whereas the long range part is taken into account by the wave function. Of course the presence of true many-body potentials will lead to multinucleon interactions as well. Consider now successive scatterings of a pair of nucleons shown in Fig.6 on several nucleons of the projectile as shown in Fig.7. One easily calculates that the contribution of the n-fold scattering $a_2^{(n)}$ has the n dependence of the form $a_2^{(n)} = (c + dn)a^n$, where a is the NN forward amplitude and c and d are some dimensionful constants. Thus it is not as easy to relate $a_2^{(n)}$ to $a_2^{(1)}$ as for the scattering of a single nucleon $a^{(n)} = a^n$ (instead $a_2^{(n)}$ can only be expressed through $a_2^{(1)}$ and $a_2^{(2)}$ and the relation involves a dimensionful constant). This argument shows that multiple scattering of a pair of nucleons may in principle lead to much more complicated expressions than the Glauber one.

Of course the picture of multinucleon interactions based on a potential model cannot be true. We expect that at small distances nucleons merge to form a structure made of quarks with some quantum levels in the spirit of the bag model. We call such a structure a multibaryon. In the simplest possible approach a multibaryon can be characterized only by its baryon number k (a $3k$ quark bag). In this picture a mutinucleon interaction may be viewed upon as shown in Fig.8a. It is then trivial to introduce multiple collisions of a multibaryon (Fig.8b) as a direct generalization of the case of a single nucleon studied in the parton picture in §2. If the number of partons in a multibaryon is distributed according to Poisson's law (with $\langle N \rangle$ depending on k) we shall obtain the Glauber relation for the n -fold scattering amplitude of a multibaryon $\mu^{(n)} = \mu^{n}$. The transition to multinucleon amplitudes will require a dimensionful constant β with a meaning of the probability for the merging of nucleons into a multibaryon:

$$a_k^{(n)} = \beta_k \mu_k^{(n)} \qquad (41)$$

From this we find $a_k^{(n)} = \beta_k^{1-n} a_k^{n}$, which generalizes the usual Glauber eikonal relation for multiple scattering. Note that this problem may also be considered in the framework of the Regge theory as a search of an appropriate simplifying factorizability condition for an n Regge vertex of k nucleons.

In the multinucleon picture the inclusion of multinucleons interactions becomes quite simple. Physically it amounts to the assumption that nuclei consist not only of single nucleons but also of multibaryons of various k with probabilities determined by β_k. All formulas obtained so far for single nucleons remain valid also for multibaryons. Omitting intermediate steps we write here only the final result

for the AA amplitude in this picture. Introduce the amplitudes μ_{kl} for scattering of two multibaryons of baryon numbers k and l ($\mu_{11} = a$). As usual $\hat{\mu}_{kl} = \mu_{kl}/\beta_k\beta_l$. Generalize the quantum field theory (23) by taking fields and φ_k, φ_k^{+} (a pair for each multibaryon) and a Lagrangian

$$\mathcal{L} = \varphi^{+} K \varphi + \sum g_k (e^{\varphi_k} - 1) + \sum g_k' (e^{\varphi_k^{+}} - 1) \qquad (42)$$

with $(K^{-1})_{ik} = \hat{\mu}_{ik}$. Then Eqs (34) and (35) remain valid if we substitute $g_k = -i(iz)^k \beta_k T_k(x_l)$ in (34) and $g_k = -iA^k \beta_k T_k(x_l)$ in (35) and similarly for the projectile.

§6. The AGK rules and inclusive cross sections

For physical applications one has to study absorptive parts of hA and AA forward amplitudes. The classification of contributions to the absorptive parts coming from different intermediate physical states is achieved by the well-known AGK rules first established for hh scattering in the multi-Regge approach /11/ and later proved for hA scattering in its nonrelativistic version /12/ and in a more general context in /13/. Here we generalize the AGK rules for AA scattering and also to include multinucleon interactions.

The validity of the AGK rules is in fact practically independent of the process under consideration and rests on the following very general property of the amplitude $\mathcal{A}$ for some collision involving composite particles. Let $i\mathcal{A}$ be a real functional of elementary amplitudes ia (which may be of different types generally but, for simplicity, we shall assume them identical), that is

$$i\mathcal{A} = F(ia), \quad F(z^*) = F^*(z) \qquad (43)$$

Then the AGK rules are valid for $\mathcal{A}$ in the same form as found in /11/. For our purpose we see that (43) is obeyed by hA and AA amplitudes with multinucleon interactions taken into account (Eqs (18) and (27)). Therefore the AGK rules hold for them.

The proof of the AGK rules from (43) is very simple. Let F_n be the n-th term in the power expansion of F in ia: $F_n = f_n (ia)^n$. The coefficients f_n are real according to (43). Let also $\mathcal{D} = 2\,\mathrm{Im}\,\mathcal{A}$ be the absorptive part of $\mathcal{A}$ and $\mathcal{D}_n$ its term corresponding to F_n: $\mathcal{D}_n = -2\,\mathrm{Re}\,F_n$. Evidently $\mathcal{D}_n = -f_n\left((ia)^n + c.c.\right)$. The AGK rules are based on the identity

$$-\left((ia)^n + c.c.\right) = (d+b)^n - b^n + b^n - \left((ia)^n + c.c.\right) \qquad (46)$$

with $d = -b = 2\,\mathrm{Im}\,a$ being the absorptive part of the elementary amplitude. Eq.(44) is trivially satisfied as $d+b = 0$. The contribution on the right-hand side of (44) proportional to d^k is naturally interpreted as coming from k physical intermediate states in a and is taken to correspond to k-fold scattering. Contributions containing b describe diffractive corrections to the physical scattering. The first two terms in (44) correspond to nondiffractive processes (with one d at least), the last two terms describe diffractive processes. The decomposition (44) directly follows the Cutkosky rules in a relativistic theory. In the nonrelativistic theory however this is not so: terms containing d^k with $k > 1$ refer to absorptive parts of non-Glauber contributions /12/.

From (44) we find the total absorptive part $\mathcal{D}$ as a sum of a nondiffractive $\mathcal{D}_{nd}$ and a diffractive $\mathcal{D}_d$ terms. The nondiffractive part is evidently

$$\mathcal{D}_{nd} = F(d+b) - F(b) = F(0) - F(-2\,\mathrm{Im}\,a) \qquad (47)$$

Note also that if d is somehow split into two parts $d = d_1 + d_2$ then the absorptive part $\mathcal{D}_1$ containing one d_1 at least according to (45) is given by

$$\mathcal{D}_1 = \mathcal{D} - \mathcal{D}_{d_1=0} = F(0) - F(-d_1) \qquad (46)$$

For instance, choosing for d_1 the inelastic contribution to d we get the absorptive part for new particle production:

$$\mathcal{D}^{prod} = F(0) - F\left(-2(\mathrm{Im}\,a)^{in}\right) \qquad (47)$$

Eqs (45),(47) are well-known for the hA Glauber scattering. As can be seen their validity is much more general and independent of the model.

For inclusive cross-sections one has to substitute in $\mathcal{D}$ for some of the absorptive parts d quantities $d^{(n)}$ corresponding to the production of n particles. Allowing for any number of such substitutions we shall have for $\mathcal{D}$

$$\mathcal{D} = F\left(b + d + \sum_{n=1} d^{(n)}\right) - F(b) = F\left(-\sum_{n=1} d^{(n)}\right) - F(-d) \qquad (48)$$

This relation evidently generalizes (45). In reality one is interested only in the part of $\mathcal{D}$ corresponding to the production of a given number of observed particles. Such contribution is obtained from (48) by expanding F in powers of $d^{(n)}$ and keeping only terms with the fixed total number of produced particles. For example for one-particle inclusive cross sections only terms linear in $d^{(1)}$ should be taken. As can be seen from (48) all contributions to particle production do not depend neither on d nor on b. This is the famous AGK cancellation of all extra interactions in inclusive cross-sections /11/.

A few words about one-particle inclusive production in hA and AA collisions. The produced particles may originate either from the interaction of active partons or from spectators. In the latter case they have small velocities in the lab. or antilab. systems. The AGK rules do not apply to such spectator particles. Their spectra crucially depend on parton distributions in hadrons and on nucleon distributions in nuclei and will not be discussed here. We shall comment only about particles produced in elementary interactions which obey the AGK rules. Kinematically they appear either in the central region or in the region of nuclear fragmentation but with velocities much higher than the typical nuclear ones. To the latter case belong the mentioned cumulative particles.

In these kinematical regions according to the AGK rules the one-particle inclusive cross-section is given by the impulse approximation diagram shown in Fig.9. The cross-section can be split into contributions I_{ik} coming from a collision of i nucleons at the same point in the projectile with k nucleons at the same point in the target. If the corresponding elementary cross-section is $\sigma^{(1)}_{ik} = a^{(1)}_{ik}/2s$ with $a^{(1)}_{ik}$ being the absorptive part of the elementary amplitude in Fig.9, we obtain

$$I_{ik} \quad \sigma^{(1)}_{ik}\, \nu_i\, \nu'_k\, i!\, k!\, (2\pi)^{2-i-k} \tag{49}$$

Here ν_i and ν'_k are defined in (22). The cross-sections $\sigma^{(1)}_{ik}$ are unknown for $i,k>1$. However they can be found from experimental data on the cumulative particle production. Indeed, the kinematical region where $\sigma^{(1)}_{ik} \neq 0$ is enlarging with i and k and in the cumulative region, where $\sigma^{(1)}_{11} = 0$, the particle production is completely due to $\sigma^{(1)}_{ik}$ with $i, k>1$. If we further assume that in a given cumulative region the cross-section is determined by the $\sigma^{(1)}_{ik}$ with

minimal allowed values of i and k, then the experimental data provide direct information on $\sigma^{(1)}_{ik}$ with $i,k>1$. Note that the dependence of I on A and A' is completely described by ν_i and ν'_k in (49).

§7. Conclusion

We presented here a formalism which allows to generalize the usual Glauber multiple scattering approach to processes involving interactions of more than two nucleons at the same point. Some argument has also been given to justify the approximate validity of the Glauber formula at high energies when successive collisions of the incoming hadron in the nuclear target become impossible. It rests on the assumption that multiple collisions at high energies are in fact parallel collisions of hadron constituents, partons, and requires that the number of partons be distributed according to the Poisson law.

The AGK rules proved for the AA scattering with multinucleon interactions allow to study inclusive production of fast secondaries. Of course the presented formalism tracts only the nuclear side of the problem, essentially the geometrical features of the process. The dynamical part remains hidden in the amplitudes for constituent interactions and should either be studied by methods appropriate to hadron dynamics or treated phenomenologically.

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FIGURE CAPTIONS

Fig.1 The parton graph for multiple scatterings in a AA collision.

Fig.2 Multiple scatterings with multinucleon interactions.

Fig.3 Graphs for NN interactions in AA collisions without loops (a) and with a loop (b).

Fig.4 The parton interpretation for vertices in Fig.3.

Fig.5 An extra contribution with multiple interactions of a given pair of nucleons to be discarded.

Fig.6 Multinucleons interactions for a pair NN potential due to large transferred momenta.

Fig.7 Multiple interactions of a given pair of nucleons at the same point in the potential picture.

Fig.8 A formation of a multibaryon (a) and its multiple interactions (b).

Fig.9 The impulse approximation graph for one-particle inclusive cross-sections.

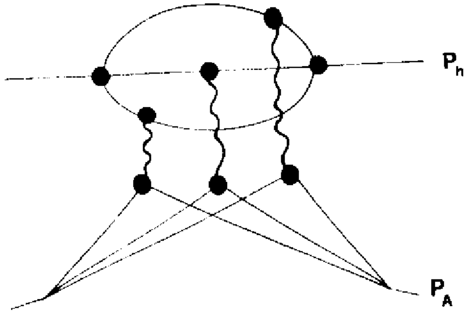

Fig.1

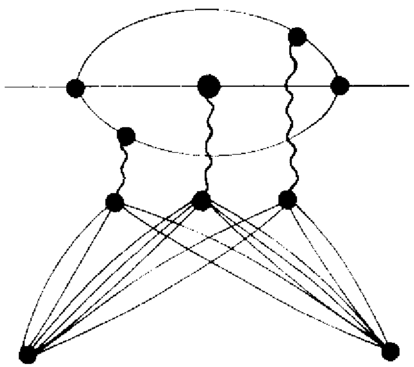

Fig.2

Fig.3

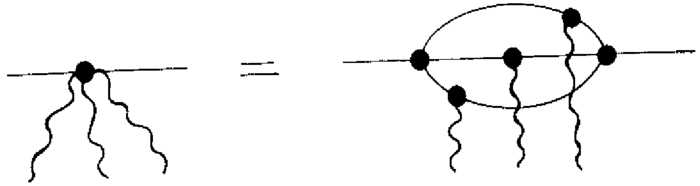

Fig.4

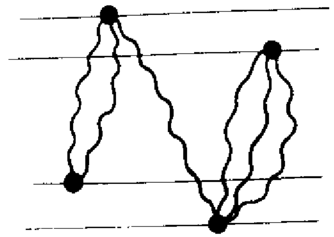

Fig.5

Fig.6

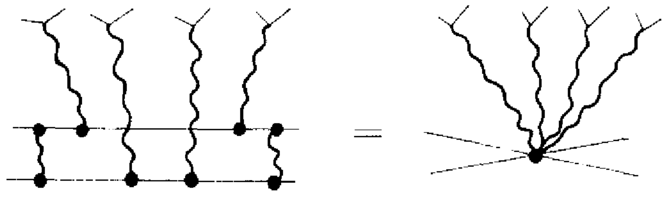

Fig.7

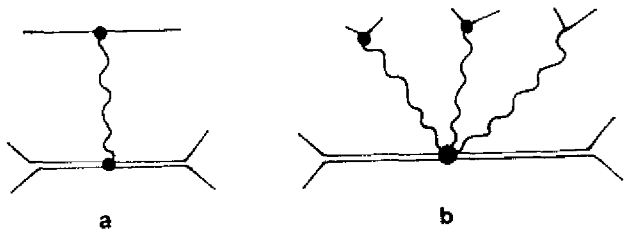

Fig.8

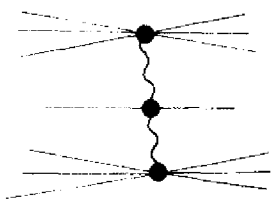

Fig.9

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