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A STUDY OF BETA DECAY ENERGIES AND ATOMIC MASSES

Leif Spanier

Department of Nuclear Physics, Lund Institute of Technology, 1988

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AND

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av

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Abstract

The Q_β energies of $^{123-128}$Cd and $^{123-131}$In have been determined using the end points of β spectra recorded in β-γ coincidence experiments. A HPGe planar detector was used to detect the β-particles and a semi-empirical response function was used when unfolding the electron distribution. The mass excesses were deduced and when they were compared with the predictions of various mass formulae, the cadmium isotopes were found to be heavier than those predicted by most of the mass formulae. The excitation energy of the $1/2^-$ proton-hole state in the odd indium isotopes was shown to be constant for all the heavy isotopes.

The Q_{EC} energies of ^{156}Dy and ^{96}Pd were determined using the β^+/EC intensity ratio method. The ratio of the intensity of the β^+ branch to the total beta decay intensity was determined by means of γ-spectroscopic methods. The mass excesses were deduced. The two-proton binding energy for the N=82 isotones showed only a small step of approximately 0.5 MeV when the doubly-magic nucleus ^{156}Gd was encountered.

A liquid drop type mass formula with deformation and shell energy corrections and with few free parameters is presented. The shell energy correction is a simple analytical expression for the equilibrium deformation of the nucleus. An analytical expression for the equilibrium nuclear deformation is also presented. The mass formula was applied to nuclei with Z and N greater than 50. The RMS deviation is 0.55 milli mass units.

The reaction ^{98}Mo(p,n)^{98}Tc was investigated through the counter ratio method, the ratio of the number of slow neutrons to the number of fast neutrons. The Q_{pn} energy value of a low-spin state in ^{98}Tc was determined. The state at 90.9 keV excitation energy is proposed to be the 14.6 μs isomer and have spin and parity 1^+.

Key words

Measured β^- decay energy, β^+/EC intensity ratio, (p,n) threshold energy. Deduced atomic mass. Liquid drop mass formula.

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Introduction

The mass is an important characteristic of the atomic nucleus reflecting the nuclear force and the nuclear structure. It also enters into many situations where we have a nuclear decay or reaction, often in the form of the difference in mass between two nuclei. Below follow some examples where the nuclear mass or mass difference is important:

* The two-particle binding energies are generally smooth functions of the proton or neutron numbers. However, when a magic number is encountered the binding energy makes a drastic jump illustrating this phenomenon.

* For delayed particle emission the mass difference determines whether the decay mode is energetically possible.

* For a beta decay with known lifetime, the mass difference or more precisely the decay energy is needed to calculate the log ft value and the matrix element to obtain greater details concerning the decay.

* In nuclear reactions the excitation energies can be determined from the atomic masses, and reaction channel cross sections may be calculated.

It is convenient to visualize the nuclear mass or, which is more common, the atomic mass as a surface in three-dimensional space. The atomic number Z and neutron number N constitute the two horizontal axes while the atomic mass is the third. The surface has the shape of a valley and the nuclides at the bottom constitute the line of beta stability. For many atoms the mass is known from experiment. In the early days of physics the masses of the stable isotopes were measured, but techniques evolved capable of determining also the masses of unstable nuclei. In the latest compilation of experimentally determined atomic masses by Wapstra et al. <WaAH88>, the 1986-87 Atomic Mass Table, 1659 nuclides, with at least one experimental determination of the mass, are listed. The nuclides range from the neutron and proton all the way up to element number 106 with the mass number 263. Since many nuclear experiments and calculations that require a knowledge of the atomic mass involve nuclei with unknown masses far away from beta stability, it is important to be able to extrapolate the atomic mass surface to unknown mass regions. This was recognized early and a large number of mass formulae have been developed since Weizsäcker <Weiz35> and Bethe and Bacher <BeBa36>

introduced the liquid drop model of the nucleus in 1935-36. As experimental techniques have improved, the accuracy of the mass measurements have increased and the masses of nuclei still further away from the line of beta stability have been determined. This has inspired the development of new and better ways of predicting unknown masses.

When newly measured atomic masses are compared with the mass formulae it is possible to test the theoretical ideas underlying the formulae and also to estimate the reliability of the various predictions. The experimental determination and theoretical calculation of atomic masses are thus closly related and encourage one another.

One situation where not even the most optimistic nuclear physicist expects it to be possible to determine the relevant atomic masses from experiment is for the nuclei that participate in the r process, the process in violent stellar events that is supposed to produce a large proportion of the stable neutron-rich elements and all elements heavier than bismuth <Clay68>. Only around the two waiting-point nuclei, ^{80}Zn and ^{130}Cd, where the r process comes closest to the line of beta stability, is the known mass surface reasonably close. The mass of ^{80}Zn is actually already known <Lund84> and the masses of ^{128}Cd and ^{131}In were also determined in this work. For the majority of these nuclei however, one is confined to estimating the mass from extrapolations. The extrapolation is long, more than 30 nuclides away from the line of beta stability for some elements, and it is important that the mass formula can be trusted.

Experimental determinations of atomic masses

Measurement of nuclei close to beta stability

There are a large number of methods in use for the determination of atomic masses, including actual mass measurements and also more indirect techniques. The most direct method is to determine the atomic mass by mass spectroscopy. The atom is first accelerated by a potential of a few tens of kV and then passes through areas with magnetic and electric fields. The deflection of the atom in the fields is measured and the mass can thus be calculated. Actually, the atomic mass of ^{12}C is defined to be 12 u, where u denotes the atomic mass

unit. All other masses are measured relative to this isotope. The precision of mass spectroscopy is increased if the mass difference between isotopes or molecules is measured and if this is as small as possible. Thus, most favourable is the situation where two different molecules with the same mass number are compared. A pair of such molecules is called a mass spectroscopic doublet and two examples are,

$$^{12}C_3 - {}^{35}ClH \quad \text{and} \quad {}^{12}C_5H_{10} - {}^{35}Cl_2$$

With the mass differences of these doublets and the definition of the ^{12}C mass, the masses of ^{35}Cl and H can be accurately calculated. A large number of such mass doublets have been measured.

The measurement of reaction Q values is another pricipal method of determining the mass difference between two isotopes. Formally a reaction is written,

$$a + A \rightarrow B + b + Q$$

where a and b are the incoming and outgoing particles and A and B are the target and residual nuclei, respectively. The incoming particle has a known energy and the outgoing particle is detected and its energy determined. From this and the geometry of the reaction the Q value can be determined. Since the masses of particles a and b are usually known with high accuracy the mass difference between A and B can be calculated.

One drawback of the methods mentioned so far is that only nuclei close to the line of beta stability are usually considered. In the latter method the nuclide A must have a long enough lifetime to be suitable as a target. In the mass spectroscopic case suitable beams of the isotopes must be formed and for radioactive nuclei this is only possible when single atoms are considered and not when complex molecules are compared.

For most radioactive nuclei one is confined to determining the atomic mass difference by means of measuring the decay energy, especially the alpha or beta energy.

The evaluation of the atomic masses from a large number of experiments like those mentioned above are done periodicaly by Wapstra <WaGo71> <WaBo76> <WaAu85> <WaAH88> .

Beta decay energy measurements

Nuclei that lie on the slopes of the beta stability valley are unstable and can decay by beta decay. For the neutron-rich nuclei, the β^- decay can be written,

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+1}Y_{N-1} + \beta^- + \bar{\nu} $$

The energy of the beta particles is continuous but if the maximum energy is determined the mass difference of the two nuclei X and Y can be calculated. For neutron-deficient nuclei there are two competing decay modes, positron decay and orbital electron capture (EC). The residual nucleus, Y, is often left in an excited state and this usually decays by gamma decay. In this thesis the beta decay energies of both neutron-rich and neutron-deficient nuclei have been determined, and in the following, the methods will be described in some detail.

Q_β energy determination of neutron-rich nuclei

Method

The OSIRIS facility <Ruds76> at the Studsvik Neutron Research Laboratory is a very powerful source of neutron-rich radioactive nuclei. A target consisting of a few grams of ^{235}U is exposed to the neutron flux of the R2-0 research reactor. The target is positioned inside the ion source and the induced fission product nuclei are rapidly ionized and accelerated. After being deflected by a magnet the mass-separated ion beam reaches the experimental site.

In the present work the ion beam was focused onto a moving aluminized tape. Two germanium detectors were positioned in front of the ion beam spot. The gamma rays were detected in an ORTEC HPGe closed-end coaxial detector while the beta particles were detected in an ORTEC HPGe planar detector with a 0.25 mm thick Be window. The detectors were positioned outside the vacuum system separated from this with a 0.08 mm thick aluminum window. The gamma detector was shielded from the

beta particles by an additional 8 mm thich aluminum absorber. The signals were processed in a standard fast-slow coincidence set-up. The fast coincidence window was set to 25 ns to include the prompt events and the accepted events were written on magnetic tape in a two-parameter list mode for further analysis. The yield of cadmium from the ion source was relatively low, but, since the cadmium isotopes often had the shorter lifetime, the moving tape could be used to remove the longer lived activity from the counting position and thus enhance the low activity. The speed of the tape was chosen so that the nuclei of interest were in the counting position for about one half-life.

In beta decay end-point energy analysis one usually constructs the beta spectrum by displaying all the electrons that are in coincidence with one of the proceeding gamma decays. In Paper I we adopted a 'reversed' method where we instead first bunched the electrons in 200 keV slices and then looked at the gamma spectra in coincidence with the electrons in each of the slices. For each slice the areas of the relevant gamma peaks were determined. When the peak area was plotted as a function of the electron energy of the slices an 'electron' spectrum was formed which should be identical to an electron spectrum deduced in the usual way. In Paper II the normal technique was used.

Response function

The use of a germanium detector as the beta particle detector has many advantages over conventional plastic or silicon beta detectors.

* The resolution is high, and the width of the full-energy peak is only around 15 keV.
* The position of the full-energy peak is very nearly proportional to the electron energy <Deck82> and the detector can be energy calibrated with standard gamma sources.
* Commercial detectors are available which are thick enough to fully stop the electrons. In our case the detector was 13 mm thick and this is enough to fully stop 10 MeV electrons <Page72>.

One severe drawback is, however, that the shape of the response function is rather complex. When Decker et al. <Deck82> parametrised the response function they used 12 energy-dependent parameters. In gamma spectroscopy this causes no problems since the gamma decay is monoenergetic and only the full-energy peak is needed for the

analysis. In beta spectroscopy the continuous beta spectrum is convoluted with the response function giving the electron distribution in the detector. Thus, the whole response function must be considered when the electron distribution is deconvoluted to a beta spectrum.

For plastic scintilators and silicon detectors a full-energy peak plus a constant background is usually used. This is also practicable for germanium detectors if the electron energy is not higher than about 4 MeV <ReMo78>, <ReMH80>. For higher energies the bremsstrahlung part of the electron slowing down processes becomes noticeable.

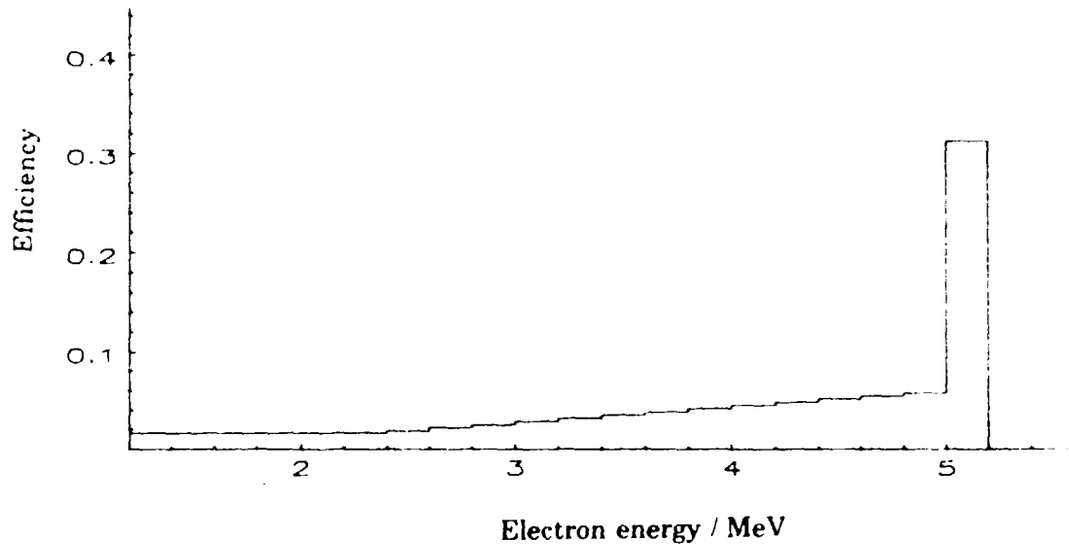

Fig. 1 The response function for 5.1 MeV electrons.

There is a probability that the bremsstrahlung photons leave the detector and this must be considered. For beta-end point energy determinations the very complex response function can be simplified and we use the semi-empirical response function of Rehfield et al. <ReMH80>. In fig. 1 the response function of 5.1 MeV electrons is shown. Besides a full-energy peak and a constant background, it also includes a bremsstrahlung part, having the shape of a triangle. The width and area of this triangle vary with electron energy. The parameter values were chosen so as to give linear Fermi-Kurie plots of the allowed beta decays of the ground states of the odd indium isotopes. The variation in the parameters had the strongest influence on the lower part of the electron spectrum. Since the upper part of the spectrum is more important when beta end-point energies are considered, the results were rather insensitive to the actual values of the parameters. When the end-point energy values obtained from

analysis with several different response functions were compared, the variation in the end-point energies was less than 30 keV.

Q_{EC} energy determination of neutron-deficient nuclei

Method

For the positron-decaying, neutron-deficient nuclei it is also possible to determine the decay energy by end-point analysis in Fermi-Kurie plots, in the same way as for neutron-rich nuclei. We have, however, adopted the completely different method of determining the Q_{EC} value from the measurement of the EC/β^+ intensity ratio. This method is based on the determination of the ratio of the orbital electron capture (EC) intensity to the intensity of the positron decay mode. This ratio depends strongly on the positron energy. The dependence is approximately W_0^{-3} where W_0 is the relativistic positron end-point energy. For allowed beta decays the theoretical calculations of the EC/β^+ ratio are believed to be very accurate <LeSh78> and a comparison between the tables of Gove and Martin <GoMa71> and the one of Dzhelepov et al. <DzZS72> used in this work shows a difference in the $EC(K)/\beta^+$ ratio of less than 2% for the ^{156}Dy decay in Paper III, and even less in the case of ^{96}Pd in Paper IV.

The strong energy dependence of the ratio implies that the relative error in the deduced positron end-point energy value is only about 30% of the experimental relative error in the ratio measurement, and this makes the method attractive. Another appealing point is that the ratio may be determined by gamma spectroscopic means without involving the detection of the positrons directly.

We have applied gamma spectroscopic methods for the determination of the EC/β^+ ratio. This essentially means that the positron decay intensity is determined by measuring the intensity of the 511 keV gamma rays that follow the annihilation of the positrons. This makes some demands on the target surroundings. The positrons should annihilate close to the target, only to two 511 keV gamma rays and without any time delay. Thus, a positron annihilator must be placed around the source. Aluminium is suitable for the following reasons.

* The path length of a 1 MeV positron is approximately 2 mm in aluminum <Page72>, and all positrons from a beta decay with an end-point energy of 1 MeV will be stopped after a few millimetres. The source of the annihilation radiation will thus be very close to the position of the sources that are later used for efficiency calibration.

* The time taken for the positrons to slow down and thermalize is short, around 10^{-10} s or less.

* When the positrons are stopped positronium is formed. In a thin medium 75% is however formed in the 140 ns long-lived triplet state. This state can only decay via a three-photon mode where the energy spectrum of the photons is continuous. In aluminum the lifetime of the triplet state is quenched by a factor of about 100 <DeBe65> and this makes the three photon decay mode negligiable and thus the result of virtually all annihilations of thermal positrons is two prompt 511 keV gamma rays.

In Paper III we used a fixed target annihilator set-up. The target was positioned between two aluminum semi-cylinders with 8 mm radius, as shown in fig. 2 One of the halves had a 6 mm wide tantalum-dressed hole through which the beam could pass. Because the positrons could escape through this hole an experiment was performed to allow corrections to be made for this loss.

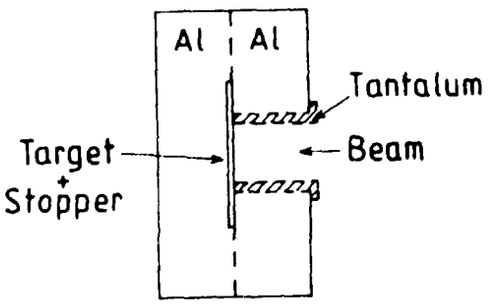

Fig. 2 The positron annihilator used in the experiment described in paper III.

A second aluminum semi-cylinder was manufactured without a hole. A source of ^{22}Na was placed in the target position and the intensity of the annihilation radiation was measured, first with the absorber without the hole and then with the absorber with the hole. The ratio of the intensities was used to correct for the positron escape through the hole. This correction was not needed in the experimental set-up

described in Paper IV. Here the ion beam with mass number 96 from the mass separator was collected on a moving tape. At 4 min intervals the tape was moved to position the activity at the counting position. The annihilator consisted, in this experiment, of two close parallel aluminium slabs and the positron escape out of the entrance and exit holes of the tape was negligiable.

Q_{pn} determination of the first 1^+ state in ^{98}Tc

The neutrino production in the sun has been the subject of interest ever since the first results from the solar neutrino flux measurements by Davies et al. <DaHH68> showed a lower flux on earth than could be explained from standard solar theory. The prospect of using the detection of solar-neutrino-produced ^{98}Tc to determine the long-term neutrino flux motivated a search for the first 1^+ state in ^{98}Tc. The method described in Paper VI is, in principle, similar to techniques frequently used for determining the atomic mass difference between two nuclei by means of a nuclear reaction. The reaction Q value to a specific state is measured and if the excitation energy of this state is known the mass difference can be calculated. In our case the mass difference was already known and instead the previously unknown excitation energy of the lowest 1^+ state in ^{98}Tc could be determined. A detailed description is given in Paper VI.

Theoretical determination of atomic masses

The importance of theoretical descriptions of atomic masses was mentioned in the introduction. In Paper V we describe a semi-empirical mass formula of the liquid drop type with very few entirely free parameters.

Atomic mass formulae

A massformula with few free parameters

The guiding principle in the construction of the mass formula was to keep the number of free parameters as small as possible and to include only necessary and physical terms. We also tried to determine the different parameter values by considering different physical quantities so that in the final fit to all the experimental masses only a few parameters were left to be determined. A mass formula is mainly a polynomial in Z and N in powers of $^1/_3$ and of course the fit to the known masses will be better if a larger number of terms and parameters are incorporated. The contribution from the different terms will, however, be uncontrollable and the result of an extrapolation uncertain. Another problem is also that the uncertainties in the experimental masses are much smaller than the expected accuracy of the mass formula. Actually, more than half of the experimental mass values are known with an error of less than 10 keV, while the typical mass formulae have an average error of about 500 keV. It is thus very difficult to apply statistical methods to determine whether terms are necessary or not. It therefore is better to keep the number of terms small and physical.

The physical philosophy behind the mass formula is the following. It consists of three parts,

$$M(Z,N,\beta) = M_w(Z,N) - E_{def}(Z,N,\beta) - E_{shell}(\beta) \ .$$

M_w is a liquid drop type expression describing the mass of a spherical nucleus with a smooth single-particle level distribution. The difference $(M_{experimental}-M_w)$ is shown in fig. 3 for some of the even-even rare earth nuclei. Two corrections must be applied to M_w.

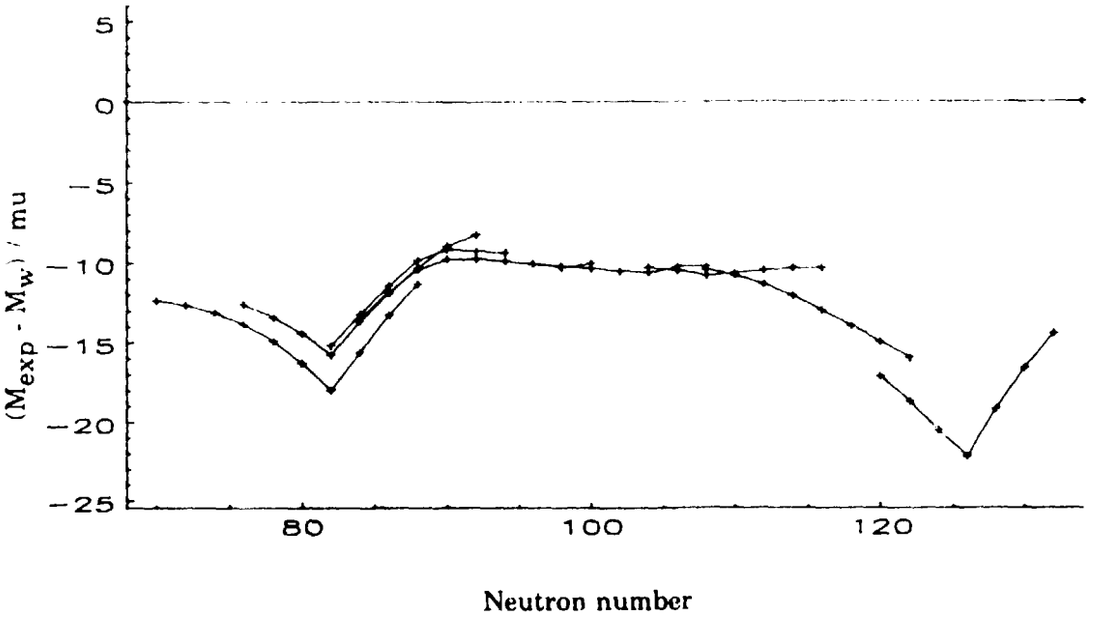

Fig. 3 The difference between the experimental mass values and the liquid drop part of the mass formula shown for some rare earth nuclei.

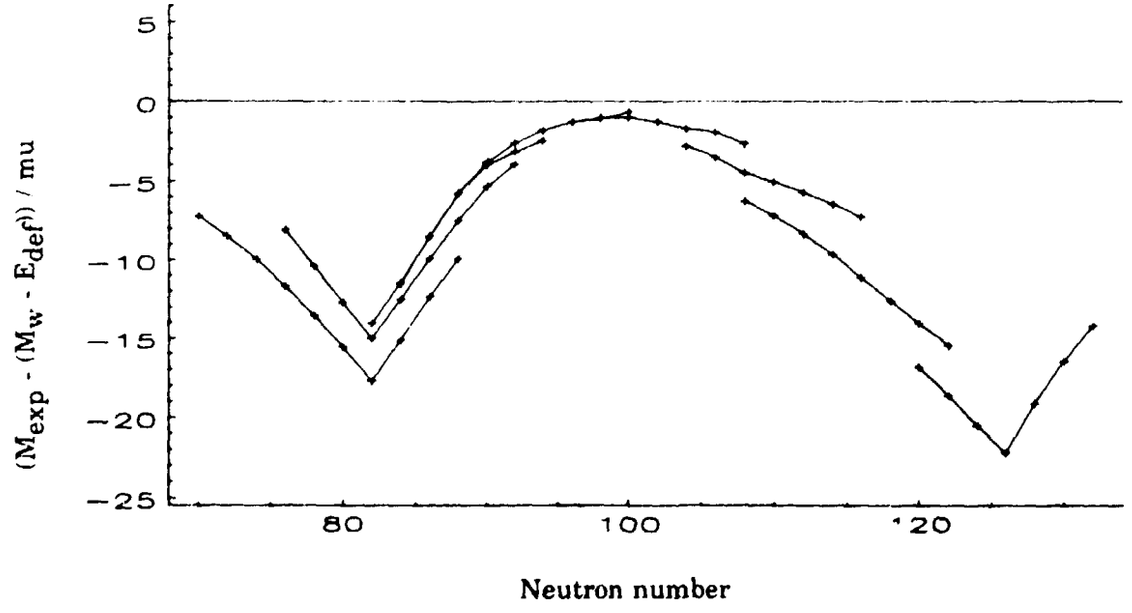

Fig. 4 The negative of the experimental shell energy, that is, the difference between the experimental mass values and the liquid drop part of the mass formula corrected for the deformation energy.

One can notice that real nuclei with a smooth single-particle level distribution are not spherical but deformed. The hypothetical mass value M_w must thus first be corrected for the nuclear deformation and this deformation energy is denoted E_{def}. In our interpretation of E_{def} there are two separate sources which contribute to the deformation energy. The well known change in the Coulomb and surface energies of the liquid drop constitutes one part. The second contribution comes from the energy change in single-particle levels when the nuclear potential is deformed. We write this term in the form $C \cdot \beta$, where C is a constant for each nucleus.

When the deformation energy is subtracted from M_w and $(M_{experiment}-(M_w-E_{def}))$ is plotted, a smooth curve is obtained as in fig. 4. When this is compared with a plot of the deformation of the nuclei, a suggestion is immediately obtained for the form of the second correction, the shell energy E_{shell}. The shapes of the curves are very similar and E_{shell} seems to be proportional to the nuclear deformation. This is also our approach and we write the shell energy, $E_{shell}=\kappa \cdot (\beta_{max}-\beta)$. The constant κ is determined from diagrams where $((M_w-E_{def})-M_{experiment})$ is plotted as a function of the deformation, but it can also be estimated from fig. 4.

In the fitting procedure, as discussed in Paper V, we actually tried to determine all parameter values, except the volume and surface parameters, by means other than fitting them directly to the atomic masses. In the final fit to the experimental masses these two parameters were determined, and the two κ values in the shell energy term and a_c in the Wigner term were also slightly adjusted. The result is a mass formula with very few free parameters and it reproduces the known masses for the non-magic nuclei above N,Z=50 with an average deviation of 0.42 keV.

The mass formula was developed over a period of years and the main novelties of our version are the following,

* There are several microscopic calculations <BrCM71>, <Groo76>, <Dutt86> indicating that the liquid drop type mass formulae overestimate the binding energy of very neutron-rich nuclei. To correct for this a term proportional to $(N-Z)^2$ was introduced to increase the mass of neutron rich nuclei, especially the heavy ones.

* The values of many parameters are given by simple expressions of the mass number A. The parameters in the shell energy term are kept constant for all nuclei that are considered.

* The nuclear deformation is treated as a variable in the mass formula. The sparcity of experimentally determined deformations <Rama87> makes a simple prescription for calculating them very useful. A simple analytical expression is developed which only depends on Z and N.

* The present form of the mass formula makes it very suitable for extrapolation into unknown regions of the mass surface. The masses of 4162 nuclei with Z and N greater than 50 were calculated and will be published in Atomic Data and Nuclear Data Tables 39, 1988.

Comparison with some new experimental values

One interesting and sensitive way to test a mass formula is to compare its predictions with the results of new experiments. There is however only a limited amount of new data available that has not been included in the 1986-87 Atomic Mass Table <WaAH88> which was used as data base when the formula was fitted to the known masses. A few examples of newly measured masses and decay energies, that were not used when determining the parameters of the mass formula in paper V, are the following.

The beta decay energy of ^{171}Re was recently determined by Runte et al. <Runt87> using the EC/β^+ intensity ratio method. Table 1 shows the predictions of some mass formulae compared with the experimental value.

The prediction of the existence of super-heavy elements with appreciable lifetimes around ^{298}114 has inspired a search for heavier elements <Munz88>. At the moment the heaviest known nucleus with a known mass is ^{266}108 <Munz87> and in Table 1 the predictions of various mass formulae are compared with the experimental mass of this nucleus. Other examples from this heavy nucleus area are the alpha decay energies of ^{265}108 <Munz87> and ^{266}109 <Munz84>.

As can be seen from Table 1, the predictions of the mass formula described in paper V are in good agreement with the experimental values.

Table 1

Comparison of various mass predictions with some new experimental data

	^{171}Re Q_{EC} (MeV)	^{264}108 Mexc (MeV)	^{265}108 Q_α (MeV)	^{266}109 Q_α (MeV)
Experimental value				
Runte <Runt87>	5.67			
Münzenberg <Munz87>		120.0		
Münzenberg <Munz87>			10.52	
Münzenberg <Munz84>				11.27
	ΔQ_{EC} (MeV)	ΔMass (MeV)	ΔQ_α (MeV)	ΔQ_α (MeV)
Predicted value				
Paper V	-0.27	0.14	-0.07	0.27
Dussel <DuCZ87>	-0.24	-0.15	-0.11	2.13
Möller <MoNi86>	-0.50	-1.09	0.22	0.36
Möller <Moll87>	-0.29	1.66	0.65	0.81
Comay <CoKZ87>	0.02	0.46	0.21	
Satpathy <SaNa87>	-0.19	0.20	-0.10	0.28
Tachibana <Tach87>	0.00	-0.78	0.15	-1.10
Jänecke <JaMa87>	-0.03			0.27
Masson <MaJa87>	-0.24			
Myers <Myer76>	0.12	-1.28	0.02	0.35
Groote <GrHT76>	0.10	-0.40	-0.07	0.26
Seeger <SeHo76>	-0.03	-0.40	0.44	0.79
Liran <LiZe76>	-0.40	-0.27	-0.25	0.03
Jänecke <Jane76>	-0.99			
Comay <CoKe76>	-0.60			
Jänecke <JaEy76>	-0.35			

Note. ΔQ means $Q_{experimental} - Q_{theoretical}$. Mexc is the mass excess and ΔMass is the difference between the experimental and calculated mass excess values

Comparison of various mass formulae.

A fairly large number of mass formulae are available today and they are based on a variety of principles. In 1975 several mass predictions were collected in the 1975 Atomic Mass Predictions <Mari75> and some of these are widely used when unknown masses are needed or when comparisons with new experimental masses are performed. In the 7th Conference on Atomic Masses and Fundamental Constants (AMCO-7) an initiative was taken by P. Haustein <Haus84> to make a collection of contemporary predictions and this will soon be published <Haus88>. It is interesting to compare these predictions with some experimentally determined masses and also to see the differences for extrapolations far from stability.

In the following I shall briefly describe the various formulae. It is possible to divide the predictions into a few groups.

First we consider formulae that are based on the liquid drop or droplet model. These can be separated into two categories according to the way in which they treat the shell and deformation energies. One way is to use a nuclear model and extract the shell and deformation corrections from this. Formulae belonging to this category are the one presented in Paper V, those of Möller and Nix <MoNi86> and Möller et al. <Moll87>. They are all included in the latest compilation <Haus88>. The formulae of Myers <Myer76>, Groote et al. <GrHT76> and Seeger and Howard <SeHo76> also belong to this category. Another method of correcting for the shell energy is by the formalism of mass relations. Relations between the shell energies of close-lying nuclei are set up. This results in large systems of equations that are solved for the shell correction. To this second category belong the mass formulae of Tashibana et al. <Tash87> and Jänecke and Eynon <JaEy76>.

The separation of the liquid drop formulae into these two categories is also motivated by the number of parameters that they contain. Formulae in the first category have about 10 to 30 parameters or less, while the formulae in the second category often include several hundred parameters.

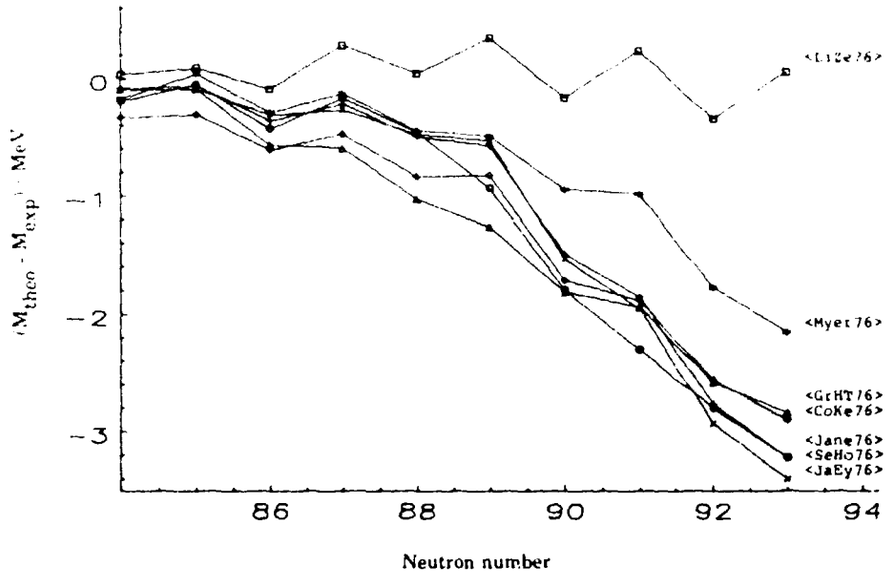

Fig. 5a The difference between the calculated mass values of various mass formulae and the experimentally determined masses are shown for the heavy Cs isotopes. The mass formulae were participating in the 1975 Atomic Mass Predictions. The symbols are the same as in the reference list.For the isotopes with neutron number greater than 87 the theoretical mass values are extrapolations.

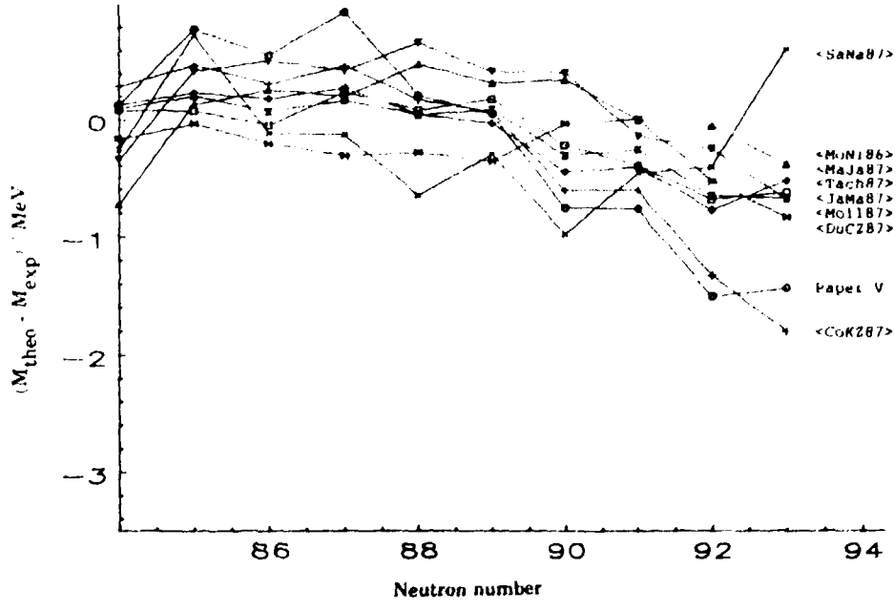

Fig. 5b The difference between the calculated mass values of various mass formulae and the experimentally determined masses for the heavy Cs isotopes. The mass formula are participating in the 1986-87 Atomic Mass Predictions.

Shell model mass equations are represented by the semi-empirical model of Liran and Zeldes <LiZe75>. This model have almost 180 parameters.

A large number of predictions are based entirely or almost entirely on mass relations. In this case the whole or a major part of the atomic mass value is considered when the relations between neighbouring nuclei are set up. The 1986-87 Atomic Mass Predictions include formulae by Comay et al. <CoKZ87>, Satpathy and Nayak <SaNa87>, Jänecke and Masson <JaMa87> and Masson and Jänecke <MaJa87>. Those of Jänecke <Jäne76> and Comay and Kelson <CoKe76> are from the 1975 predictions. The one of Monahan and Serduke <MoSe78> is also a mass relation. As mentioned above, the number of parameters in this kind of mass formulae are very large, sometimes exceding 500.

There are of course several other mass formulae but most of these are rather old or have been superseded by one of those mentioned above.

When the RMS values of the deviations for a number of different mass formulae are compared it can be seen that, as a general trend, the RMS value decreases as the number of parameters increases. Typical RMS values for liquid drop type formulae are 0.5 to 1 MeV while the multi-parameter mass relations have RMS values from 0.5 down to 0.1 MeV. The low RMS value is, however no guarantee that the extrapolated mass predictions will be equally good.

One demonstrative example is the caesium isotopes. During the last decade the masses of some eight new heavy isotopes have been determined. In fig. 5 the predictions of several mass formulae are compared with experimentally determined masses of the heaviest Cs isotopes <WaAH87>. Around 1975, the heaviest isotope with experimentally determined mass was ^{144}Cs with 87 neutrons. In fig. 5a the predictions of some mass formulae, from this period of time, are shown. As can be seen, the calculated mass values from most of the formulae are close to the masses that were experimentally known in 1975. The extrapolated mass values deviate, however, for most of the mass formulae, shortly beyond neutron number 87 which was the heaviest isotope with known mass at that time. The calculated masses from some mass formulae from the 1986-87 Atomic Mass Predictions are shown in fig. 5b. At the time when the parameters of those mass formulae were determined, the masses of all Cs isotopes shown, were experimentally known and included in the data base used. For most of the formulae,

the calculated mass values are close to the experimental ones that were used in the determination of the parameters.

This is actually no surprise. It is a mathematical fact that with a large number of parameters, one can get a good fit to any set of experimental data. It is, however, dangerous to use an expression with many parameters for extrapolation purposes, because, in some cases, it may deviate considerably beyond the experimental data points. The reason is the following. With a large number of parameters, the different parameters can interact in an unforeseen way when they are fitted to the experimental data. Even if the individual parameters can be given physical meaning, unless the parameters are determined independently, the actual values may not reflect the true physics, and the expression can behave in an unexpected way when it is used for extrapolations. In the case of the mass formula described in Paper V, it is quite feasible to reduce the RMS deviation considerably by increasing the number of parameters. If, for example, the mass formula is fitted separately within each nuclear region bound by the magic numbers, a RMS value of only 0.35 keV is obtained. We believe, however, that by having a low number of parameters, and by determining most of them in an independent way, we avoid the problem of interacting parameters mentioned above. The fit to the known masses is somewhat worse than could be obtained, but the extrapolated mass values should be more reliable.

Very neutron rich nuclei

In r process calculations of element synthesis in violent stellar events the atomic mass is one of the most important parameters <MaWa85>. The extrapolation is very long and a comparison of the results of different mass predictions shows a wide spread in the mass values. In fig. 6 various mass predictions are compared with results from the mass formula presented in Paper V for isotopes of Cs (Z=55) and Rn (Z=86). One sees that for nuclei that lie on the r-process path (N=110 and N=165) the difference between various mass predictions amounts to more than 25 MeV. In the static r process calculation the important property is the neutron binding energy, rather than the actual mass of the nuclei. Fig. 7 shows the neutron binding energy of the odd neutron isotopes of the neutron-rich Cs and Rn nuclei. The actual r process path is supposed to pass through nuclei with neutron binding energies between 1 and 2 MeV. Assuming a neutron binding

energy of 1.5 MeV, the different mass formulae predict paths that differ by 10 to 15 neutrons. The choice of mass formula should thus be carefully considered for this application. From fig. 6 one can generally say that for very neutron-rich nuclei the liquid drop type mass formulae predict more strongly bound nuclei than do the mass relation type of formulae.

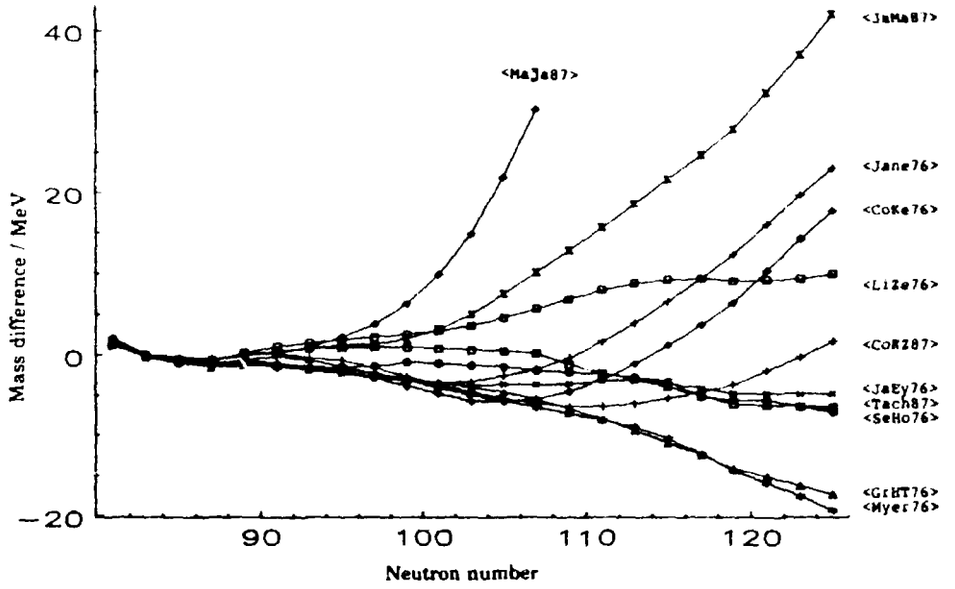

Fig. 6a The difference between the extrapolated mass values of various mass formulae and the formula described in Paper V shown for the very neutron-rich Cs isotopes. Only the odd-neutron isotopes are shown.

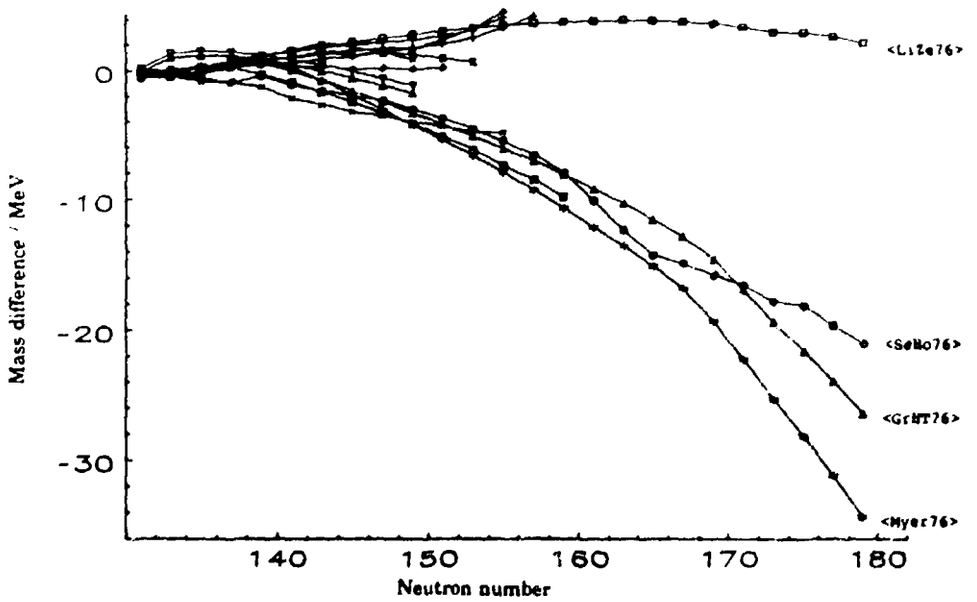

Fig. 6b Same as figure 6a but for the very neutron-rich Rn isotopes.

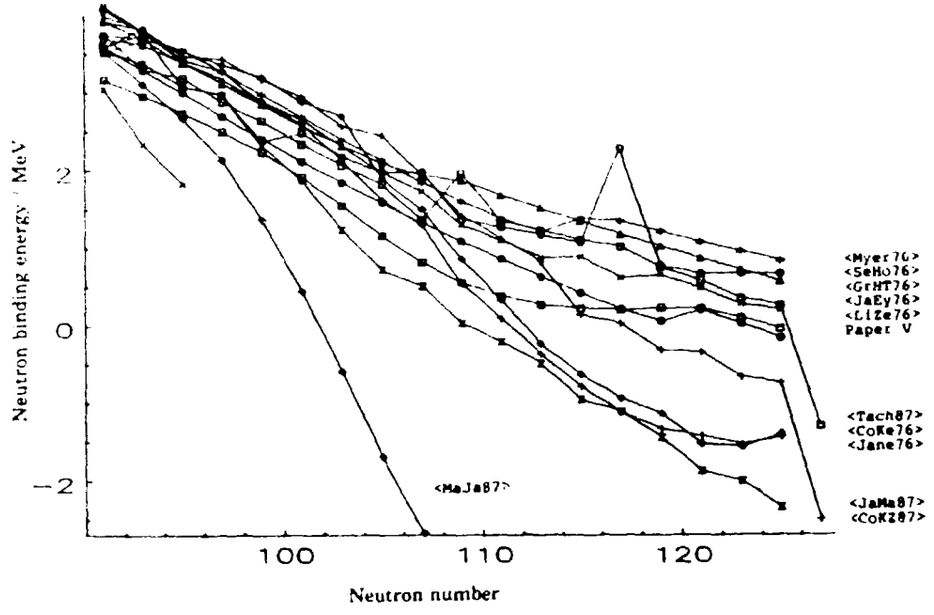

Fig. 7a The neutron binding energy for the odd-neutron neutron-rich Cs isotopes as predicted by various mass formulae.

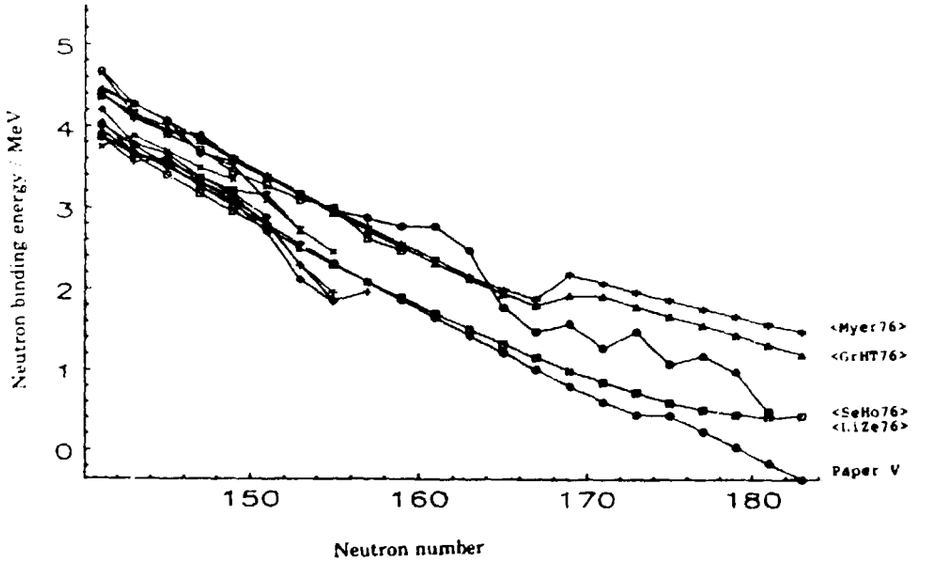

Fig. 7b Same as figure 7a but for the very neutron-rich Rn isotopes.

List of the papers

This thesis is based on the following papers,

I THE Q_β VALUES OF THE HEAVY Cd AND In ISOTOPES
(in collaboration with K. Aleklett, B. Ekström and B. Fogelberg)
Nucl. Phys A474 (1987) 359

II A DETERMINATION OF PROTON HOLE ENERGIES IN ^{131}In
(in collaboration with B. Fogelberg and Ye Zongyuan)
Research report NFL-58, 1988

III THE MASS OF THE NUCLEUS ^{148}Dy
(in collaboration with S.Z. Gui, H. Hick and E. Nolte)
Z. Phys. A299 (1981) 113

IV THE GAMOW-TELLER TRANSITION IN THE ^{96}Pd —> ^{96}Rh DECAY
(in collaboration with K. Rykaczewski, I.S. Grant, R. Kirchner, O. Klepper, V.T. Koslowsky, P.O. Larsson, E. Nolte, G. Nyman, E. Roeckl, D. Schart, P. Tidemann-Petersson, E.F. Zganjar and J. Zylicz)
Z. Phys. A322 (1985) 263

V A MODIFIED BETHE-WEIZSÄCKER MASS FORMULA WITH DEFORMATION AND SHELL CORRECTIONS AND FEW FREE PARAMETERS
(in collaboration with S.A.E. Johansson)
to be published in Atomic Data and Nuclear Data Tables 39 (1988)

VI THE LOWEST 1^+ STATE IN ^{98}Tc
(in collaboration with H. Morinaga)
Nucl. Phys. Rep. LUTFD2/(TFKF-3053) 1-12/ (1988)

Summary of the papers

Paper I

In the first work we turn to the neutron-rich side of the beta stability line to determine the masses of the heavy cadmium and indium isotopes. This work was performed at the OSIRIS on-line mass separator at Studsvik. The Q_β values were determined from the beta end-point energies deduced from the analysis of Fermi-Kurie plots.

The beta decays of $^{123-128}$Cd and $^{123-130}$In were studied with a β-γ coincidence method. The gamma rays were detected in a coaxial germanium detector while the beta particles were detected in a planar HPGe detector. Though the response function of germanium detectors is rather complex for beta particles it was found that a semi-empirical response function could be used in the unfolding of the beta spectra.

The previously unknown Q_β values of $^{123-128}$Cd were determined and improved data for $^{123-130}$In were obtained. From these data the mass excesses for these nuclei could be determined. The mass excesses of these nuclei were compared with several theoretical mass predictions. From this comparison we concluded that most mass formulae underestimate the masses of the heavy indium and cadmium isotopes.

For the even indium isotopes more accurate values of the excitation energies of the isomers were obtained and these showed that the high-spin isomer is the ground state in ^{126}In and ^{128}In. Also the excitation energies of the isomers in ^{127}In and ^{129}In were determined.

Paper II

In this work we look into the details of the beta decays of ^{131}In. The OSIRIS mass separator in Studsvik was again used. The Q_β values of the ground state and the two known isomers in ^{131}In were determined from Fermi-Kurie analysis of gamma- gated beta decay branches. The total Q_β energy of the ground state was determined to be 9184±23 keV. The separation energy of the $g_{9/2}^{-1}$ and $p_{1/2}^{-1}$ proton-hole states was found to be about 360 keV. This is almost the same as in all the heavy odd indium isotopes, showing the purity of the $p_{1/2}^{-1}$ single-particle state.

Paper III

In the study reported in the third paper the mass excess of the very neutron-deficient nucleus ^{150}Dy was determined. The experiment was performed at the post accelerator of the Munich Tandem Laboratory. The intensity of the β^+ branch was determined from the intensity of the 511 keV annihilation radiation in coincidence with the 620 keV line, while the total beta decay intensity was determined from the singles intensity of the 620 keV line. From this, the EC(K)/β^+ intensity ratio was calculated to be 14.7±2.7. By comparing this experimental result with theoretical calculations the positron end-point energy was deduced to be 1.16±0.06 MeV and from this, the mass excess could be obtained. The result is ME(^{150}Dy)=-67.84±0.10 MeV. Also, the mass excesses of the four alpha-decaying nuclei in the alpha decay chain ending with ^{150}Dy could be determined. These masses were compared with theoretical mass predictions.

The two-particle separation energy often shows a considerable gap when a magic number is encountered. This work gave the first point, on the S_{2p} line, for N=82 isotones, beyond the proposed Z=64 magic number. The energy gap was, however, found to be only around 0.5 MeV and this is considerably smaller than in comparable situations.

Paper IV

In this work the beta decay of ^{96}Pd was studied. The experiment was performed at the on-line mass separator at GSI. The β^+ branch was determined from the intensity of the 511 keV annihilation radiation in coincidence with the 500 keV line de-exciting the 1275 keV level. The total beta intensity was determined from the the sum of the intensities of the 599 and 723 keV lines, both in coincidence with the 500 keV line. The $\beta^+/(\beta^+$+EC) intensity ratio was calculated and found to be 0.257±0.028, which corresponds to a Q_{EC} of 3.45±0.15 MeV. Finally, the mass excess of ^{96}Pd was calculated to be -76.23±0.15 MeV.

Paper V

In this work we turn from the experimental determination of atomic masses to the problem of the theoretical prediction of atomic mass excesses. A mass formula with very few free parameters is described and the advantages of a small number of parameters are discussed. The expression for the mass formula consists of three parts:

$$M(Z,N,\beta) = M_w(Z,N) - E_{def}(Z,N,\beta) - E_{shell}(\beta)$$

M_w is a Weizsäcker type liquid drop expression. E_{def} is the deformation energy of the nucleus and it contains contributions from the deformation of both the charged liquid drop and the single-particle potential. E_{shell} is the shell energy which, in our treatment, only depends on the nuclear deformation. Microscopic calculations indicate that liquid drop type mass formulae underestimate the masses of very neutron-rich nuclei. With this in mind we introduced a term proportional to $(N-Z)^4$. This term does not affect the fit to the known masses but makes the very neutron-rich nuclei heavier.

In our mass formula the nuclear deformation is an independent variable. Since not many experimental deformations are known it is necessary to calculate them. For this purpose we developed a simple expression for the nuclear deformation. In the fit to the known atomic masses the mean deviation is 0.45 milli mass units (RMS 0.55 mu) and several of the descrepancies are explained in terms of single-particle level fluctuations.

Paper VI

The aim of this last study was to determine tne position of the first 1^+ state in ^{98}Tc. This has both astrophysical and nuclear structure applications. The work was performed at the Pelletron laboratory in Lund.

The counter ratio method was used to to detect a low-spin state at 88±8 keV exitation energy in ^{98}Tc. The neutrons are detected as a function of the energy of the incomming protons. The number of low-energy neutrons is measured with four BF_3 counters and the number of neutrons with higher energy is measured with a longcounter. When the

proton energy reaches a p,n threshold the number of low energy neutrons increases and the ratio shows a sharp increase.

Our result together with several previous studies allows us to conclude that the low-spin state we detected is a 1^+ state and that it is also the 14.6 μs isomer known to lie somewhere in this excitation energy region. We also put forward a new level scheme for the low-energy part which is slightly different from previous suggestions.

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During the course of this work I have met and collaborated with many people. I would like to take this opportunity to express my gratitude to them.

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